INTERACTION OF NOTIVARG WITH EXTERNAL WEYL CURRENT*

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The canonical analysis of *(i)* the free notivarg theory and *(ii)* the theory of the notivarg interacting with the external Weyl current is performed.

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1. Introduction

In the previous paper [1] we have investigated the conditions providing consistency of the theory of the notivarg interacting with the external current. The conditions, that the external current must obey, have been found. In the present paper we carry out, with details, the canonical analysis of

(i) the free notivarg theory,

(ii) the theory of the notivarg interacting with the Weyl external current.

Let us recall that the canonical analysis performed in Ref. [2] has been aimed at explanation of the physical contents of the theory of the free notivarg. The present analysis can be regarded as arrangements for quantization of the free notivarg theory as well as the theory of the notivarg interacting with the classical current.

2. Free notivarg

2.1. Constraints

We discuss the free notivarg theory described by the action [2,3]

$$I = \int d^4x \left(-\left(\partial_\sigma K^{\sigma\nu\alpha\beta}\right)^2 + \left(\partial_\sigma K^{\sigma\nu\alpha}{}_\nu\right)^2 \right) , \tag{1}$$

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where the potential $K^{\mu\nu\alpha\beta}$ has the symmetry properties of the Riemann tensor: $K^{\mu\nu\alpha\beta} = K^{\alpha\beta\mu\nu} = -K^{\mu\nu\beta\alpha}$, $\varepsilon_{\mu\nu\alpha\beta}K^{\mu\nu\alpha\beta} = 0$. Using the decomposition (see Ref. [1])

$$K^{\mu\nu\alpha\beta} = \left(T^{ij},\ S^{ij},\ K^{ij},\ A^{i}\right)$$

we can rewrite the action (1) in the form

$$I = \int d^4x \mathcal{L}\left(T^{ij},\ S^{ij},\ K^{ij},\ A^{i},\ \partial^0 T^{ij},\ \partial^0 S^{ij}\right), \tag{2}$$

where the Lagrangian density is

$$\begin{aligned} \mathcal{L} = &- 2\left(\partial^0 T^{ki}\right)^2 + \left(\partial^0 T\right)^2 - 4\partial^0 T^{ki}\varepsilon_{jkm}\partial^j S^m_i \\ &- 8\partial^0 T^{ki}\partial_i A_k + 2\left(\partial^0 S^{ki}\right)^2 - \left(\partial_m T^{mi}\right)^2 \\ &+ 4\varepsilon_{mkt}\partial^0 S^k_s \partial^m K^{ts} + 2\left(\partial^i S^{kj}\right)^2 - 4\left(\partial^k A^i\right)^2 \\ &+ 4\left(\partial_i A^i\right)^2 - 8\varepsilon^{ijm}\partial^k S_{km}\partial_i A_j - 2\left(\partial^m K^{ij}\right)^2 \\ &+ 3\left(\partial_m K^{mi}\right)^2 + \left(\partial^i K\right)^2 - 2\partial^m K_{mp}\partial^p K \\ &+ 2\partial_m T^{km}\partial^i K_{ik}, \end{aligned} \tag{3}$$

($T \equiv T^m_m$ and $K \equiv K^m_m$). We note that the variables K^{ij} and A^i occur without their time derivatives.

Let us

(i) define the canonical momenta

$$\begin{aligned} \Pi^{ki} &\equiv \frac{\partial \mathcal{L}}{\partial \partial^0 T_{ki}} = -4\partial^0 T^{ki} + 2g^{ki}\partial^0 T \\ &\qquad +2\left(\varepsilon^{kjm}\partial_j S^i_m + \varepsilon^{ijm}\partial_j S^k_m\right) - 4\left(\partial^k A^i + \partial^i A^k\right), \\ P^{ki} &\equiv \frac{\partial \mathcal{L}}{\partial \partial^0 S_{ki}} = 4\partial^0 S^{ki} - 2\left(\varepsilon^{kmp}\partial_m K^i_p + \varepsilon^{imp}\partial_m K^k_p\right); \end{aligned} \tag{4}$$

(ii) perform the Legendre transformation

$$I = \int d^4x \left(\Pi^{ik}\partial^0 T_{ik} + P^{ik}\partial^0 S_{ik} - \mathcal{H}_c\right)$$

to obtain the canonical Hamiltonian density

$$\begin{aligned} \mathcal{H}_c = &- \tfrac{1}{8}\left(\Pi^{ik}\right)^2 + \tfrac{1}{8}\Pi^2 + \Pi_{ki}\varepsilon^{kpm}\partial_p S^i_m \\ &- 2\Pi^{kl}\partial_k A_l + 2\Pi\partial_m A^m + 8\varepsilon^{ijm}\partial^k S_{km}\partial_i A_j \\ &+ 4\varepsilon^{kpm}\partial_p S^i_m \partial_i A_k - 3\left(\partial_m S^{mi}\right)^2 + \left(\partial_m T^{mi}\right)^2 \\ &- 2\partial_m T^{mk}\partial^p K_{pk} + \tfrac{1}{8}\left(P^{ij}\right)^2 + P^k_i\varepsilon^{imp}\partial_m K_{kp}, \end{aligned} \tag{5}$$

where $\Pi \equiv \Pi^m_m$.

The momenta conjugated to K^{ij} and A^i vanish because the velocities $\partial^0 K^{ij}$ and $\partial^0 A^i$ do not occur in the Lagrangian density (3)

$$p^i_A \equiv \frac{\partial \mathcal{L}}{\partial \partial^0 A_i} = 0\,, \qquad p^{ij}_K \equiv \frac{\partial \mathcal{L}}{\partial \partial^0 K_{ij}} = 0\,. \tag{6}$$

So, there are the following primary constraints

$$\Phi^i_{(1)} = p^i_A\,, \tag{7}$$

$$\Phi^{ij}_{(2)} = p^{ij}_K\,. \tag{8}$$

We introduce the total Hamiltonian [4]

$$H_{\text{tot}} = \int d^3x \Big(\mathcal{H}_c + \lambda_i \Phi^i_{(1)} + \lambda_{ij}\Phi^{ij}_{(2)}\Big)\,, \tag{9}$$

where λ_i and λ_{ij} are Lagrange multipliers. The dynamics is expressed by

$$\partial^0 a = \{a,\ H_{\text{tot}}\}_{\Phi_{(1)}=\Phi_{(2)}=0}\,, \tag{10}$$

where $\{\ldots,\ldots\}$ is the Poisson bracket and a is a function of dynamical variables.

The theory is consistent if constraints hold for all times. This leads to the secondary constraints

$$\begin{aligned}
\Phi^i_{(3)} &= \partial_m \Pi^{mi} - \partial^i \Pi + 2\varepsilon^{ims}\partial_m \partial^k S_{ks}\,,\\
\Phi^{ij}_{(4)} &= \partial^i \partial_m T^{mj} + \partial^j \partial_m T^{mi} + \tfrac{1}{2}\big(\varepsilon^{ims}\partial_m P^j_s + \varepsilon^{jms}\partial_m P^i_s\big)\,,\\
\Phi^{ij}_{(5)} &= \Delta \widetilde{\Pi}^{ij} + \partial^i \partial_m \widetilde{\Pi}^{mj} + \partial^j \partial_m \widetilde{\Pi}^{mi} + \tfrac{1}{2}\Big(\frac{\partial^i \partial^j}{\Delta} - g^{ij}\Big)\partial_m \partial_n \widetilde{\Pi}^{mn}\,,
\end{aligned} \tag{11}$$

where $\widetilde{\Pi}^{ij} \equiv \Pi^{ij} - \frac{1}{3} g^{ij}\Pi$. The dynamics of the constraints is

$$\begin{aligned}
\partial^0 \Phi^i_{(1)} &= \big\{\Phi^i_{(1)},\ H_{\text{tot}}\big\}_{\Phi_{(1)}=\Phi_{(2)}=0} = -2\Phi^i_{(3)}\,,\\
\partial^0 \Phi^{ij}_{(2)} &= -\Phi^{ij}_{(4)}\,,\\
\partial^0 \Phi^i_{(3)} &= \partial_m \Phi^{mi}_{(4)} - \partial^i \Phi^m_{(4)m}\,,\\
\partial^0 \Phi^{ij}_{(4)} &= \Phi^{ij}_{(5)} - \tfrac{1}{2}\big(\partial^i \Phi^j_{(3)} + \partial^j \Phi^i_{(3)}\big) - \tfrac{1}{2}\frac{\partial^i \partial^j}{\Delta}\partial_m \Phi^m_{(3)}\,,\\
\partial^0 \Phi^{ij}_{(5)} &= 0\,.
\end{aligned} \tag{12}$$

All the constraints obey the relations

$$\{\Phi_{(i)}, \Phi_{(j)}\} = 0. \tag{13}$$

So, we have the theory with the first class constraints.

We point out that

(i) only five constraints $\Phi_{(4)}$ are independent because there exists the identity

$$\partial_i \partial_j \Phi^{ij}_{(4)} + \Delta \Phi_{(4)m}^{\ \ \ \ m} \equiv 0.$$

Using the formulae of Appendix we obtain the following decomposition of $\Phi_{(4)}$ into independent parts

$$\Phi^{ij}_{(4)} = \tilde{\bar{\Phi}}^{ij}_{(4)}(\pm 2) + \tilde{\bar{\Phi}}^{ij}_{(4)}(\pm 1) - \frac{\partial^i \partial^j}{\Delta} \Phi_{(4)m}^{\ \ \ \ m},$$

where

$$\tilde{\Phi}^{ij}_{(4)} = \Phi^{ij}_{(4)} - \tfrac{1}{3} g^{ij} \Phi_{(4)m}^{\ \ \ \ m};$$

(ii) only two constraints $\Phi_{(5)}$ are independent because (according to Appendix)

$$\Phi^{ij}_{(5)} = \Delta \widetilde{\Pi}^{ij}(\pm 2).$$

2.2 New Hamiltonian

Because the first class constraints can be added to Hamiltonian [5], we construct the new Hamiltonian density

$$\mathcal{H}^{\text{new}} = \mathcal{H}_0 + V_i \Phi^i_{(3)} + V^{(4)}_{ij} \Phi^{ij}_{(4)} + V^{(5)}_{ij} \Phi^{ij}_{(5)}, \tag{14}$$

where V_i, $V^{(4)}_{ij}$ and $V^{(5)}_{ij}$ are the Lagrange multipliers, and $\mathcal{H}_0$ has the following form

$$\mathcal{H}_0 = -\tfrac{1}{8}\left(\Pi^{ik}\right)^2 + \tfrac{1}{8}\Pi^2 + \Pi_{ki}\varepsilon^{kpm}\partial_p S^i_m - 3\left(\partial_m S^{mi}\right)^2 + \left(\partial_m T^{mi}\right)^2. \tag{15}$$

We observe that the variables K^{ij} and A^i disappear in the new description.

Let us note that the field $V^{(4)}_{ij}$ has only five independent components

$$V^{(4)}_{ij} = \widetilde{V}^{(4)}_{ij}(\pm 2) + \widetilde{V}^{(4)}_{ij}(\pm 1) - \frac{\partial_i \partial_j}{\Delta} V^{(4)m}_{\ \ \ \ \ m};$$

where

$$\widetilde{V}^{(4)}_{ij} = V^{(4)}_{ij} - \tfrac{1}{3} g_{ij} V^{(4)m}_{\ \ \ \ \ m},$$

and the field $V_{ij}^{(5)}$ has only two independent components

$$V_{ij}^{(5)} = V_{ij}^{(5)}(\pm 2) .$$

The Hamiltonian density $\mathcal{H}^{\rm new}$ is derivable from the phase-space Lagrangian density [6]

$$\mathcal{L}^{\rm new} = P_{ij}\partial^0 S^{ij} + \Pi_{ij}\partial^0 T^{ij} - \mathcal{H}_0 - V_i \Phi^i_{(3)} - V_{ij}^{(4)}\Phi^{ij}_{(4)} - V_{ij}^{(5)}\Phi^{ij}_{(5)} , \qquad (16)$$

where the Lagrange multipliers V_i, $V_{ij}^{(4)}$ and $V_{ij}^{(5)}$ are treated as dynamical variables. So, passing to the canonical formalism, we find the primary constraints

$$\pi^i = 0 \quad \pi^{ij}_{(4)} = 0 , \quad \pi^{ij}_{(5)} = 0 ,$$

where $\pi^i, \pi^{ij}_{(4)}$ and $\pi^{ij}_{(5)}$ are the canonical momenta conjugated to V_i, $V_{ij}^{(4)}$ and $V_{ij}^{(5)}$ respectively. Thus the new total Hamiltonian is

$$H^{\rm new}_{\rm tot} = \int d^3x \left(\mathcal{H}^{\rm new} + \lambda_i \pi^i + \lambda_{ij}^{(4)}\pi^{ij}_{(4)} + \lambda_{ij}^{(5)}\pi^{ij}_{(5)} \right) , \qquad (17)$$

where λ's are the Lagrange multipliers.

The dynamics is expressed by

$$\partial^0 a = \{a , \ H^{\rm new}_{\rm tot}\}_{\pi=0} . \qquad (18)$$

In particular we have

$$\partial^0 \pi^i = -\Phi^i_{(3)} \quad \partial^0 \pi^{ij}_{(4)} = -\Phi^{ij}_{(4)} , \quad \partial^0 \pi^{ij}_{(5)} = -\Phi^{ij}_{(5)} .$$

The time derivative of $\Phi_{(3)}$, $\Phi_{(4)}$ and $\Phi_{(5)}$ are given by Eqs (12).

2.3. Gauge transformations and noncovariant gauge conditions

We are now ready to discuss the gauge invariance of the Lagrangian

$$L = \int d^3x \mathcal{L}^{\rm new} . \qquad (19)$$

The generator of the gauge transformation is

$$G = \int d^3x \left(\alpha_i \pi^i + \alpha_{ij}^{(4)}\pi^{ij}_{(4)} + \alpha_{ij}^{(5)}\pi^{ij}_{(5)} + \eta_i \Phi^i_{(3)} + \eta_{ij}^{(4)}\Phi^{ij}_{(4)} + \eta_{ij}^{(5)}\Phi^{ij}_{(5)} \right) ,$$

where α's and η's are gauge functions ($\alpha^{(4)}$ and $\eta^{(4)}$ have such a structure as $\Phi_{(4)}$ has, and $\alpha_{ij}^{(5)} = \alpha_{ij}^{(5)}(\pm 2)$, $\eta_{ij}^{(5)} = \eta_{ij}^{(5)}(\pm 2)$).

The generator obeys the consistency condition [6]

$$\frac{d}{dt}G = 0\,.$$

Using Eq. (18) we obtain

$$\begin{aligned}
\alpha_i &= \partial^0\eta_i + \partial^j\eta^{(4)}_{ij} - \partial^i\eta^{(4)m}{}_m\,,\\
\alpha^{(4)}_{ij} &= \partial^0\eta^{(4)}_{ij} - \tfrac{1}{2}\left(\partial_i\eta_j + \partial_j\eta_i\right) - \frac{\partial_i\partial_j}{\Delta}\partial_m\eta^m\,,\\
\alpha^{(5)}_{ij} &= \partial^0\eta^{(5)}_{ij} + \eta^{(4)}_{ij}(\pm 2).
\end{aligned}$$

The infinitesimal gauge transformations are

$$\begin{aligned}
&\delta V_i \equiv \{V_i\,,\, G\} = \alpha_i\,, \quad \delta V^{(4)}_{ij} = \alpha^{(4)}_{ij}\,, \quad \delta V^{(5)}_{ij} = \alpha^{(5)}_{ij}\,,\\
&\delta T^{ij} = -\tfrac{1}{2}\left(\partial^i\eta^j + \partial^j\eta^i\right) + g^{ij}\partial_m\eta^m + \Delta\eta^{(5)ij}\,,\\
&\delta \Pi^{ij} = -\left(\partial^i\partial_m\eta^{(4)mj} + \partial^j\partial_m\eta^{(4)mi}\right),\\
&\delta S^{ij} = \tfrac{1}{2}\left(\varepsilon^{imp}\partial_m\eta^{(4)j}{}_p + \varepsilon^{jmp}\partial_m\eta^{(4)i}{}_p\right),\\
&\delta P^{ij} = \varepsilon^{imk}\partial_m\partial^j\eta_k + \varepsilon^{jmk}\partial_m\partial^i\eta_k\,.
\end{aligned} \tag{20}$$

We point out that there are ten independent gauge functions: three η_i, five $\eta^{(4)}_{ij}$ and two $\eta^{(5)}_{ij}$. To fix completely the gauge freedom we can impose the following noncovariant conditions $\chi_{(i)}$ $(i = 1, \ldots, 10)$

$$\begin{aligned}
T^{ij} - \tfrac{1}{3}g^{ij}T &= 0 \qquad &&(5 \text{ conditions}),\\
\varepsilon^{ipm}\partial_p S^j_m + \varepsilon^{jpm}\partial_p S^i_p &= 0 \qquad &&(4 \text{ conditions}),\\
\Pi &= 0 \qquad &&(1 \text{ condition}).
\end{aligned}$$

We note that

$$\left\{\chi_{(i)}\,,\, \chi_{(j)}\right\} = 0\,. \tag{21}$$

2.4. Physical Lagrangian

Solving the constraints $\Phi_{(i)}$ $(i = 3, 4, 5)$ we get [2] (see Appendix too)

$$\begin{aligned}
&\widetilde{\Pi}^i_{\mathrm{T}} = -2\varepsilon^{ijk}\partial_j S_{\mathrm{T}k}\,, \quad \widetilde{\Pi}_{\mathrm{L}} = -\tfrac{2}{3}\Pi\,, \quad P^{ij}(\pm 2) = 0\,,\\
&\widetilde{T}_{\mathrm{L}} = \tfrac{1}{3}T\,, \quad P^i_{\mathrm{T}} = -2\varepsilon^{ijk}\partial_j\widetilde{T}_{\mathrm{T}k}\,, \quad \widetilde{\Pi}^{ij}(\pm 2) = 0\,.
\end{aligned}$$

Inserting these solutions to the action (19) we obtain the action describing only one physical degree of freedom [2]

$$I = \int d^4x \mathcal{L}_{\rm phys} , \tag{22}$$

where

$$\mathcal{L}_{\rm phys} = \tfrac{3}{2} P_{\rm L} \partial^0 S_{\rm L} - \tfrac{3}{16} P_{\rm L}^2 + 3\left(\partial^i S_{\rm L}\right)^2 , \tag{23}$$

or, in the configuration space,

$$\mathcal{L}_{\rm phys} = \tfrac{1}{2} \partial_\mu \varphi \partial^\mu \varphi ,$$

where

$$\varphi = \sqrt{6} S_{\rm L} .$$

3. Interaction with the Weyl external current

3.1. Weyl external current [1]

We assume that the interaction is described by

$$\mathcal{L}_{\rm int} = \tfrac{1}{4} K^{\mu\nu\alpha\beta} j_{\mu\nu\alpha\beta} , \tag{24}$$

where the external source $j_{\mu\nu\alpha\beta}$ has symmetry of the Weyl tensor. Using the decomposition of the Weyl tensor

$$j^{\mu\nu\alpha\beta} = \left(\tau^{ij} , \sigma^{ij}\right) ,$$

where τ^{ij} and σ^{ij} are symmetric and traceless, we rewrite the interaction Lagrangian density in the following form

$$\mathcal{L}_{\rm int} = \tau_{ij}\left(T^{ij} - K^{ij}\right) - 2\sigma_{ij} S^{ij} . \tag{25}$$

The Weyl external current obeys the condition

$$\partial_\mu \partial_\alpha j^{\mu\nu\alpha\beta} = 0 ,$$

or, in components,

$$\begin{aligned} &\tau_{\rm L} = 0 , \\ &\partial^0 \tau^i_{\rm T} = \varepsilon^{imn} \partial_m \sigma_{{\rm T}n} , \\ &\left[(\partial^0)^2 + \Delta\right] \tau^{ij}(\pm 2) = \partial^0 \left(\varepsilon^{ipk} \partial_p \sigma^j_k(\pm 2) + \varepsilon^{jpk} \partial_p \sigma^i_k(\pm 2)\right) . \end{aligned} \tag{26}$$

3.2. Constraints

The Lagrangian density is now

$$\mathcal{L}' = \mathcal{L} + \mathcal{L}_{\rm int}\,,$$

where $\mathcal{L}$ and $\mathcal{L}_{\rm int}$ are given by Eq. (3) and Eq. (25) respectively. Because the interaction term does not contain any derivative, the canonical momenta do not change and the canonical Hamiltonian density is

$$\mathcal{H}'_{\rm c} = \mathcal{H}_{\rm c} - \tau_{ij}\big(T^{ij} - K^{ij}\big) + 2\sigma_{ij}S^{ij}\,, \tag{27}$$

where $\mathcal{H}_{\rm c}$ is given by Eq. (5).

The primary constraints are the same as in the free case (Eqs (7) and (8)) and they lead to the secondary constraints $\Phi'_{(i)}$ $(i = 3,\, 4,\, 5)$

$$\begin{aligned}
\Phi'^{i}_{(3)} &= \Phi^{i}_{(3)}\,,\\
\Phi'^{ij}_{(4)} &= \Phi^{ij}_{(4)} + \tau^{ij}\,,\\
\Phi'^{ij}_{(5)} &= \Phi^{ij}_{(5)} + \partial^0\tau^{ij}(\pm 2) - \big(\varepsilon^{ims}\partial_m\sigma^{j}_{s}(\pm 2) + \varepsilon^{jms}\partial_m\sigma^{i}_{s}(\pm 2)\big)\,.
\end{aligned} \tag{28}$$

We note that $\tilde{\Phi}'^{ij}_{(4)}(0) \equiv 0$ for $\tau_{\rm L} = 0$ (see Eq. (26)). The system of the constraints is consistent what can be seen from the following relations

$$\begin{aligned}
\partial^0\Phi^{i}_{(1)} &= \big\{\Phi^{i}_{(1)}\,, H'_{\rm tot}\big\}_{\Phi_{(1)}=\Phi_{(2)}=0} = -2\Phi'^{i}_{(3)}\,,\\
\partial^0\Phi^{i}_{(2)} &= -\Phi'^{ij}_{(4)}\,, \qquad \partial^0\Phi'^{i}_{(3)} = \partial_m\Phi'^{mi}_{(4)}(\pm 1)\,,\\
\partial^0\Phi'^{ij}_{(4)} &= \Phi'^{ij}_{(5)} - \tfrac{1}{2}\big(\partial^i\Phi'^{j}_{(3)} + \partial^j\Phi'^{i}_{(3)}\big) - \tfrac{1}{2}\frac{\partial^i\partial^j}{\Delta}\partial_m\Phi'^{m}_{(3)}\,,\\
\partial^0\Phi'^{ij}_{(5)} &= 0\,,
\end{aligned} \tag{29}$$

where

$$H'_{\rm tot} = H_{\rm tot} + \int d^3x\,\big[-\tau_{ij}\big(T^{ij} - K^{ij}\big) + 2\sigma_{ij}S^{ij}\big] \tag{30}$$

with $H_{\rm tot}$ given by Eq. (9). The current conditions (26) must be taken into account to obtain Eqs (29).

3.3. New Hamiltonian

We construct the new Hamiltonian density

$$\begin{aligned}
\mathcal{H}'^{\rm new} &= \mathcal{H}^{\rm new} - \tau_{ij}\big(T^{ij} - V^{(4)ij}\big) + 2\sigma_{ij}S^{ij}\\
&\quad + V^{(5)}_{ij}\big[\partial^0\tau^{ij}(\pm 2) - \big(\varepsilon^{ims}\partial_m\sigma^{j}_{s}(\pm 2) + \varepsilon^{jms}\partial_m\sigma^{i}_{s}(\pm 2)\big)\big]\,,
\end{aligned} \tag{31}$$

where $\mathcal{H}^{\text{new}}$ is given by Eq. (14), and $V^{(4)}_{ij}$ and $V^{(5)}_{ij}$ have the same structure as in the free case. The Hamiltonian (31) and all the constraints are derivable from the phase – space Lagrangian density

$$\mathcal{L}'^{\text{new}} = P_{ij}\partial^0 S^{ij} + \Pi_{ij}\partial^0 T^{ij} - \mathcal{H}'^{\text{new}} . \tag{32}$$

It is invariant under the gauge transformation (20).

3.4. Physical Lagrangian

To solve the constraints $\Phi'_{(i)}$ $(i = 3, 4, 5)$ we must use the current conditions (26). We obtain:

$$\begin{aligned}
&\text{from} \quad \Phi'_{(3)} = 0 : \quad \widetilde{\Pi}^i_{\mathrm{T}} = -2\varepsilon^{ijk}\partial_j S_{\mathrm{T}k} , \quad \widetilde{\Pi}_{\mathrm{L}} = -\tfrac{2}{3}\Pi ; \\
&\text{from} \quad \Phi'_{(4)} = 0 : \quad \widetilde{T}_{\mathrm{L}} = \tfrac{1}{3}T , \quad P_{ij}(\pm 2) = \tfrac{1}{2\Delta}\big(\varepsilon_{ipm}\partial^p \tau^m_j(\pm 2) \\
&\qquad\qquad\qquad\quad + \varepsilon_{jpm}\partial^p \tau^m_i(\pm 2)\big) , \\
&\qquad\qquad\qquad\quad \widetilde{T}^i_{\mathrm{T}} = \tfrac{1}{2\Delta}\varepsilon^{ims}\partial_m P_{\mathrm{T}s} + \tfrac{1}{\Delta}\tau^i_{\mathrm{T}} ; \\
&\text{from} \quad \Phi'_{(5)} = 0 : \quad \widetilde{\Pi}^{ij}(\pm 2) = \tfrac{1}{\Delta}\big(\varepsilon^{ims}\partial_m \sigma^j_s(\pm 2) + \varepsilon^{jms}\partial_m \sigma^i_s(\pm 2)\big) \\
&\qquad\qquad\qquad\quad - \tfrac{1}{\Delta}\partial^0 \tau^{ij}(\pm 2) .
\end{aligned}$$

Inserting these solutions to the action

$$I = \int d^4x \mathcal{L}'^{\text{new}}$$

we obtain

$$I = \int d^4x \mathcal{L}_{\text{phys}} , \tag{33}$$

where

$$\begin{aligned}
\mathcal{L}_{\text{phys}} = &\tfrac{3}{2}P_{\mathrm{L}}\partial^0 S_{\mathrm{L}} - \tfrac{3}{16}\big(P_{\mathrm{L}}\big)^2 + 3\big(\partial^i S_{\mathrm{L}}\big)^2 - 3\sigma_{\mathrm{L}} S_{\mathrm{L}} \\
&- \tfrac{1}{4}\sigma^n_m(\pm 2)\tfrac{1}{\Delta}\sigma^m_n(\pm 2) - \tfrac{1}{8}\big(\tfrac{1}{\Delta}\partial^0\tau^{ij}(\pm 2)\big)\big(\tfrac{1}{\Delta}\partial^0\tau_{ij}(\pm 2)\big) \\
&+ \tfrac{3}{8}\tau^{ij}(\pm 2)\tfrac{1}{\Delta}\tau_{ij}(\pm 2) + \big(\tfrac{1}{\Delta}\tau^i_{\mathrm{T}}\big)\big(\tfrac{1}{\Delta}\tau_{\mathrm{T}i}\big) .
\end{aligned} \tag{34}$$

The last four terms are similar to the Coulomb term $\rho\frac{1}{\Delta}\rho$ in the electrodynamics. We see that only one degree of freedom (described by S_{L}) is propagated, but to describe the interaction in the consistent way the further degrees of freedom must be taken into account.

4. Final remarks

We have obtained the canonical description of the free notivarg as well as the notivarg interacting with the external Weyl current. The quantum theory of the notivarg will be given in next papers.

We thank Drs M. Majewski and L.C. Papaloucas for their interest in this work.

Appendix

The well known decomposition of a vector a^i into transversal and longitudinal parts is

$$a^i = a^i_{\mathrm{T}} + a^i_{\mathrm{L}}\,,$$

where

$$a^i_{\mathrm{T}} = a^i + \partial^i a_{\mathrm{L}}\,, \quad a^i_{\mathrm{L}} = -\partial^i a_{\mathrm{L}}\,, \quad a_{\mathrm{L}} = \tfrac{1}{\Delta}\partial_i a^i\,, \quad \Delta = -\partial_i\partial^i\,.$$

The analogous decomposition of a symmetric traceless tensor a^{ij} is

$$a^{ij} = a^{ij}(\pm 2) + a^{ij}(\pm 1) + a^{ij}(0)\,,$$

where

$$\begin{aligned} a^{ij}(\pm 2) &= a^{ij} + \tfrac{1}{\Delta}\left(\partial^i\partial_m a^{mj} + \partial^j\partial_m a^{mi}\right) + \tfrac{1}{2\Delta}\left(\tfrac{\partial^i\partial^j}{\Delta} - g^{ij}\right)\partial_m\partial_n a^{mn}\,,\\ a^{ij}(\pm 1) &= -\tfrac{1}{\Delta}\left(\partial^i\partial_m a^{mj} + \partial^j\partial_m a^{mi}\right) - \tfrac{2}{\Delta}\tfrac{\partial^i\partial^j}{\Delta}\partial_m\partial_n a^{mn}\,,\\ a^{ij}(0) &= \tfrac{1}{2\Delta}\left(3\tfrac{\partial^i\partial^j}{\Delta} + g^{ij}\right)\partial_m\partial_n a^{mn}\,. \end{aligned}$$

The parts $a^{ij}(\pm 1)$ and $a^{ij}(0)$ can be expressed by the transversal and longitudinal parts of the vector $a^i = \partial_j a^{ij}$

$$\begin{aligned} a^{ij}(\pm 1) &= -\tfrac{1}{\Delta}\left(\partial^i a^j_{\mathrm{T}} + \partial^j a^i_{\mathrm{T}}\right)\,,\\ a^{ij}(0) &= \tfrac{3}{2}\left(\tfrac{1}{\Delta}\partial^i\partial^j + \tfrac{1}{3}g^{ij}\right)a_{\mathrm{L}}\,. \end{aligned}$$

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