ON AN APPROXIMANT FOR THE SPACING DISTRIBUTION USING NONLINEAR MAPPING

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A nonlinear mapping which maps a variable having range from 0 to ∞ onto another variable having range 0 to unity is used to find an approximant for the distribution of the spacing of levels of compound nucleus. It is shown to provide a very good approximation over the entire range of spacing.

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1. Introduction

In an earlier paper [1] we had shown that an identity which expresses a determinant in terms of trace of log of the matrix can be used in a simple way to study the distribution of the level spacing of compound nucleus. As we emphasized in [1] the level spacing distribution is an important quantity as it is a measure of the fluctuation of the system and is intimately related to the two point correlation function. Since the probability density of level spacing is obtained by taking the second derivative of the probability of finding an interval which is free of any levels, one therefore derives expression for the probability distribution of the empty interval.

The analytic expressions for the probability distribution of the empty interval which have been derived in the past [1, 2, 3] are either for small values of the empty interval or for very large values. At present there is no single expression which is valid over the entire range of the values of empty interval.

Recently Fernández and Ogilvie [4] have used a non linear mapping to obtain approximants which provide a very good approximation to the given function over the entire range of the variable. The purpose of the present work is to use this kind of nonlinear mapping to obtain an approximant for the distribution of the empty interval. It will be shown that this approximant provides a very good approximation over the entire range of the empty interval.

The formulation is presented in Section 2 and concluding remarks are presented in Section 3.

2. Formulation

As shown in [1], the probability distribution $E(t)$ for the empty interval t for small and large values of t is given by

$$E(t) = \exp\left(-t - \tfrac{1}{2}t^2 - \tfrac{1}{3}\left(1 - \tfrac{\pi^2}{12}\right)t^3 + \tfrac{1}{4}\left(\tfrac{\pi^2}{9} - 1\right)t^4 - \left(\tfrac{1}{5} - \tfrac{\pi^2}{36} + \tfrac{\pi^4}{1200}\right)t^5 + \cdots\right), \quad t \to 0, \qquad (1)$$

$$E(t) = \exp\left(-\tfrac{\pi^2}{16}t^2 + \cdots\right), \quad t \to \infty. \qquad (2)$$

Our aim is to find an approximant for $\log E(t)$ which matches (1) and (2). This is achieved by introducing the following nonlinear mapping

$$t = \frac{Ku}{1-u}, \qquad (3)$$

where K is a constant and the new variable u takes on values between 0 and 1 when t goes from 0 to ∞.

The approximant $F(t)$ which approximates the function $\log E(t)$ over the entire range of t can then be written as

$$F = (1-u)^{-2}\sum_{n=1}^{3} g_n u^n + A\ln(1-u)^{-1}, \qquad (4)$$

where g_n $(n = 1, 2, 3)$ and A are constants.

It can be easily shown by expanding (4) using (3) for small and large values of t that it correctly reproduces the exact $\ln E(t)$ given by (1) and (2). We have kept three unknown constants g_n in (4), but one could increase this number if more accurate values of $E(t)$ are needed. The unknown constants g_n $(n = 1, 2, 3)$ and A are expressed in terms of K by comparing $F(t)$ with the exact expression $\log E(t)$ given by (1) while K is obtained by writing the first term in the asymptotic expansion of $F(t)$ and using its exact value of $-\frac{\pi^2}{16}$ as given by (2). Thus we get the following values of the unknown constants.

$$g_1 = -0.5662\,, \quad g_2 = 0.0519\,, \quad g_3 = 0.0600\,,$$
$$A = -0.2920\,, \quad K = 0.8582\,. \tag{5}$$

In Table I we have shown the calculated values of $E(t)$ using the approximant given by (4) and the above values of the unknown constants. In the same Table we have also shown the exact values of $E(t)$ as given in Mehta's book [5]. From this Table we find that the approximant given by (4) has reproduced the values of $E(t)$ correct to the third place of decimal. Thus the approximant provides an extremely good approximation to $E(t)$ over entire range of t.

TABLE I

Calculated values of the probability density function $E_{\rm cal}(t)$ using approximant (4), and the corresponding exact values $E_{\rm ex}(t)$ for various values of the empty interval t.

t	$E_{\rm cal}(t)$	$E_{\rm ex}(t)$
0.064	0.9361	0.9364
0.318	0.6906	0.6903
0.637	0.4267	0.4269
1.273	0.1155	0.1152
1.910	0.0194	0.0192
2.546	0.0020	0.0020
3.183	0.0001	0.0001

3. Concluding Remarks

An approximant with five unknown constants has been shown to provide a very good approximation to the distribution of the empty interval over the entire range of the empty interval. The unknown constants in the approximant are obtained by the known expansion of $\log E(t)$ for small values of t and the first term in the asymptotic expansion of $\log E(t)$.

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