ESTIMATES FOR A HYPOTHETICAL MAGNETIC-TYPE INTERACTION OF NUCLEONS

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The new magnetic-type interaction for nucleons, following from a hypothetical Abelian composite structure of quarks is further discussed. A particular model is explored, where the quark is composed of a spin-1/2 preon (existing in two flavors) and a spin-0 preon (existing in three colors) bound together by a new Abelian gauge field producing a Coulombic attraction $V = -e^{(u)\,2}/r$ with $O(e^{(u)\,2}) = 1$. Denoting the resulting new magnetic-type moment of the nucleon by $\mu_N^{(u)} = 3\,e_{\rm eff}^{(u)}/2m_N$, we are able to show that $9\,e_{\rm eff}^{(u)\,2} \ll e^{(u)\,2}$. Hfs experiments for H_2 molecules set the upper limit $9\,e_{\rm eff}^{(u)\,2} < 2 \times 10^{-7}$. The ordinary magnetic moment of the nucleon comes out $\mu_N = (3 \text{ or } -2)e/2m_N$ for N = p or n, in good agreement with the experiment.

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In a recent paper [1] we considered consequences of the hypothesis that quarks (strictly speaking, current quarks) are not elementary but rather composed [2] of some more elementary constituents (preons) bound by a new *Abelian* gauge force ("ultraelectromagnetic force" mediated by "ultraphotons" Γ). Then quarks, though expected to be neutral with respect to the corresponding new Abelian charge ("ultracharge"), should display new magnetic-type moments ("ultramagnetic moments") leading to a new magnetic-type interaction ("ultramagnetic interaction") of quarks with ultraphotons and, consequently, with other quarks. Hence, also nucleons, as composed of quarks, should get resulting ultramagnetic moments implying an ultramagnetic interaction of nucleons with ultraphotons and other nucleons [3]. If the ultramagnetic moment of spin-1/2 preons is $\mu_{\rm preon}^{(u)} = e^{(u)}/2m_{\rm preon}$, where $\alpha^{(u)} = e^{(u)\,2}$ denotes the preon ultraelectromagnetic coupling constant, then the ultramagnetic moments of ultracharge-neutral quarks are $\mu_q^{(u)} = e_{\rm eff}^{(u)}/2m_q^{\rm const}$ with an effective quark ultramagnetic coupling constant $\alpha^{(\mathrm{u})}_{\mathrm{eff}} = e^{(\mathrm{u})\,2}_{\mathrm{eff}}$ which should be expected much smaller than $\alpha^{(\mathrm{u})} = O(1)$ ($\mu^{(\mathrm{u})}_{\mathrm{q}}$ are structural anomalous moments for neutral nearly point-like particles). Here, in the definition of $e^{(\mathrm{u})}_{\mathrm{eff}}$, the quarks masses are chosen as quark constituent masses $m^{\mathrm{const}}_{\mathrm{q}}$. In this case the resulting ultramagnetic moments of nucleons become $\mu^{(\mathrm{u})}_{\mathrm{N}} \simeq 3e^{(\mathrm{u})}_{\mathrm{eff}}/2m_{\mathrm{N}}$, where $m_{\mathrm{N}} \simeq 3m^{\mathrm{const}}_{\mathrm{q}}$ are nucleon masses (it is so, since $\mu^{(\mathrm{u})}_{\mathrm{N}} = \mu^{(\mathrm{u})}_{\mathrm{q}}$, *cf.* Eqs. (24) and (30)).

In the present paper we calculate $e^{(\mathrm{u})}_{\mathrm{eff}}$ in a particular model of composite quarks, mentioned in the conclusion of Ref. [1] (*cf.* also Refs. [3,4]). In this model, the u and d current quarks are relativistic bound states of a spin-1/2 preon (existing in two flavors) and a spin-0 preon (existing in three colors) held together by an Abelian attraction described, on the potential-theory level, by the Coulombic potential $V = -\alpha^{(\mathrm{u})}/r$ (the static one-ultraphoton-exchange force between two preons of opposite ultracharges $e^{(\mathrm{u})}$ and $-e^{(\mathrm{u})}$).

To this end, let us consider the relativistic two-body wave equation introduced in Ref. [5] for a system of one spin-1/2 particle and one spin-0 particle. In the case of an internal and/or external potential $V = V(\vec{r}_1, \vec{r}_2)$ and external ultramagnetic 3-potential $\vec{A}^{(\mathrm{u})}(\vec{r})$, this equation takes the form

$$\left\{\left[E - V - \vec{\alpha}_1\cdot\left(\vec{p}_1 - e^{(\mathrm{u})}_1\vec{A}^{(\mathrm{u})}(\vec{r}_1)\right) - \beta m_1\right]^2 - \left(\vec{p}_2 - e^{(\mathrm{u})}_2\vec{A}^{(\mathrm{u})}(\vec{r}_2)\right)^2 - m_2^2\right\}\phi = 0 \tag{1}$$

or

$$\Big\{E - V - 2\left[\vec{\alpha}_1\cdot\left(\vec{p}_1 - e^{(\mathrm{u})}_1\vec{A}^{(\mathrm{u})}(\vec{r}_1)\right) + \beta m_1\right] + \frac{1}{\sqrt{E-V}}\Big[\left(\vec{p}_1 - e^{(\mathrm{u})}_1\vec{A}^{(\mathrm{u})}(\vec{r}_1)\right)^2$$

$$+ m_1^2 - \left(\vec{p}_2 - e^{(\mathrm{u})}_2\vec{A}^{(\mathrm{u})}(\vec{r}_2)\right)^2 - m_2^2 + \tfrac{1}{2}\sigma_{kl}F^{(\mathrm{u})}_{kl}(\vec{r}_1)\Big]\frac{1}{\sqrt{E-V}}\Big\}\sqrt{E-V}\,\phi = 0\,, \tag{2}$$

where $\sigma_{kl} = -\frac{i}{2}[\alpha_k, \alpha_l] = \epsilon_{klm}\sigma_m$, $F^{(\mathrm{u})}_{kl}(\vec{r}) = \partial_k A^{(\mathrm{u})}_l(\vec{r}) - \partial_l A^{(\mathrm{u})}_k(\vec{r}) = -\epsilon_{klm}B^{(\mathrm{u})}_m(\vec{r})$ and $\sigma_{kl}F^{(\mathrm{u})}_{kl} = -2\vec{\sigma}\cdot\vec{B}^{(\mathrm{u})}$ with $\vec{B}^{(\mathrm{u})}(\vec{r}) = \mathrm{rot}\,\vec{A}^{(\mathrm{u})}(\vec{r})$. The wave equation (2) implies that the probability density is given by

$$\psi^\dagger\psi \quad \text{with} \quad \psi = \sqrt{E-V}\,\phi\,. \tag{3}$$

In the case of a constant external ultramagnetic field $\vec{B}^{(\mathrm{u})}$ giving $\vec{A}^{(\mathrm{u})}(\vec{r}) = \frac{1}{2}(\vec{B}^{(\mathrm{u})}\times\vec{r})$ (in the gauge with $\mathrm{div}\vec{A}^{(\mathrm{u})}(\vec{r}) = 0$), the wave equation (2) can be rewritten as

$$\Big\{E - V - 2\Big[\vec{\alpha} \cdot \Big(\vec{p}_1 - e_1^{(u)} \frac{\vec{B}^{(u)} \times \vec{r}_1}{2}\Big) + \beta m_1\Big] + \frac{1}{\sqrt{E-V}}\Big[\vec{p}_1 + m_1^2$$

$$- \vec{p}_2 - m_2^2 - e_1^{(u)}(\vec{\sigma} + \vec{L}_1) \cdot \vec{B}^{(u)} + e_2^{(u)} \vec{L}_2 \cdot \vec{B}^{(u)}\Big] \frac{1}{\sqrt{E-V}}\Big\}\psi = 0\,, \qquad (4)$$

where terms of the order $O(B^{(u)2})$ are neglected. Here, $\vec{L}_i = \vec{r}_i \times \vec{p}_i$ $(i = 1, 2)$.

In the equal-mass case of $m_1 = m_2 \equiv m$, in terms of the centre-of-mass and relative coordinates, where

$$\vec{r}_1 = \vec{R} + \tfrac{1}{2}\vec{r}, \qquad \vec{r}_2 = \vec{R} - \frac{1}{2}\vec{r},$$

$$\vec{p}_1 = \tfrac{1}{2}\vec{P} + \vec{p}, \qquad \vec{p}_2 = \tfrac{1}{2}\vec{P} - \vec{p}, \qquad (5)$$

the wave equation (4) reads

$$\Big\{E - V - \vec{\alpha} \cdot \Big(\vec{P} - 2\frac{e_1^{(u)} + e_2^{(u)}}{2} \frac{\vec{B}^{(u)} \times \vec{R}}{2} - \frac{e_1^{(u)} - e_2^{(u)}}{2} \frac{\vec{B}^{(u)} \times \vec{r}}{2}\Big)$$

$$- 2\vec{\alpha} \cdot \Big(\vec{p} - \frac{e_1^{(u)} - e_2^{(u)}}{2} \frac{\vec{B}^{(u)} \times \vec{R}}{2} - \frac{1}{2} \frac{e_1^{(u)} + e_2^{(u)}}{2} \frac{\vec{B}^{(u)} \times \vec{r}}{2}\Big) - 2\beta m$$

$$+ \frac{1}{\sqrt{E-V}}\Big[2\vec{P} \cdot \vec{p} - (e_1^{(u)}\vec{\sigma} + e_1^{(u)}\vec{L}_1 - e_2^{(u)}\vec{L}_2) \cdot \vec{B}^{(u)}\Big] \frac{1}{\sqrt{E-V}}\Big\}\psi = 0\,. \; (6)$$

Here, $V = V(\vec{R}, \vec{r})$ and

$$\vec{L}_{1,2} = \tfrac{1}{2}(\vec{R} \times \vec{P}) + \tfrac{1}{2}(\vec{r} \times \vec{p}) \pm (\vec{R} \times \vec{p}) \pm \tfrac{1}{4}(\vec{r} \times \vec{P})\,. \qquad (7)$$

In Eq. (6) the combinations $\vec{P} - \ldots$ and $\vec{p} - \ldots$ are kinetic momenta corresponding to the canonical momenta $\vec{P}$ and $\vec{p}$. Note that for an internal potential $V = V(\vec{r})$ and an external ultramagnetic field parallel to the z axis $\vec{B}^{(u)} = (0, 0, B^{(u)})$ the component P_z is a constant of motion. When $V = V(\vec{r})$ and $\vec{B}^{(u)} = 0$, then the whole vector $\vec{P}$ is a constant of motion. For a weak ultramagnetic field $\vec{B}^{(u)}$ and a slow motion of the centre of mass, two factors $1/\sqrt{E-V}$ in Eq. (6) can be consistently approximated by $1/\sqrt{E^{(0)} - V}$, where $E^{(0)}$ is an eigenvalue of Eq. (6) with $V = V(\vec{r})$, $\vec{B}^{(u)} = 0$ and $\vec{P} = 0$:

$$[E^{(0)} - V(\vec{r}) - 2(\vec{\alpha} \cdot \vec{p} + \beta m)]\psi^{(0)} = 0 \qquad (8)$$

(it implies the use of centre-of-mass frame: $\vec{p}_1 = -\vec{p}_2 \equiv \vec{p}$). In Eq. (8) with a central potential $V = V(r)$, the angular momentum $\vec{J}^{(0)} = \vec{\sigma}/2 + \vec{r} \times \vec{p}$ is a constant of motion.

It is easily seen that in the ultracharge-neutral case of $e_1^{(u)} = -e_2^{(u)} \equiv e^{(u)}$ and for Coulombic potential $V = -\alpha^{(u)}/r$ the spectrum $E^{(0)}$ is given by the Sommerfeld-type formula

$$E^{(0)} = 2m\left[1 + \left(\frac{\alpha^{(u)}/2}{n_r + \gamma}\right)^2\right]^{-1/2} \quad \text{with } \gamma = \sqrt{(j+1/2)^2 - (\alpha^{(u)}/2)^2}, \tag{9}$$

where $n_r = 0, 1, 2, \ldots$ and $j = 1/2, 3/2, 5/2, \ldots$. Hence, for the ground state ($n_r = 0$, $j = 1/2$) one gets $E_0^{(0)} = 2m\gamma_0$ with $\gamma_0 = \sqrt{1 - (\alpha^{(u)}/2)^2}$. Thus, $\gamma_0 \to 0$ and $E_0^{(0)} \to 0$ for $\alpha^{(u)} \to 2$ ($\alpha^{(u)} = 2$ is the critical value of $\alpha^{(u)}$ leading to the Klein paradox at $r = 0$). In the convenient representation where

$$\vec{\alpha} = \begin{pmatrix} 0 & i\vec{\sigma}_P \\ -i\vec{\sigma}_P & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1_P & 0 \\ 0 & -1_P \end{pmatrix}, \quad \vec{\sigma} = \begin{pmatrix} \vec{\sigma}_P & 0 \\ 0 & \vec{\sigma}_P \end{pmatrix} \tag{10}$$

with $\vec{\sigma}_P$ and 1_P denoting four Pauli matrices, the ground-state wave function corresponding to $m_j = +1/2$ is of the form

$$\psi_0^{(0)}(\vec{r}) = \left[\frac{\left(2m\sqrt{1-\gamma_0^2}\right)^{2\gamma_0+1}(\gamma_0+1)}{2\Gamma(2\gamma_0+1)}\right]^{1/2} r^{\gamma_0-1} e^{-m\sqrt{1-\gamma_0^2}\,r}$$
$$\times \begin{pmatrix} Y_{00}(\hat{r}) \\ 0 \\ \sqrt{\frac{1}{3}\frac{1-\gamma_0}{1+\gamma_0}}\, Y_{10}(\hat{r}) \\ -\sqrt{\frac{2}{3}\frac{1-\gamma_0}{1+\gamma_0}}\, Y_{11}(\hat{r}) \end{pmatrix}, \tag{11}$$

where $\psi_0^{(0)}(\vec{r}) \equiv \sqrt{E_0^{(0)} + \alpha/r}\,\phi_0^{(0)}(\vec{r})$ and $\langle \psi_0^{(0)} \,|\, \psi_0^{(0)} \rangle = 1$. Note that, after some calculations, the wave function (11) implies the formulae

$$\langle \psi_0^{(0)} \,|\, \sigma_z \,|\, \psi_0^{(0)} \rangle = \tfrac{2}{3}\gamma_0 + \tfrac{1}{3} \xrightarrow{\alpha^{(u)} \to 2} \tfrac{1}{3}, \tag{12}$$

$$\langle \psi_0^{(0)} \,|\, (\vec{r} \times \vec{p})_z \,|\, \psi_0^{(0)} \rangle = \tfrac{1}{3}(1 - \gamma_0) \xrightarrow{\alpha^{(u)} \to 2} \tfrac{1}{3}, \tag{13}$$

$$\langle \psi_0^{(0)} \,|\, \tfrac{1}{2}(\vec{r} \times \vec{\alpha})_z \,|\, \psi_0^{(0)} \rangle = \frac{1}{6m}\frac{\Gamma(2\gamma_0+2)}{\Gamma(2\gamma_0+1)} \xrightarrow{\alpha^{(u)} \to 2} \frac{1}{6m}, \tag{14}$$

and

$$\langle\psi_0^{(0)}|\frac{1}{E_0^{(0)}+\alpha^{(u)}/r}\left[\sigma_z+(\vec{r}\times\vec{p})_z\right]|\psi_0^{(0)}\rangle$$

$$=\frac{1}{6m}\frac{\gamma_0+2}{\Gamma(2\gamma_0+1)}\int_0^\infty dx\,\frac{x^{2\gamma_0+1}e^{-x}}{\gamma_0 x+2(1-\gamma_0)}\xrightarrow{\alpha^{(u)}\to 2}\frac{1}{6m} \tag{15}$$

with $x\equiv 2m\sqrt{1-\gamma_0^2}\,r=\alpha^{(u)}m\,r$.

Now, let us consider a simple quark model of the nucleon, where three of our composite quarks are bound around the nucleon centre of mass by the sum of potentials $V(\vec{R}_1,\vec{R}_2,\vec{R}_3)$ and $\sum_{i=1}^{3}\beta_i\,S(R_i)$ with

$$S(R)=\begin{cases} m_q^{\rm const} & \text{for } R<\text{nucleon radius}\\ \infty & \text{for } R>\text{nucleon radius}\end{cases} \tag{16}$$

being the confining factor. Here, $m_q^{\rm const}$ denotes the quarks constituent mass, while $m_q^{\rm current}\equiv E_0^{(0)}=2m\gamma_0$ is the quark current mass where $m_{\rm preon}\equiv m$ stands for the preon mass. The internal potential for preons within quarks is $\sum_{i=1}^{3}V(r_i)$ with $V(r)=-\alpha^{(u)}/r$. Then, three quarks confined in the nucleon obey the wave equation

$$\Big\{E-V(\vec{R}_1,\vec{R}_2,\vec{R}_3)-\sum_{i=1}^{3}\left[\vec{\alpha}_i\cdot\left(\vec{P}_i-e^{(u)}\frac{\vec{B}^{(u)}\times\vec{r}_i}{2}\right)+\beta_i S(R_i)\right]$$
$$-2\sum_{i=1}^{3}\left[\vec{\alpha}_i\cdot\left(\vec{p}_i-e^{(u)}\frac{\vec{B}^{(u)}\times\vec{R}_i}{2}\right)+\beta_i m+\tfrac{1}{2}V(r_i)\right] \tag{17}$$
$$+\sum_{i=1}^{3}\frac{1}{E_0^{(0)}-V(r_i)}\left[2\vec{P}_i\cdot\vec{p}_i-e^{(u)}(\vec{\sigma}_i+\vec{R}_i\times\vec{P}_i+\vec{r}_i\times\vec{p}_i)\cdot\vec{B}^{(u)}\right]\Big\}\psi=0$$

because in Eq. (6) $e_1^{(u)}=-e_2^{(u)}\equiv e^{(u)}$ ($e_q^{(u)}\equiv e_1^{(u)}+e_2^{(u)}=0$ and $e_1^{(u)}-e_2^{(u)}=2e^{(u)}$ with $e^{(u)}\equiv e_1^{(u)}$ being the ultracharge of spin-1/2 preon).

In the nonrelativistic approximation of the nucleon, where three quark centres of mass are supposed to move slowly enough around the nucleon centre of mass, we can approximate in Eq. (17) the relativistic terms

$$\vec{\alpha}_i\cdot\left(\vec{P}_i-e^{(u)}\frac{\vec{B}^{(u)}\times\vec{r}_i}{2}\right)+\beta_i\,m_q^{\rm const} \tag{18}$$

by their nonrelativistic counterparts

$$m_{\rm q}^{\rm const} + \frac{1}{2m_{\rm q}^{\rm const}}\left[\vec{\alpha}_i \cdot \left(\vec{P}_i - e^{(\rm u)}\frac{\vec{B}^{(\rm u)} \times \vec{r}_i}{2}\right)\right]^2$$
$$= m_{\rm q}^{\rm const} + \frac{1}{2m_{\rm q}^{\rm const}}\left[\vec{P}_i^2 - e^{(\rm u)}(\vec{r}_i \times \vec{P}_i)\cdot\vec{B}^{(\rm u)}\right]. \qquad (19)$$

Then, the nucleon mass $m_{\rm N} \simeq 3\,m_{\rm q}^{\rm const}$. Notice that in Eq. (19) there is no spin coupling term $(e^{(\rm u)}/2\,m_{\rm q}^{\rm const})\vec{\sigma}_i \cdot \vec{B}$, what follows from the relation $[\vec{P}_i, \vec{r}_i] = 0$ implying that

$$\left[\vec{\alpha}_i \cdot \left(\vec{P}_i - e^{(\rm u)}\frac{\vec{B}^{(\rm u)} \times \vec{r}_i}{2}\right)\right]^2 = \left(\vec{P}_i - e^{(\rm u)}\frac{\vec{B}^{(\rm u)} \times \vec{r}_i}{2}\right)^2. \qquad (20)$$

In contrast to the quark external motion within the nucleon, the quark internal motion is highly relativistic, what can be seen from the formula

$$\langle\psi_0^{(0)}\,|\,\beta\,|\,\psi_0^{(0)}\rangle = \gamma_0 \xrightarrow{\alpha^{(\rm u)} \to 2} 0 \qquad (21)$$

corresponding classically to $\sqrt{1 - v_{\rm cl}^2} \to 0$ *i.e.* , to $v_{\rm cl}^2 \to 1$. Performing the nonrelativistic substitution (18)→(19), we obtain from Eq. (17) the following approximate wave equation for three composite quarks confined in the nucleon:

$$\Big\{E - V(\vec{R}_1, \vec{R}_2, \vec{R}_3) - 3m_{\rm q}^{\rm const} - \sum_{i=1}^{3}\frac{1}{2m_{\rm q}^{\rm const}}\left[\vec{P}_i^2 - e^{(\rm u)}(\vec{r}_i \times \vec{P}_i)\cdot\vec{B}^{(\rm u)}\right]$$
$$- 2\sum_{i=1}^{3}\left[\vec{\alpha}_i \cdot \left(\vec{p}_i - e^{(\rm u)}\frac{(\vec{B}^{(\rm u)} \times \vec{R}_i}{2}\right) + \beta_i m + \tfrac{1}{2}V(r_i)\right]$$
$$+ \sum_{i=1}^{3}\frac{1}{E_0^{(0)} - V^{in}(r)}$$
$$\times \left[\vec{P}_i \cdot \vec{p}_i - e^{(\rm u)}(\vec{\sigma}_i + \vec{R}_i \times \vec{P}_i + \vec{r}_i \times \vec{p}_i)\cdot\vec{B}^{(\rm u)}\right]\Big\}\psi = 0\,, \qquad (22)$$

where $\psi = 0$ outside the nucleon. This equation will be a basis for our conclusions.

From Eq. (22) we can read of the form of the operator for (internal) ultramagnetic moment of a composite quark confined in the nucleon:

$$\vec{\mu}_{\rm q}^{(\rm u)} = -\frac{e^{(\rm u)}}{2m\gamma_0 + \alpha^{(\rm u)}/r}(\vec{\sigma} + \vec{r} \times \vec{p}). \qquad (23)$$

Making use of Eqs. (23) as well as (12) and (15) we get the following formula for the ground-state expectation value for (internal) ultramagnetic moment of our composite quark:

$$\mu_{\rm q}^{({\rm u})} = \frac{\langle \psi_0^{(0)} \,|\, \mu_{q\,z}^{({\rm u})} \,|\, \psi_0^{(0)} \rangle}{\langle \psi_0^{(0)} \,|\, \sigma_z \,|\, \psi_0^{(0)} \rangle}$$

$$= -\frac{e^{({\rm u})}}{2m}\frac{\gamma_0 + 2}{\Gamma(2\gamma_0 + 1)} \int\limits_0^\infty dx \, \frac{x^{2\gamma_0+1} e^{-x}}{\gamma_0 x + 2(1-\gamma_0)} \xrightarrow{\alpha^{({\rm u})} \to 2} -\frac{({\rm sgn}\, e^{({\rm u})})\sqrt{2}}{2m}, \quad (24)$$

where $\gamma_0 = \sqrt{1 - (\alpha^{({\rm u})}/2)^2} = E_0^{(0)}/2m$ and

$$e^{({\rm u})} = ({\rm sgn}\, e^{({\rm u})})\sqrt{\alpha^{({\rm u})}} = ({\rm sgn}\, e^{({\rm u})})\sqrt{2}(1-\gamma_0^2)^{1/4} \xrightarrow{\alpha^{({\rm u})} \to 2} ({\rm sgn}\, e^{({\rm u})})\sqrt{2}\,. \quad (25)$$

From Eq. (24) we readily obtain the formula for $e_{\rm eff}^{({\rm u})}$:

$$e_{\rm eff}^{({\rm u})} = 2m_{\rm q}^{\rm const}\mu_{\rm q}^{({\rm u})} \xrightarrow{\alpha^{({\rm u})} \to 2} -({\rm sgn}\, e^{({\rm u})})\sqrt{2}\frac{m_{\rm q}^{\rm const}}{m} = O\left(\frac{m_{\rm N}}{m}\right), \quad (26)$$

where $m_{\rm N} \simeq 3m_{\rm q}^{\rm const}$ and $m_{\rm preon} \equiv m$. Thus, due to Eqs. (25) and (26), $3e_{\rm eff}^{({\rm u})} : e^{({\rm u})} \simeq m_{\rm N} : m$ if $\alpha^{({\rm u})} = 2$.

An argument analogical to the above enables us to find step by step the ordinary (intrinsic) magnetic moment of the composite quark confined in the nucleon. In this case $e_1^{({\rm u})} \to e_1 = (2/3$ or $-1/3)e$, $e_2^{({\rm u})} \to e_2 = 0$ and $\vec{B}^{({\rm u})} \to \vec{B}$, where $\alpha = e^2 \simeq 1/137$ ($e_{\rm q} \equiv e_1 + e_2 = e_1$ and $e_1 - e_2 = e_1$ with e_1 being the charge of spin-1/2 preon). Then, the operator for this magnetic moment takes the form

$$\vec{\mu}_{\rm q} = e_{\rm q}\left[\frac{1}{2m_{\rm q}^{\rm const}}\vec{\sigma} + \frac{1}{4}\vec{r} \times \vec{\alpha} - \frac{1}{2m\gamma_0 + \alpha^{({\rm u})}/r}\left(\vec{\sigma} + \frac{1}{2}\vec{r} \times \vec{p}\right)\right]. \quad (27)$$

Making use of Eqs. (27) as well as (12), (14) and

$$\langle \psi_0^{(0)} \,|\, \frac{1}{E_0^{(0)} + \alpha^{({\rm u})}/r}\left[\sigma_z + \frac{1}{2}(\vec{r} \times \vec{p})_z\right] \,|\, \psi_0^{(0)} \rangle$$

$$= \frac{1}{8}\frac{1}{\Gamma(2\gamma_0 + 1)} \int\limits_0^\infty dx \, \frac{x^{2\gamma_0+1} e^{-x}}{\gamma_0 x + 2(1-\gamma_0)} \xrightarrow{\alpha^{({\rm u})} \to 2} \frac{1}{8m}, \quad (28)$$

we get the following formula for the ground-state expectation value for (intrinsic) magnetic moment of our composite quark:

$$\mu_{\rm q} = \frac{\langle \psi_0^{(0)} \,|\, \mu_{q\,z} \,|\, \psi_0^{(0)} \rangle}{\langle \psi_0^{(0)} \,|\, \sigma_z \,|\, \psi_0^{(0)} \rangle} \xrightarrow{\alpha^{({\rm u})} \to 2} e_{\rm q}\left(\frac{1}{2m_{\rm q}^{\rm const}} - \frac{1}{8m}\right) = \frac{e_{\rm q}}{2m_{\rm q}^{\rm const}}\left(1 + \frac{1}{4}\frac{m_{\rm N}}{3m}\right), \quad (29)$$

where $e_{\rm q} = (2/3 \ \text{ or } \ -1/3)\sqrt{\alpha}$.

With the moments $\mu_{\rm q}^{(\rm u)}$ and $\mu_{\rm q}$ as given in Eqs. (24) and (29), the nucleon ultramagnetic moment and nucleon magnetic moment becomes, respectively,

$$\left.\begin{array}{ll}\mu_{\rm p}^{(\rm u)} & = \frac{2}{3}(2\mu_{\rm u}^{(\rm u)} - \mu_{\rm d}^{(\rm u)}) + \frac{1}{3}\mu_{\rm d}^{(\rm u)} \\ \mu_{\rm n}^{(\rm u)} & = \frac{2}{3}(2\mu_{\rm d}^{(\rm u)} - \mu_{\rm u}^{(\rm u)}) + \frac{1}{3}\mu_{\rm u}^{(\rm u)}\end{array}\right\} \xrightarrow{\alpha^{(\rm u)}\to 2} -\frac{(\text{sgn } e^{(\rm u)})\sqrt{2}}{2m}$$

$$= -\frac{(\text{sgn } e^{(\rm u)})\sqrt{2}}{2m_{\rm N}}\frac{m_{\rm N}}{m} \tag{30}$$

and

$$\begin{aligned}\mu_{\rm p} &= \tfrac{2}{3}(2\mu_{\rm u} - \mu_{\rm d}) + \tfrac{1}{3}\mu_{\rm d} \xrightarrow{\alpha^{(\rm u)}\to 2} \frac{\sqrt{\alpha}}{2m_{\rm q}^{\rm const}} \simeq 3\,\frac{\sqrt{\alpha}}{2m_{\rm N}}, \\ \mu_{\rm n} &= \tfrac{2}{3}(2\mu_{\rm d} - \mu_{\rm u}) + \tfrac{1}{3}\mu_{\rm u} \xrightarrow{\alpha^{(\rm u)}\to 2} -\tfrac{2}{3}\frac{\sqrt{\alpha}}{2m_{\rm q}^{\rm const}} \simeq -2\,\frac{\sqrt{\alpha}}{2m_{\rm N}},\end{aligned} \tag{31}$$

since the nonrelativistic nucleon expectation value of σ_z is $+1$ (for $m_j = +1/2$). It is seen that the predictions (31) agree nicely with the experimental values $\mu_{\rm p}^{\rm exp} = 2.8\,(\sqrt{\alpha}/2m_{\rm N})$ and $\mu_{\rm n}^{\rm exp} = -1.9\,(\sqrt{\alpha}/2m_{\rm N})$. Note a pretty large difference in magnitude between $\mu_{\rm N}^{(\rm u)}$ and $\mu_{\rm N}$, since certainly $m_{\rm N} \ll m$ ($m \equiv m_{\rm preon}$) though $\sqrt{2} > \sqrt{\alpha} = 0.00854$. For instance, when $m_{\rm preon} = O(1 \text{ TeV}) \div O(10 \text{ TeV})$, one obtains $\mu_{\rm N}^{(\rm u)}/\mu_{\rm N} = O(10^{-2}) \div O(10^{-3})$ if $\alpha^{(\rm u)} = 2$.

Thus, we can conclude this discussion on our Abelian model of composite quarks as follows. For $\alpha^{(\rm u)}$ increasing to its critical value $\alpha^{(\rm u)} \to 2$ the quark current mass $m_{\rm q}^{\rm current} \equiv E_0^{(0)} = 2m_{\rm preon}\gamma_0$ decreases to 0, while the nucleon ultramagnetic moment $\mu_{\rm N}^{(\rm u)}$ and nucleon magnetic moment $\mu_{\rm N}$ tends to $-[(\text{sgn } e^{(\rm u)})\sqrt{2}/2m_{\rm N}](m_{\rm N}/m_{\rm preon})$ and $(3 \text{ or } -2)\sqrt{\alpha}/2m_{\rm N}$, respectively, where $m_{\rm q}^{\rm const} \simeq m_{\rm N}/3$ is the quark constituent mass. Therefore, the strength "$\alpha^{(\rm u)}$" of ultramagnetic interaction of nucleons with ultraphotons and other nucleons, discussed in Ref. [1] (*cf. e.g.* Eq. (3) there), should be now reinterpreted as $9\,\alpha_{\rm eff}^{(\rm u)} = \alpha^{(\rm u)}(m_{\rm N}/m_{\rm preon})^2$ if $\alpha^{(\rm u)} = 2$ (since $3e_{\rm eff}^{(\rm u)} = e^{(\rm u)}(m_{\rm N}/m_{\rm preon})$ if $e^{(\rm u)} = (\text{sgn } e^{(\rm u)})\sqrt{2}$). So, we can see that the strength $9\,\alpha_{\rm eff}^{(\rm u)}$ of nucleon ultramagnetic interaction is pretty weak for any reasonable preon mass $m_{\rm preon}$ which should be large to guarantee a small quark size $\sim 1/m_{\rm preon}$ (*cf.* the shape-factor $\exp\,(-m_{\rm preon}\sqrt{1-\gamma_0^2}\,r)$ in the quark wave function (11)). For instance, when $m_{\rm preon} = O(1 \text{ TeV}) \div O(10 \text{ TeV})$, one gets $9\,\alpha_{\rm eff}^{(\rm u)} = O(10^{-6}) \div O(10^{-8})$ if $\alpha^{(\rm u)} = 2$. Hence, quark size $\sim 1/m_{\rm preon} = O(10^{-16} \text{ cm}) \div O(10^{-17} \text{ cm})$. Note that the deviation of the actual $\alpha^{(u)}$ from $\alpha^{(u)} = 2$ should be extremely small since by means of the relation $\alpha^{(u)} = 2\sqrt{1 - (E_0^{(0)}/2m_{\rm preon})^2}$ one estimates $2 - \alpha^{(u)} = O(10^{-11})$ for $E_0^{(0)} \equiv m_q^{\rm current} = O(10\ {\rm MeV})$. On the other hand, we can see (to our satisfaction) that an analogical argument gives the nucleon ordinary magnetic moment nicely consistent with the experiment, what is, of course, a minimal necessary check of the model.

In our opinion, however, the phenomenological discussion of the hypothetical ultramagnetic interaction and experimental search suggested for it in Ref. [1] might be considered as independent of the detailed conclusions about the magnitude of the strength $9\,\alpha_{\rm eff}^{(u)}$ of nucleon ultramagnetic interaction, drawn in the present paper on the ground of a particular model of composite quarks. At any rate, this paper provides us with a strong argument that $9\,\alpha_{\rm eff}^{(u)} \ll O(1)$, what makes the experimental search for nucleon ultramagnetic interaction difficult. Note that the decays $\pi^0 \to 2\Gamma$ and $\Sigma^0 \to \Lambda\Gamma$ are forbidden (independently of the magnitude of $\alpha_{\rm eff}^{(u)}$) by the isospin symmetry, as Γ is a isospin scalar.

In Ref. [1] we called the reader's attention to the molecular radiofrequency experiments (1953) determining H_2 rotational levels in external magnetic field [6]. They measured hfs effects in H_2 molecules fully consistent with the ordinary magnetic dipole-dipole interaction of two protons involved. This agreement sets an upper limit on our hypothetical ultramagnetic dipole-dipole interaction between two protons:

$$V_{\rm umag}(\vec{r}) = -\frac{\mu_{\rm p}^{(u)\,2}}{r^3}[3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2]\,, \tag{32}$$

where $\mu_{\rm p}^{(u)\,2} = 9\,\alpha_{\rm eff}^{(u)}/(2m_{\rm p})^2$. According to an analysis in Ref. [7] the experiments of Ref. [6] leave such a margin for $V_{\rm umag}(\vec{r})$ that there must hold the upper limit

$$\mu_{\rm p}^{(u)\,2} \left\langle \frac{1}{r^3} \right\rangle_{H_2} < 3 \times 10^{-19}\ {\rm MeV} \tag{33}$$

with

$$\left\langle \frac{1}{r^3} \right\rangle_{H_2} = \frac{1}{r_{\rm eq}^3}, \quad r_{\rm eq} = 0.74 \times 10^{-8}\ {\rm cm}. \tag{34}$$

Hence, we get the upper bound $9\,\alpha_{\rm eff}^{(u)} < 2 \times 10^{-7}$ which is consistent with the rough model-estimation $9\alpha_{\rm eff}^{(u)} = O(10^{-6}) \div O(10^{-8})$ valid for $m_{\rm preon} = O(1\,{\rm TeV}) \div O(10\,{\rm TeV})$ and $\alpha^{(u)} = 2$. Note that in our argument the ultraphoton rest mass is zero (then, the ultraphoton is the gauge boson of a new unbroken U(1) local symmetry generated by the ultracharge). The upper bound $9\,\alpha_{\rm eff}^{(u)} < O(10^{-7})$ might increase drastically, if the ultraphoton developed a nonnegligible rest mass m_Γ (in a process of spontaneously breaking our U(1) symmetry [8]), introducing the Yukawa exponent $\exp(-m_\Gamma r)$ to the interaction (32). For instance, in the case of $m_\Gamma = 5, 15, 20, 25$ keV one would get

$$9\,\alpha^{(u)}_{\rm eff} < 0.0004,\ 0.02,\ 0.1,\ 1,$$

respectively [7].

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