NUCLEAR TRANSIENTS

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Nuclear scattering is classically analysed using realistic conservative interactions. Due to chaoticity of motion, the projectile can stay in the vicinity of the target for a long time. Such phenomena are called transients. We investigate properties of transients: fractal dimensions, Lyapunov exponents and mean lifetimes. We conclude that the lifetime of transients is similar to that of single-particle resonances. It rises substantially when we consider a more general system, with larger number of degrees of freedom.

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1. Introduction

Classical mechanics has been frequently applied to describe heavy-ion collisions. It has been argued that masses of colliding systems and angular momenta involved are large enough to justify such approach. Deep-inelastic and fusion reactions are especially well suited to be described by classical means.

The easiest way to model heavy-ion reactions is to assume only two-body dynamics. Conservative systems lead to a split with reality because any loss of the relative energy is impossible in this case. The system can fuse only for a single initial condition when the trajectory hits the top of the Coulomb barrier. This particular impact parameter corresponds to the singularity of deflection function. For all other impact parameters the deflection function is regular. Introduction of a friction force changes the picture. The system transfers the energy of the relative motion to internal degrees of freedom of projectile and target, as one used to interpret the friction force. Zero of the relative energy is a token of fusion. Despite many apparent successes of classical models utilizing both conservative and non-conservative interactions in reproducing experimental data, some important features of nuclear reactions remain unexplained. In particular, all fused systems live infinitely long whereas one could expect rather an exponential decay of the compound system with time (a resonance).

A proper handling of the internal degrees of freedom of nucleus requires taking into account its many-body structure *explicite*. Some efforts in this direction has been already undertaken with noticeable success [1–3]. From the mathematical point of view, the main difference between two-body and many-body dynamical systems arises from the fact that the latter is non-integrable and in the most cases reveals a very complicated, chaotic, behaviour.

In this paper we discuss properties of a simple scattering system exhibiting chaotic behaviour and show how resonance phenomena can emerge. In Section 2 we introduce some general definitions concerning properties of chaotic systems. The notion of transient is especially important when one considers scattering problems. Its origin traces back to a subtle interplay between stable and unstable manifolds. They have Cantor set structure and can be characterized by a sequence of fractal dimensions, defined in Section 3. We also show there how lifetime of transient is connected with the fractal dimension and Lyapunov exponent. The concept of transient chaos is applied to a nuclear system defined in Section 4. We calculate the fractal dimension and the Lyapunov exponent for the nuclear transient determining its average lifetime in Section 5. A similarity to single-particle quantum resonances is discussed in Section 6. We also consider there possible generalizations of the system presented in this paper.

2. Dynamics of chaotic systems and transients

Let us consider a conservative dynamical system. Studying the behaviour of trajectories around hyperbolic (saddle) fixed points is of crucial importance to decide about eventual regularity or chaoticity of motion (see *e.g.* [4,5]). Points positioned along unstable directions expand exponentially with time, whereas points along stable directions converge exponentially to the fixed point. In general, unstable (stable) directions form unstable (stable) manifolds. Any point on the stable manifold requires infinitely long time to reach the fixed point, *i.e.* it remains in the vicinity of the fixed point forever. The system is called chaotic if the stable and unstable manifolds cross each other in infinitely many points, called homoclinic points. The resulting behaviour of such system is extremely complicated.

One of the consequences of chaotic behaviour of a dynamical system is the sensitive dependence on initial conditions. The distance between two close trajectories, initially equal to δ_0, rises exponentially with time t, $\delta = \delta_0 \exp(\lambda t)$, when t is large. The rate of divergence, λ, is called the

Lyapunov exponent and is positive. In practice, λ can be obtained by the simultaneous solving of the original and linearized equations of motion.

Regularity and chaoticity of many dynamical systems (not necessarily conservative) is connected with their dependence on a parameter. For some values of this parameter we deal with the regular regime, otherwise the system is chaotic[1]. However, the chaotic behaviour can manifest itself also in the regular domain. During the evolution, points belonging to the regular basin converge to a periodic orbit (the attractor) and, as an ultimate outcome, the Lyapunov exponent is zero. But we can be interested in the convergence process itself. For some initial conditions the attractor is reached almost instantly, for other the evolution must be very long. Looking at the behaviour of the system for those long trajectories, we find a striking similarity to strange attractors (attractors in the chaotic regime): Lyapunov exponents converge to a positive number and the pattern drown by the trajectory resembles the shape of the strange attractor. Consequently, those transitory phenomena are called semi-attractors or transients [6,7]. More precisely, a trajectory reveals the transient behaviour when it remains outside a small neighbourhood of the periodic orbit. The time the trajectory needs to fall into this neighbourhood is the lifetime of the transient.

The scattering problem concerning conservative systems can be similarly formulated [8]. The phase space is open and trajectories, coming from infinity, fall into the interaction region where the motion is chaotic. The trajectories stay there for some time revealing transient behaviour. Transients decay when they leave the interaction region and escape to infinity.

The time a particular trajectory abides within the interaction region depends on the position in the phase space of initial conditions in respect to the stable and unstable manifolds. The escape of points on the unstable manifold is very rapid. On the contrary, trajectories located on the stable manifold remain in the interaction region for the infinite time. Therefore, it is crucial for the determination of the lifetimes of transients to ask how dense in the phase space the stable manifold is.

3. Dimensions of manifolds

A trajectory crossing a plane during the evolution draws a more or less complicated structure on it (the Poincaré section), built from isolated points. One can ask about dimension of this set of points when time goes to infinity. It appears that sometimes the points do not form a line, a plain figure, etc. and they are also not isolated: the dimension of the set is fractal.

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[1] Equations describing flows, depending on the Reynolds number, can serve as an example.

Let us assume that the set is embedded in the d-dimensional Euclidean space. If we need $N(\epsilon)$ d-dimensional cubes of size ϵ to cover the set completely, the fractal dimension D is defined as

$$N(\epsilon) \sim \epsilon^{-D}, \quad \epsilon \to 0. \tag{1}$$

The most famous example of the fractal set is Cantor set. In order to construct it, the unit interval is divided into three equal parts and the middle segment is removed. The same is done with the remaining two segments. The procedure goes on to infinity, and finally one gets the Cantor set. It is easy to check that (1) implies $D = \log 2/\log 3$.

The definition (1) can be generalized. We define a family of dimensions by [9]

$$D^{(q)} = \frac{1}{1-q} \lim_{\epsilon\to 0} \frac{\ln \sum p_i^q}{\ln(1/\epsilon)}, \tag{2}$$

where p_i is the probability that a point is found in the i-th cube. Formula (2) resolves itself to (1) for $q = 0$; $D^{(0)}$ is called the capacity or Hausdorff dimension. A special importance have also $D^{(1)}$ (information dimension) and $D^{(2)}$ (correlation dimension).

The measure of the fractal set is zero. Indeed, the total capacity of the cubes needed to cover it is proportional to $\epsilon^d N(\epsilon) = \epsilon^{d-D}$ and goes to zero ($\epsilon \to 0$).

Stable and unstable manifolds connected with the scattering of chaotic systems have (the same) fractal dimensions. The structure of the stable manifold is clearly recognizable from the deflection function. Changing the impact parameter one crosses the stable manifold what results in the singularity of the deflection angle, accompanied by a rapid growth of time a trajectory spends inside the interaction region. Subsequent magnifications of the impact parameter intervals show that singularities split and their number rises in a constant ratio, allowing to calculate the dimension. Since the dimension is fractal, the probability to hit the stable manifold by a random sampling of the impact parameter is zero. In practice, one gets always a finite lifetime.

The average lifetime of transients can be assessed from the Lyapunov exponents and fractal dimensions. The speed of their decay is exponential. More precisely, the probability that a randomly chosen trajectory has not yet escaped after time t is $\exp(-t/\tau)$ where

$$1/\tau = \sum_{\lambda_i>0} \lambda_i(1 - D_i); \tag{3}$$

λ_i denotes the spectrum of Lyapunov exponents and D_i's are partial information dimensions [10].

The interpretation of (1) is following [7]. A typical trajectory continues to abide within the interaction region unless it falls into an interval between two folds of the stable manifold. The total density of these holes is proportional to $1 - D_i$. The Lyapunov exponent λ_i is just the mean velocity of the escaping flow in i-th unstable direction. The escaping rates in stable and neutral directions are slower than exponential. The total escaping flow we get by summation over all unstable directions.

4. Description of the system

We consider the scattering of an alpha particle on ^{12}C treated as an alpha-cluster nucleus [1,2], *i.e.* the whole dynamical system consists of four alpha particles. They interact via two-body interactions [11]:

$$V(r) = \begin{cases} \alpha/r + a_1 \exp\left[-\left(\frac{r-a_2}{a_3}\right)^2\right] + a_4 \exp\left[-\left(\frac{r-a_5}{a_6}\right)^2\right], & r > r_{\min} \\ a_7 + a_8(r - r_{min})^2, & r < r_{\min} \end{cases} \tag{4}$$

with parameters $a_1 = -5.673$ MeV, $a_2 = 3.781$ fm, $a_3 = 1.23$ fm, $a_4 = 1.6$ MeV, $a_5 = 4.351$ fm, $a_6 = 0.896$ fm, $a_7 = -3.16419$ MeV, $a_8 = 4.0041$ MeV/fm^2; $r_{\min} = 3.6355$ fm is the minimum of the potential. The parameter α stands for the Coulomb parameter.

The potential (4) has been obtained from adiabatic time-dependent Hartree-Fock (ATDHF) calculations. Since the ATDHF is unable to find an effective two-body interaction in the density overlap region, the original potential has been derived only up to the minimum. The far left part has been assumed as parabolic with parameters determined by matching conditions at the minimum [12]. The sketch of the potential is presented in ref.[13]. Using the $\alpha - \alpha$ interaction (4) enables for a construction of alpha-cluster nuclei (^{12}C, ^{16}O, ^{20}Ne etc.), properly reproducing radial density distributions in the position and momentum spaces, as well as experimental binding energies and separation energies of subsequent alpha particles.

To enable any serious analysis of our scattering problem we have been compelled to drastic simplifications. All degrees of freedom within the target has been frozen. The target alpha-particles have been fixed at equal distances so chosen to ensure the proper binding energy of ^{12}C ($E_B = -7.2747$ MeV). The collision has been assumed planar. In this way the problem is reduced to scattering by a two-dimensional potential. The motion is then restricted to the three-dimensional manifold, embedded in the four-dimensional phase space. The total potential we get by summation of the two-body potentials over all alpha particles. Its contour plot is presented in Fig.1. The potential possesses three hills surrounded by Coulomb barrier

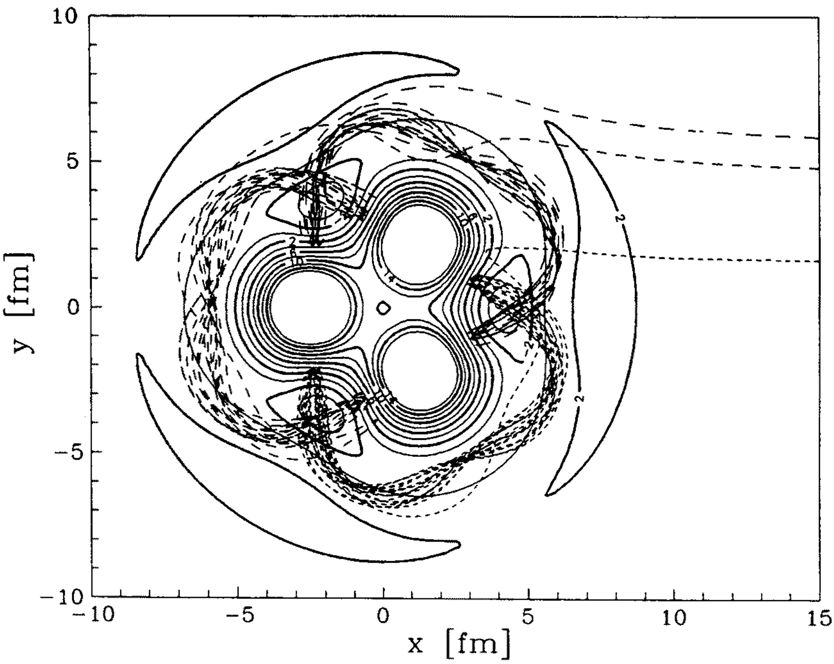

Fig. 1. Topographical map of $\alpha+^{12}$C potential (solid lines) and trajectories (dashed lines) corresponding to three singular regions (see text) of impact parameter, $E =$ 3.5 MeV

forming three heights. The hills and the Coulomb barrier are separated by a trench.

Equations of motion are of the form

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{m}\begin{pmatrix} p_x \\ p_y \end{pmatrix},$$
$$\frac{d}{dt}\begin{pmatrix} p_x \\ p_y \end{pmatrix} = -\begin{pmatrix} \partial/\partial x \\ \partial/\partial y \end{pmatrix} V(x,y), \tag{5}$$

where m is reduced mass of the system. For a given bombarding energy, initial conditions are fully determined by a choice of impact parameter. We have solved the system (5) numerically, using the Runge-Kutta-Fehlberg method [14] with integration step size adjusted automatically.

5. Properties of nuclear transient

An uncountable set of stable manifolds connected with orbits localized within the Coulomb barrier and stretched out to the asymptotic region has fractal dimension. Every trajectory which starts from one of those manifolds stays within the Coulomb barrier forever. Since the measure of the set of initial conditions connected with the stable manifolds is zero, we can consider only trajectories close to them. They also live for a long time and are attracted by localized orbits. Fig. 2 presents a section of the stable manifold, followed by a single trajectory starting at infinity, by a surface $y = x/\sqrt{3}$. After a long evolution ($t = 7.5 \cdot 10^{-18}$ s) the trajectory slips off the manifold and then escapes to infinity never crossing the plane again. We have rejected the last 65 points (to get the section through the *exact* stable manifold) and the figure consist of 3500 points. The plot resembles 5-cycle quasiperiodic orbit. Isolated points between them originate from the incoming branch of the manifold.

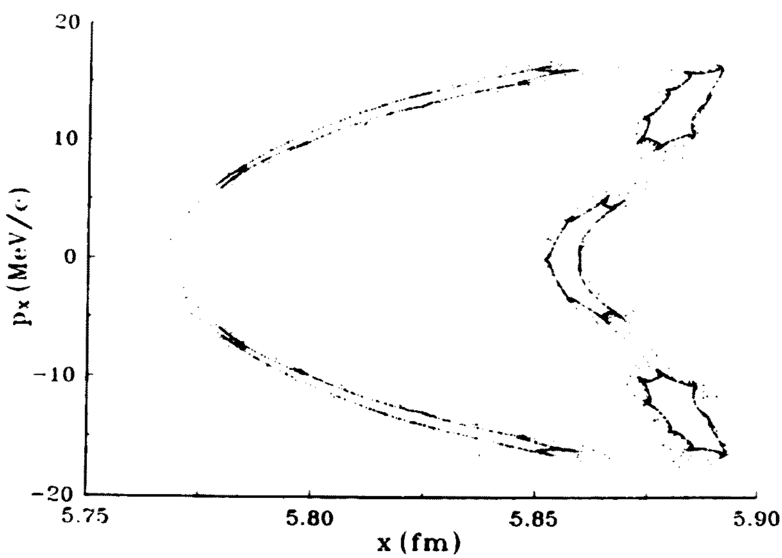

Fig. 2. The section of the stable manifold connected with the trajectory: impact parameter $b = 5.3525262978$ fm, $E = 3.5$ MeV

Fig. 3 shows the deflection function. One can distinguish three singular regions: $b < 2$ fm, $b \approx 4.4$ fm and $b \approx 5.4$ fm, separated by regular intervals. They correspond to different localizations of trajectories captured between the Coulomb barrier and two hills. In the absence of Coulomb potential all trajectories can be located in the same (central) area only [15]. Fig. 1 shows how long-living trajectories characterized by various impact parameters are positioned in the configuration space. Successive stretchings of the horizontal axis in Fig. 3a reveal the self-similar structure (Fig. 3b,c) and is an indication of the fractal dimension of the set of singularities.

Our aim is to determine the average lifetime of the transient, applying

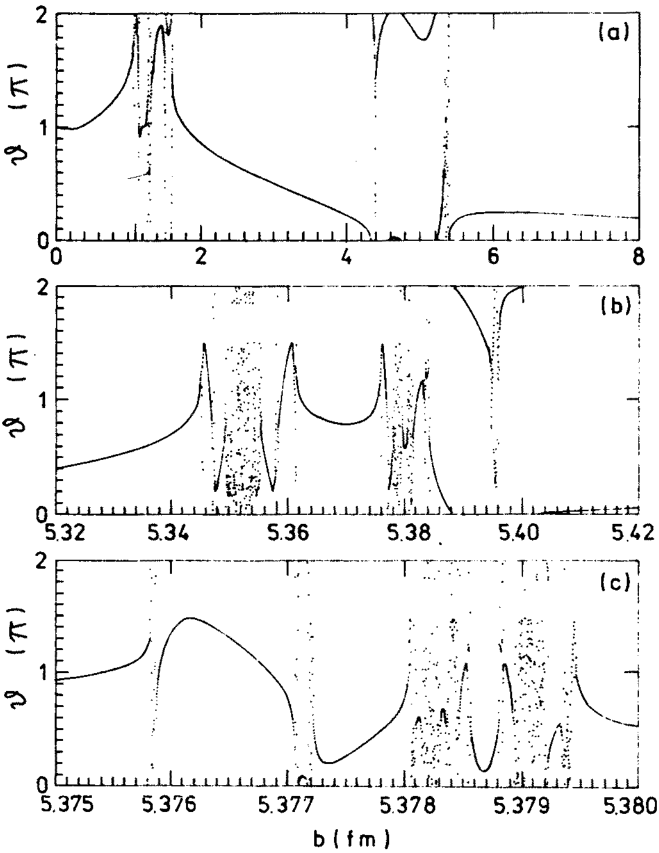

Fig. 3. The deflection function (scattering angle θ vs. impact parameter b) for $E = 3.5$ MeV presented with different impact parameter resolutions

(3). For this purpose we need the values of the dimension and the Lyapunov exponent. In order to calculate the information dimension $D^{(1)}$ we adopt the "uncertainty exponent" technique [16,17]. It has been invented for determination of the boundary of the basin of an attractor. The properties of the system are different on the both sides of that boundary. In our case, the crossing of boundary manifests itself by a rapid change of the deflection angle. The initial condition is called uncertain with uncertainty ϵ if it leads to the final angle in a different half-plane than the same condition but perturbed by an amount of ϵ. The fraction of trajectories uncertain in that sense $f(\epsilon)$ scales with ϵ like

$$f(\epsilon) \sim \epsilon^{1-D^{(1)}} \tag{6}$$

for sets with fractal structure. If the boundary of the set is regular, $f(\epsilon)$

is proportional to ϵ. Fig. 4 indicates that in our case relation (6) holds and $D^{(1)}$ can be precisely extracted. The information dimension calculated according to Eq. (6) for four energies is presented in Table I. As expected [18], it declines with energy.

TABLE I

The dimension $D^{(1)}$, Lyapunov exponents λ and mean lifetime τ for four energies.

E[MeV]	$D^{(1)}$	λ [$10^{22}\,s^{-1}$]	τ [$10^{-22}\,s$]
3.0	0.68	0.607	5.2
3.5	0.66	0.513	5.7
4.5	0.47	0.425	4.4
5.5	0.22	$\sim$ 0.56	$\sim$ 2.3

Our system possesses only one positive Lyapunov exponent connected with the unstable direction. To determine the exponent we need a long lived trajectory. The time evolution in the tangent space is governed by the set of linear equations

$$\frac{d}{dt}\begin{pmatrix}\delta x\\ \delta y\end{pmatrix} = \frac{1}{m}\begin{pmatrix}\delta p_x\\ \delta p_y\end{pmatrix},$$
$$\frac{d}{dt}\begin{pmatrix}\delta p_x\\ \delta p_y\end{pmatrix} = -\begin{pmatrix}\frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial x \partial y}\\ \frac{\partial^2}{\partial y \partial x} & \frac{\partial^2}{\partial y^2}\end{pmatrix} V(x,y)\begin{pmatrix}\delta x\\ \delta y\end{pmatrix}. \tag{7}$$

The arguments of potential $V(x,y)$ second derivatives follow the trajectory determined by the original equations of motion (5). The Lyapunov exponent is obtained as average eigenvalue of the evolution matrix of eq.(7) corresponding to the unstable direction. Fig. 5 shows the convergence of the exponent for the trajectory depicted in Fig. 2 indicating its precise determination. Looking for long trajectories at energies higher than 4.5 MeV is much more difficult since the dimension of stable manifolds becomes smaller. Table I summarizes the results. For $E = 5.5$ MeV the value of the exponent has been obtained from the trajectory living 10^{-20} s and is only approximate.

Finally, Table I contains the mean lifetime τ of the transients derived from equation (1). Well above the Coulomb barrier it declines sharply with the energy but for E below 3.5 MeV stabilizes. The influence of the Coulomb barrier eliminates all long trajectories below $E = 3$ MeV. The lifetime of the transient can be also directly determined by summing up the trajectories which still dwell within the interaction region after a given time. For this purpose, we have sampled the impact parameter interval (0, 5.4 fm) uniformly to get a statistical ensemble. Then we have calculated

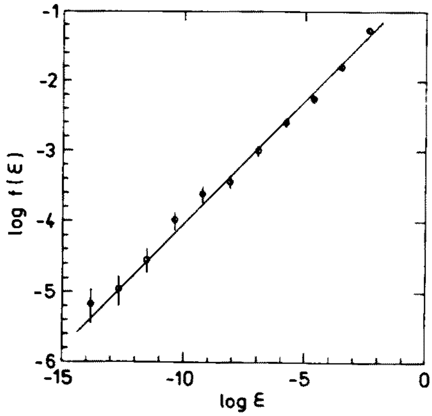

Fig. 4. The uncertainty exponent $f(\epsilon)$ for $E = 3.5$ MeV

the time after which trajectory leaves the region the transient is located and taken the average over the ensemble. The result of this experiment for $E = 3.5$ MeV is presented in Fig. 6. Due to the specific symmetry of the transient, some escaping directions (and, consequently, escaping times) are less probable then the others and the dependence of the number of surviving trajectories on time is not strictly exponential. The straight line with the slope determined from the Lyapunov exponent and the fractal dimension (taken from Table I) is also shown in the figure. The straight line agrees with the overall behaviour of the curve obtained from the direct calculations.

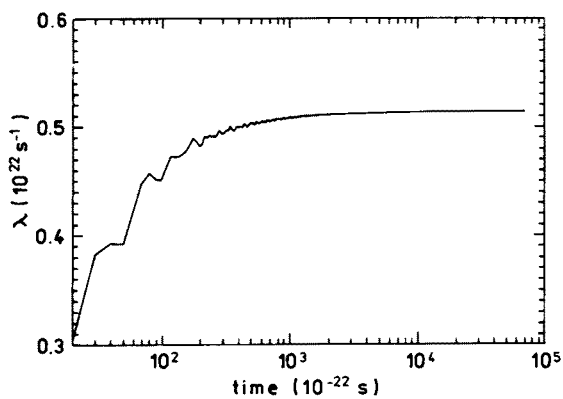

Fig. 5. The Lyapunov exponent calculated for the trajectory presented in Fig. 1

6. Summary and outlook

We have shown that the simple model of nuclear scattering system reveals chaotic behaviour. The motion is governed by the shape of the stable and unstable manifolds which can be characterized by fractal dimensions. The system possesses positive Lyapunov exponent. In contrast to two-body dynamics, trajectories do not leave the interaction region immediately but reveal transient behaviour. The number of trajectories abiding within the range of the nuclear potential drops exponentially with time. In the absence of transient this number declines faster than exponentially. The exponential law of decay resembles that of quantum mechanical resonance. All classical two-body models commonly used to describe nuclear collisions fail to produce such phenomenon. The transient can be interpreted in that sense as a single-particle state. Despite all simplifications, we have got lifetimes close to those observed experimentally as they correspond to the width of about 1–2 MeV, a typical value for the single-particle resonance [19]. Longer lifetimes, characteristic for a compound nucleus, are connected with a huge number of degrees of freedom. Since we have taken into account only two of them, one should not expect a formation and decay of the compound nucleus our present analysis is able to reveal.

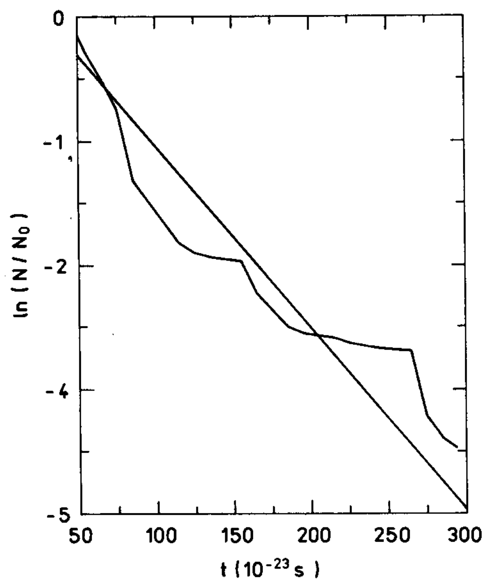

Fig. 6. Relative number of trajectories dwelling in the interaction region up to a given time

In order to convince ourselves that the exponential law holds for more general alpha-cluster systems and its lifetime rises with the number of degrees of freedom involved, we have considered a more realistic target. Now the alpha particles in the target have been enabled to move in the three-dimensional configuration space. The phase space is 24-dimensional — the energy, total momentum and angular momentum conserved. Calculations have been performed for a randomly chosen internal configuration of the target (with the binding energy E_B). We cannot repeat the whole analysis for such complicated system but the direct estimation of lifetime of a composite system is possible. Fig. 7 demonstrates this, analogously to Fig.6. The number of surviving trajectories declines exponentially, indeed. The average lifetime is $2.9 \cdot 10^{-20}$ s, 50 times larger than that for the simplified system. Further investigations of that general system are in progress.

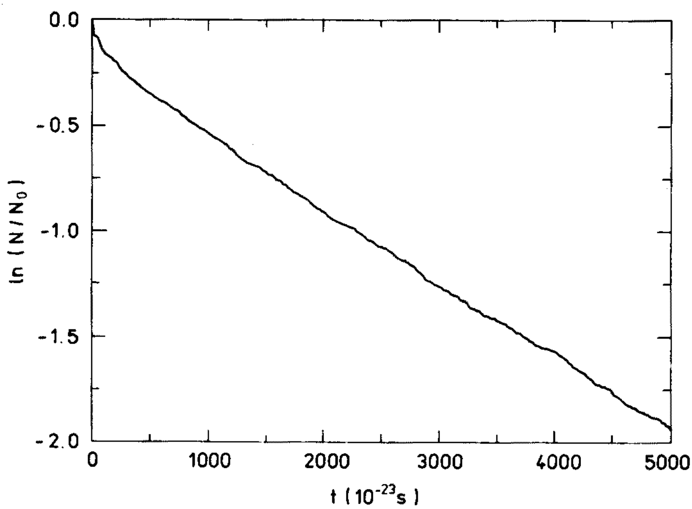

Fig. 7. Same as Fig. 6 but for a generalized system: α-particle scattered by ^{12}C nucleus built up of three interacting α clusters

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