RUNNING COUPLINGS AND THE HIGGS BOSON MASS

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We consider upper bounds on the Higgs boson mass based on the perturbative unitarity and an Argand diagram analysis. The bounds show a maximum sensitivity of 10–20 GeV when different running Higgs boson couplings are used.

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In a recent paper [1] one-loop corrections were included in the calculation of the $j = 0$ partial–wave amplitudes for the high-energy scattering of longitudinal W's, Z's and Higgs bosons in the standard model, and it has been shown that an Argand diagram analysis leads to an upper bound of 350–400 GeV on the Higgs mass, $m_{\rm H} \lesssim$ 350–400 GeV. This bound is much smaller than the ones obtained in tree-level calculations based on perturbative unitarity [2,3].

In what follows we would like to make two remarks. Namely, the analysis in Ref. [1] leads to an upper bound of $m_{\rm H} \leq$ 300–310 GeV if one disregards of the scatterings of W's. Furthermore, all these bounds are stable within 10–20 GeV if one changes the form of the running scalar self coupling.

We confine ourselves to applying the partial-wave unitarity to the channels $\frac{1}{\sqrt{2}}{\rm Z_L Z_L}$, $\frac{1}{\sqrt{2}}{\rm HH}$, ${\rm HZ_L}$, at $s \gg m_{\rm H}^2 \gg m_{\rm Z}^2$ taking into account one-loop corrections [1]. Diagonalizing the $j = 0$ partial-wave matrix of the $2 \to 2$ processes we get three eigenamplitudes, the largest one giving the strongest restriction on the Higgs mass is

$$a = -\frac{1}{4\pi}\lambda(s, m_{\rm H}^2) + \left(\frac{\lambda(s, m_{\rm H}^2)}{16\pi}\right)^2 \left(\frac{80}{\pi} + 8\sqrt{3} + 20i\right) \tag{1}$$

to $O(\lambda^2)$ where $\lambda(s, m_{\rm H}^2)$ means the running Higgs self coupling. The amplitude a is drawn in Fig. 1 as a function of the running coupling $\lambda(s, m_{\rm H}^2)$.

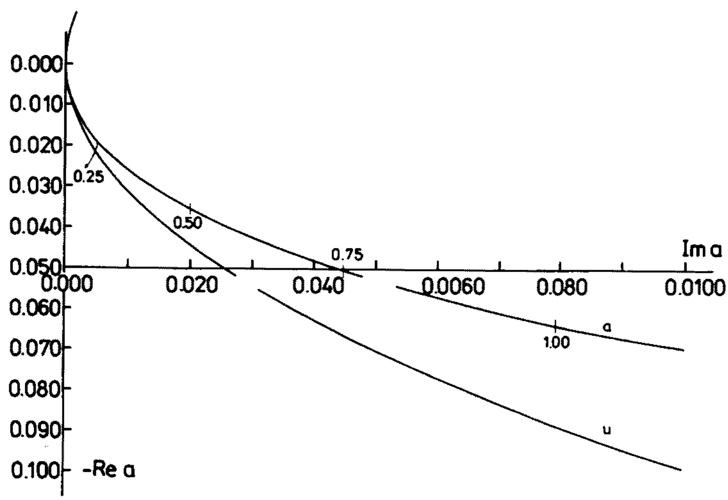

Fig. 1. The Argand diagram of the $j = 0$ eigenamplitude a and a segment u of the unitarity circle. Change of $\lambda(s, m_{\rm H}^2)$ is illustrated on the curve a

The curve u is a segment of the unitarity circle. The curve a bends considerably from the unitarity circle already at low values of $\lambda(s, m_{\rm H}^2)$. A deviation of 100% in the imaginary part of the amplitude a may be considered as a sign of the violation of elastic unitarity. In this way we get the bound $\lambda(s, m_{\rm H}^2) \lesssim 1$ to $O(\lambda^2)$. The critical value of λ obtained in Ref. [1] is 1.5-2. Assuming the standard model is valid up to $\sqrt{s} \approx 2$ TeV, and for the running of $\lambda(s, m_{\rm H}^2)$ making use of the summed up version of the Sirlin–Zucchini coupling [4],

$$\lambda(s, m_{\rm H}^2) = \lambda\left(1 - \frac{\lambda}{16\pi^2}\left(24\ln\frac{\sqrt{s}}{m_{\rm H}} + 25 - \frac{9\pi}{\sqrt{3}}\right)\right)^{-1}, \tag{2}$$

where $\lambda = G_F m_H^2/\sqrt{2}$ and $s \gg m_H^2 \gg m_W^2$, we are led to the bound $m_H \leq$ 300–310 GeV. The function (2) is drawn in Fig. 2 at $\sqrt{s} =$ 2, 3, 5, 10 TeV. A relative deviation of 60% of Ima from the unitarity circle gives $m_H \leq 240$ GeV for $\sqrt{s} \leq 2$ TeV. Given a critical $\lambda(s, m_H^2)$, Fig. 2 shows that the upper bound on m_H decreases with increasing cut off energy defining the region of validity of the standard model. In our case, however, this change is modest being $\lambda(s, m_H^2) \lesssim 1$. For instance, at the critical energy 5 TeV, $m_H \lesssim 290$ GeV (while for $\lambda = 2$ [1], $m_H \lesssim 360$ GeV).

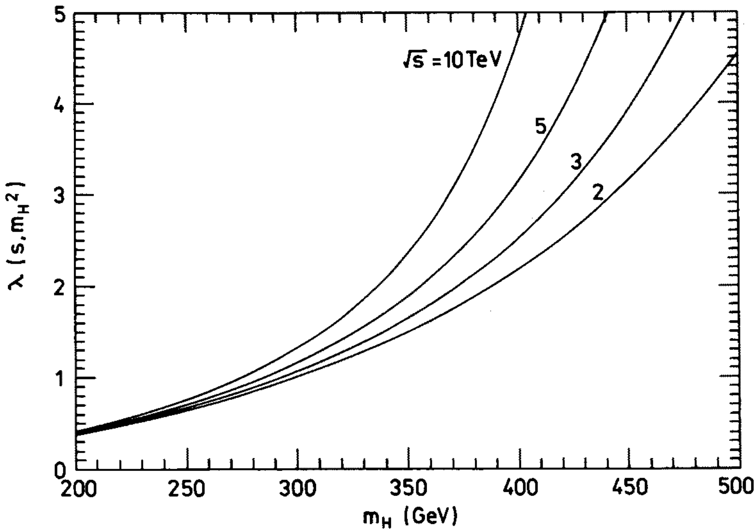

Fig. 2. Summed up version of the Sirlin–Zucchini running coupling *vs* m_H at $\sqrt{s} = 2, 3, 5, 10$ TeV

In general, the summation contained in (2) increases $\lambda(s, m_H^2)$ compared to its $O(\lambda^2)$ approximation shown in Fig. 3. The change is large at large s and m_H^2, and at a fixed s the bound on m_H increases going from (2) to the $O(\lambda^2)$ approximation. However, at $\sqrt{s} = 2$ TeV the change is only 10 GeV for $\lambda(s, m_H^2) = 1$ and the bound of 350 GeV in Ref. [1] becomes slightly smaller than 370 GeV.

Next, let us look at the role of the constant term $25 - 9\pi/\sqrt{3}$ of Eq. (2). Clearly, neglecting the constant term increases the bound on m_H. This is seen from Fig. 4 where the new running coupling is shown. For $\lambda = 1$ (1.5) and $\sqrt{s} = 2$ TeV we get, however, a small change, $m_H \lesssim 310$ (360) GeV.

In our case the two-loop corrections to the β function have a moderate influence on the running of $\lambda(s, m_H^2)$ defined by

$$\frac{d\,\lambda(s,m_H^2)}{d\ln\sqrt{s}} = \beta\big(\lambda(s,m_H^2)\big)\,,$$
$$\lambda\big(m_H^2, m_H^2\big) = \lambda = \frac{G_F m_H^2}{\sqrt{2}}\,, \tag{3}$$

where $\sqrt{s} \geq m_H$. The two-loop β function is [5]

$$\beta(\lambda) = \frac{3\lambda^2}{2\pi^2}\left(1 - \frac{13\lambda}{16\pi^2}\right) \tag{4}$$

(gauge and Yukawa interactions are neglected).

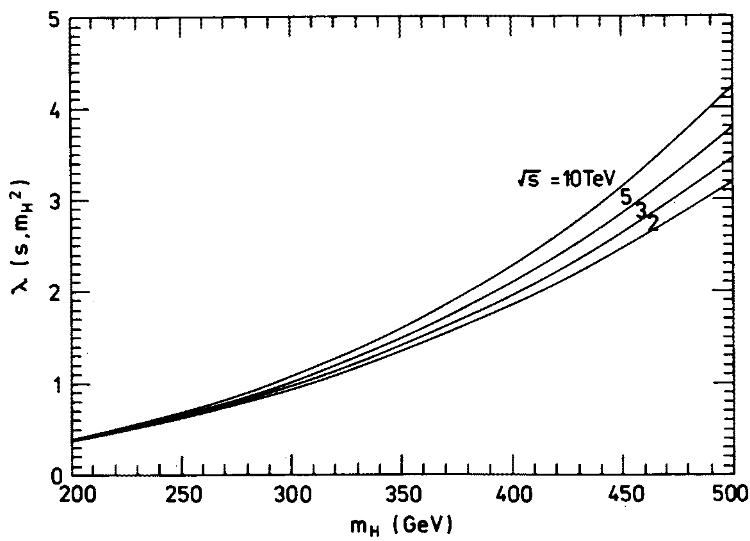

Fig. 3. $O(\lambda^2)$ approximation of Eq. (2) *vs* m_H at $\sqrt{s} = 2, 3, 5, 10$ TeV

Writing

$$\lambda(s, m_H^2) = \frac{\lambda}{g(\ln\sqrt{s})}\,, \quad g(\ln m_H) = 1\,, \tag{5}$$

(3) leads to the differential equation

$$-\frac{d\,g(\ln\sqrt{s})}{d\ln\sqrt{s}} = \frac{3\lambda}{2\pi^2}\left(1 - \frac{13\lambda}{16\pi^2 g(\ln\sqrt{s})}\right)\,. \tag{6}$$

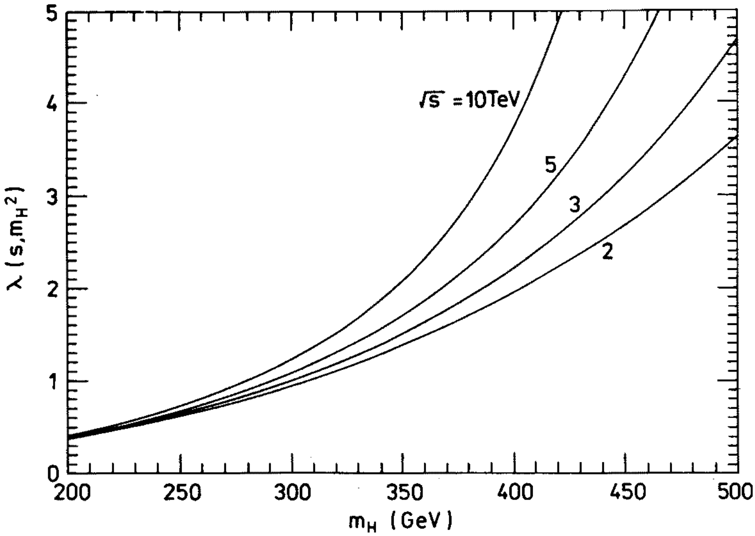

Fig. 4. Eq. (2) without the term $25 - 9\pi/\sqrt{3}$ *vs* $m_{\rm H}$ at $\sqrt{s} = 2, 3, 5, 10$ TeV

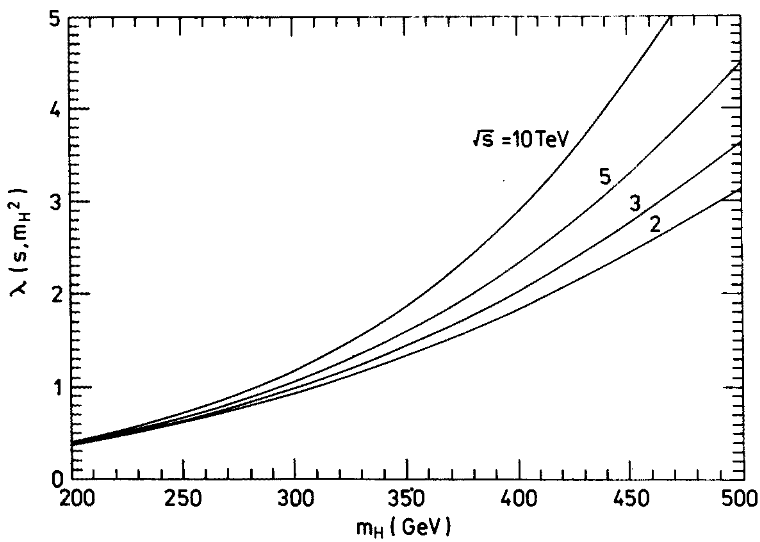

Fig. 5. Running coupling $\lambda(s, m_{\rm H}^2)$ including effects from the two-loop β function (4) *vs* $m_{\rm H}$ at $\sqrt{s} = 2, 3, 5, 10$ TeV

Integrating this equation yields

$$g(\ln\sqrt{s}) = 1 - \frac{3\lambda}{2\pi^2}\ln\frac{\sqrt{s}}{m_{\rm H}} + \frac{13\lambda}{16\pi^2}\ln\frac{1-13\lambda/16\pi^2}{g(\ln\sqrt{s})-13\lambda/16\pi^2}. \qquad (7)$$

Here $g(\ln\sqrt{s}) > 0$, since the perturbative form of β, (4), is positive and $d\lambda/d\ln\sqrt{s} > 0$. Furthermore, as $\sqrt{s}$ increases from $\sqrt{s} = m_{\rm H}$, $g(\ln\sqrt{s})$ decreases from 1. Positivity of β results in $g(\ln\sqrt{s}) > 13\lambda/16\pi^2$. (At $\sqrt{s} = m_{\rm H}$ this gives an upper bound on $m_{\rm H}$ which is a factor of $6\pi/13$ times weaker than that of Ref. [3].) The third term is positive on the right hand side of (7), therefore, at a fixed s and $m_{\rm H}^2$ $\lambda(s, m_{\rm H}^2)$ becomes smaller (see Fig. 5) compared to the case of Fig. 4. However, the change is modest for the bounds treated here. Indeed, the bound of 300 GeV will be 310 GeV and that in Ref. [1], 350 GeV, goes over into 370 GeV.

In conclusion, based on $O(\lambda^2)$ perturbation theory and an Argand diagram analysis, we have found the bound $m_{\rm H} \lesssim 300$ GeV which is not very sensitive to the form of the running of the Higgs self coupling because λ is rather small.

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