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ON THE POSSIBILITY OF RESONANCES IN LONGITUDINALLY POLARIZED VECTOR BOSON SCATTERING

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Assuming that no Higgs has been found below 1 TeV, we study the physics of longitudinally polarized vector boson (W_L) scattering in the TeV region, using a one loop calculation and partial wave analysis. We show that the occurrence of a resonance in the isospin $I = 1$ channel depends on a certain parameter called β, which is measured near threshold. We investigate the similarity between low energy $\pi\pi$ scattering and high energy $W_L W_L$ scattering, as suggested by the equivalence theorem.

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1. Introduction

The physics up to energies of around the vector boson mass is very accurately described by the Standard Model Lagrangian. The Standard Model (SM) is currently being tested at the LEP to an accuracy of about 1% and so far, up to about 100 GeV, the SM predictions are in complete agreement with the experimentally obtained results [1].

Apart from the top quark, the only thing that remains to be verified experimentally is the existence of the Higgs particle. The SM contains the linear σ-model as the Higgs sector, which is needed to ensure the renormalizability of the theory, and consequently requires the Higgs particle. Through the Higgs mechanism the vector bosons and fermions acquire their masses, but at the same time we end up with a non-zero cosmological constant. In order to agree with experiment, which says that this constant is zero, one could, for example, introduce the same constant into the Lagrangian but with the opposite sign. This, however, does not seem to be a very satisfactory solution to the problem. Based on these considerations the Higgs sector may be considered suspect and there is thus the possibility that the Higgs particle is very heavy or it may not even exist.

The foregoing discussion motivates us to consider the SM (after all, its success cannot be disregarded!) but without the Higgs particle. This may be achieved by taking the large Higgs mass limit. Of course, at this point the SM has become non-renormalizable, meaning that we are now dealing with problems like perturbation theory breakdown and violation of tree unitarity. It is obvious that if the Higgs particle does not exist, new physics will have to take its place.

In principle, evidence of this new physics may already be obtained from low energy processes by considering the one-loop corrections due to a heavy Higgs. However, as it turns out, for processes which take place at a center of mass energy less then 100 GeV, the magnitude of these one-loop corrections are at best 0.5% [2–3] and are at this point not observable. This is a result of the screening theorem [3], which states that for any process the one-loop corrections due to a heavy Higgs depend at most logarithmically on the Higgs mass. We see now that below 100 GeV the SM in the large Higgs mass limit is a very good approximation to the new physics, since in this energy domain details concerning the Higgs system, or whatever goes for it, are yet to be observed.

On the other hand, we may look at high energy processes and determine from the tree unitarity limit where new physics has to come in, in the absence of the Higgs. An example of such a process is $W_L W_L$ scattering (W_L = longitudinally polarized neutral or charged vector boson); the tree amplitude grows like the energy squared and the unitarity limit is reached at around 1 TeV [4–6]. This value of 1 TeV should be considered only as an indication since perturbation theory is not valid. All we know is that somewhere around 1 TeV new physics will have to show up in processes involving the longitudinally polarized vector boson and processes involving the top quark, since the Yukawa coupling is not to be neglected.

Here we would like to remark that according to lattice theory non-perturbative results of the ϕ^4-theory indicate that the Higgs mass cannot be larger than 630 GeV, which is to be considered only as a rough estimate [7]. It is thus indeed acceptable to assume that if no Higgs has been found below 1 TeV, then new physics will show up at around 1 TeV.

The energy region between 100 GeV and 1 TeV proves to be very interesting; it is in this energy region that we may assume that the one-loop corrections due to a heavy Higgs, being of the order of 10% [6, 8-10], are still a reasonable approximation. The importance of this energy region is understood; it still makes sense to talk about one-loop corrections, which, of course, cannot be said about the energy region above 1 TeV. Furthermore, if we are able to measure one-loop effects below 1 TeV, then we may be able to predict what will happen above 1 TeV. As we will show, this is the case for $W_L W_L$ scattering; within the framework of partial wave analysis the prediction of a resonance in the $I = 1$ channel above 1 TeV depends on a certain parameter, called β. This parameter is derived from the one-loop amplitude below 1 TeV. Because one measures cross-sections and not amplitudes, we will show in Section 7 how β may be expressed in terms of a linear combination of the differential and total cross-sections for $W_L^\pm/W_L^0$ vector boson scattering.

In the study of W_L interactions the equivalence theorem [5, 11-13] proves to be a very useful tool. For $W_L W_L$ scattering, where the interacting W_L's have an energy E much larger than their mass M, the statement is

$$A(W_L W_L \to W_L W_L) = A(\phi\phi \to \phi\phi) + O\left(\tfrac{M}{E}\right). \tag{1.1}$$

Here ϕ is the corresponding Higgs ghost. Thus, the leading energy term for the W_L amplitude is found by calculating the leading energy term for the corresponding ϕ amplitude. The above equation is valid in all orders of perturbation theory and is independent of the Higgs mass or the top quark mass [13]. The equivalence theorem may thus also be applied to the case of the SM in the heavy Higgs mass limit.

It is interesting to observe that in the heavy Higgs mass limit the tree amplitude for $\phi\phi$ scattering is exactly like the tree amplitude for $\pi\pi$ scattering if we replace the v.e.v. $v = 250$ GeV by $F_\pi = 98$ MeV. This has led many people to consider for example technicolor [14] in the study of $W_L W_L$ scattering in the energy region above 1 TeV, assuming that no Higgs has been found below 1 TeV. In this model there is one very notable feature, namely the prediction of a resonance in the $I = 1$ channel at around 1.5–2 TeV [11, 15–17]. The reason is that for $\pi\pi$ scattering there exist a resonance in this channel, called the ρ resonance, at an energy of 0.77 GeV. Then technicolor will predict the techni-ρ with a mass of

$$m_{\rho_T} = m_\rho \cdot \left(\tfrac{v}{F_\pi}\right) = 2\,\mathrm{TeV}. \tag{1.2}$$

In Section 4 it will be shown that according to the partial wave analysis, the occurrence of such a resonance corresponds to

$$\beta = 5. \tag{1.3}$$

Besides technicolor, other models have been considered as well in the prediction of a resonance for W_LW_L scattering in the $I = 1$ channel at around 2 TeV [18–19].

It is maybe interesting to see what value for β we obtain if we consider the SM in the large Higgs mass limit. We know of 2 different ways that the large Higgs mass may be taken (to be defined in Section 2) and the results are

$$\beta_a = -0.32\,, \quad \beta_b = \tfrac{1}{3}\,. \tag{1.4}$$

We can now remark the following. The first value for $\beta(\beta_a)$ has the wrong sign and cannot give a resonance at any energy, while the second value for $\beta(\beta_b)$ is far too small to produce a resonance at around 2 TeV. Although according to the partial wave analysis a resonance at around 2 TeV cannot occur in either case, the result does seem to depend in which way that we take the large Higgs mass limit.

It is clear that details concerning the one-loop amplitude are important, but this seems to be contradictory to the analysis done by Brown and Goble [20]. Some 20 years ago they produced the ρ resonance through the use of the bootstrap analysis and knowing only the tree amplitude. Their results were found to be in reasonable agreement with the experimental values. The conclusion would be that the existence of a ρ-like resonance around 2 TeV is *guaranteed* and is, in fact, independent of the underlying new physics [16]. However, this conclusion is not correct, simply because the bootstrap analysis is not correct and the success obtained by Brown and Goble is thus misleading. This was first noted by Lehmann [21]. The analysis of Ref. [20] has recently been questioned again in the literature [17, 19, 22].

But then how can we explain the ρ-resonance? For low energy $\pi\pi$ scattering the effective Lagrangian contains πN interaction terms and when considering the amplitude at one loop this interaction needs to be taken into account. Diagrams containing a Nucleon loop do contribute. This is precisely the analysis that was considered in Ref. [21] and in there the ρ-resonance was obtained to within 30% accuracy ($\beta = 3.5$) with the experimental value ($\beta = 5$).

The upshot is thus the following. The prediction of a resonance depends on the details concerning the one-loop amplitude. For low energy $\pi\pi$ scattering the resonance can be explained by the πN interaction. For high energy W_LW_L scattering the SM Lagrangian does not contain such a heavy Nucleon contribution. From Eq. (1.4) we see that the manifestation of a ρ-like resonance at around 2 TeV becomes questionable if the physics below 1 TeV is described by the effective Lagrangian of the SM in the heavy Higgs mass limit.

This paper is organized as follows. In Section 2 we define in what ways the heavy Higgs mass limit may be taken. In Section 3 we give the tree amplitude for W_LW_L scattering and show the analogy with low energy $\pi\pi$ scattering. In Section 4 we review the physics of low energy $\pi\pi$ scattering. The analysis by Brown and Goble [20] is briefly discussed, followed by the analysis by Lehmann [21]. In Section 5 we go back to high energy W_LW_L scattering and derive the results as obtained from one-loop perturbation theory. In Section 6 we discuss the arbitrariness of the large Higgs mass limit. In Section 7 we derive β as a function of the cross-sections for $W_L^{\pm}/W_L^0$ vector boson scattering and finally Section 8 contains a summary and a discussion of the results. For simplicity we consider the simple SU(2) model, which is achieved by taking the weak mixing angle θ_w equal to zero in the SM Lagrangian. The Lagrangian is given in Appendix A. In Appendix B we give the one-loop calculation for $\phi\phi$ scattering (ϕ is the Higgs ghost) in the non-linear σ-model.

Our metric is such that $p^2 = -m^2$ for a particle on mass shell with mass m and momentum p.

2. The heavy Higgs mass limit

In this paper we are investigating the amplitude for W_LW_L scattering in the three isospin channels $I = 0, 1, 2$ according to the SU(2) model in absence of the Higgs particle.

The Higgs particle may be removed from the theory by taking the heavy Higgs mass limit, which may be done in the following two different ways;

(a) One may do a leading Higgs mass expansion. Thus, when calculating the amplitude for some process, only the leading Higgs mass terms are kept. We are still dealing with the linear σ-model.

(b) One may take the large Higgs mass limit in the tree level Lagrangian and then calculate the amplitude. We are now dealing with the non-linear σ-model.

The Lagrangian of the SU(2) model is given in Appendix A and we show briefly how the non-linear model is obtained.

At the tree level there is, of course, no difference between limit (a) and limit (b). Let us now consider the one-loop amplitude for W_LW_L scattering in the large Higgs mass limit. When the energy of the interacting W_L's is much larger than their mass, the ratio of the one-loop amplitude A^1 and the tree amplitude A^0 is proportional to the energy squared [6, 9–10, 13]. Thus

$$R(W_L) = \frac{A^1}{A^0} = \frac{g^2}{16\pi^2} \cdot \frac{p^2}{M^2} \cdot c \tag{2.1}$$

(for the moment we will not worry about isospin indices). Here M is the vector boson mass, p is a typical momentum with $M^2 \ll p^2$ and c is a dimensionless expression, found by explicit calculation.

When considering the linear model (limit(a)), the expression for c may contain $\ln(m^2)$ terms. We note that quadratic Higgs mass dependence shows up only at the two-loop level, in accordance with the screening theorem [3].

When considering the non-linear model (limit(b)), divergencies remain. In the dimensional regularization scheme these are the $1/(n-4)$ terms and they correspond to the $\ln(m^2)$ terms of the linear case.

3. The tree amplitude for $\mathrm{W_L W_L}$ scattering

The process $\mathrm{W_L W_L}$ scattering is displayed in Fig. 1. All the momenta are taken to be ingoing, thus $p_1 + p_2 + p_3 + p_4 = 0$. We have $p_1^2 = p_2^2 = p_3^2 = p_4^2 = -M^2$, where M is the vector boson mass.

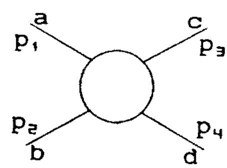


Fig. 1. The process $\mathrm{W_L^a W_L^b \to W_L^c W_L^d}$.

In the limit $M^2 \ll s,\ t,\ u\ (s,\ t$ and u to be defined in Eq. (3.2)) the tree amplitude is given by

$$A^0(\mathrm{W_L^a W_L^b \to W_L^c W_L^d}) = \frac{g^2}{4M^2} \cdot \left\{ \delta_{ab}\delta_{cd}\left(s + \frac{s^2}{-s+m^2}\right) \right.$$

$$\left. \delta_{ac}\delta bd\left(u + \frac{u^2}{-u+m^2}\right) + \delta_{ad}\delta_{bc}\left(t + \frac{t^2}{-t+m^2}\right)\right\} . \qquad (3.1)$$

Here $a, b, c, d = 1, 2, 3$ are the isospin indices and s, t and u are the Mandelstam variables. We have

$$s = -(p_1+p_2)^2 , \quad t = -(p_1+p_4)^2 , \quad u = -(p_1+p_3)^2 , \quad s+t+u = 4M^2 . \qquad (3.2)$$

In the large Higgs mass limit, thus for

$$M^2 \ll s, t, u \ll m^2 , \qquad (3.3)$$

we may ignore the terms containing the Higgs mass m in the denominators of Eq. (3.1) and we obtain

$$A^0(\mathrm{W_L^a W_L^b \to W_L^c W_L^d}) = \frac{g^2}{4M^2}\{\delta_{ab}\delta_{cd}s + \delta_{ac}\delta_{bd}u + \delta_{ad}\delta_{bc}t\} . \qquad (3.4)$$

Thus the tree amplitude grows like s, the energy squared, and the unitarity limit is reached at around 1 TeV [4–6]. Concerning the one-loop amplitude for $W_L W_L$ scattering we would at this point only like to remark the following: from Eq. (2.1) we see that since the tree amplitude grows like s, the one-loop amplitude grows like s^2.

Of course, we also could have used the equivalence theorem (see Eq. (1.1)) and derive the leading energy term by calculating the amplitude for $\phi\phi$ scattering (ϕ = Higgs ghost). The equivalence theorem is valid in the energy region of Eq. (3.3), since this theorem is also valid in the large Higgs mass limit [13], as long as the energy of the interacting vector bosons is much larger than their mass.

Consider now $\phi\phi$ scattering in the non-linear model at the tree level. The only term of the Lagrangian of Eq. (A.6) contributing to this order is given by

$$-\frac{1}{8}\left(\frac{g}{2M}\right)^2(\partial_\mu \phi^2)^2 \tag{3.5}$$

and the tree amplitude is again given by Eq. (3.4). We see that this is exactly like $\pi\pi$ scattering if we replace $v = 2M/g$ by F_π;

$$A^0(\pi^a \pi^b \to \pi^c \pi^d) = \frac{1}{F_\pi^2}\{\delta_{ab}\delta_{cd}s + \delta_{ac}\delta_{bd}u + \delta_{ad}\delta_{bc}t\}\,. \tag{3.6}$$

4. Low energy $\pi\pi$ scattering

In this Section we will review the physics of $\pi\pi$ scattering. We first discuss the partial wave analysis, needed to study the energy region where the ρ resonance is located. We then proceed by discussing the analysis considered by Brown and Goble [20]. Derived from the tree amplitude of Eq. (3.6) and the bootstrap method, they obtained the σ resonance in the $I = 0$ and the ρ resonance in the $I = 1$ channel to within 30% of the experimental values. Due to its success, this analysis, therefore, suggests that details concerning higher order interactions are not relevant. We then discuss the analysis of Ref. [21]. It is then shown that the analysis of Ref. [20] is not correct and that the prediction of the resonances do depend on higher order interactions. Indeed, as shown in Ref. [21] the resonances may be derived, by considering the πN interactions, to within 30% accuracy of the experimental values.

4.1 Partial wave analysis

In the $I = 1$ channel for $\pi\pi$ scattering it was found experimentally that there is a resonance, called the ρ resonance, located at an energy of

0.77 GeV, while the width is found to be 0.15 GeV. At an energy well below the ρ resonance, the physics of $\pi\pi$ scattering may be described by an effective Lagrangian, containing not only the non-linear σ-model but also πN interactions [21]. In order to consider the energy region up to about 1 GeV, which includes the ρ resonance, the $\pi\pi$ amplitude may be expanded into partial waves, which is unitarized. Thus

$$T(I) = 32\pi \sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)\cdot t_l^I(s)\,,$$
$$t_l^I(s) = (\cot\delta_l^I(s) - i)^{-1}\,. \tag{4.1}$$

Here I = isospin index. The isospin amplitudes $T(I)$ are defined as follows: Let the amplitude be given by (compare with Eq. (3.6))

$$A = \delta_{ab}\delta_{cd}F_1 + \delta_{ac}\delta_{bd}F_2 + \delta_{ad}\delta_{bc}F_3\,, \tag{4.2}$$

then

$$\begin{aligned} T(0) &= 3F_1 + F_2 + F_3 && I = 0 \text{ channel}\,,\\ T(1) &= F_3 - F_2 && I = 1 \text{ channel}\,,\\ T(2) &= F_2 + F_3 && I = 2 \text{ channel}\,. \end{aligned} \tag{4.3}$$

Next we need to derive the expressions for $t_l^I(s)$ of Eq. (4.1). This may be done by using the effective range approximation and $\cot\delta_l^I$ is expanded at $s = 0$. In the expansion of Eq. (4.1) we only need to include the $I = 0, 2$ S-waves ($l = 0$) and the $I = 1$ P-wave ($l = 1$). The expressions for the $t_l^I(s)$ are given by

$$t_l^I(s) = \frac{1}{2}\frac{1}{32\pi}\int_{-1}^{1} dx P_l(x)T(I)\,, \tag{4.4}$$

where $t_l^I = t_0^0,\, t_1^1,\, t_0^2$. The t_l^I's need to be determined to order s^2 and we may write

$$t_l^I(s) = A_l^I(s)\cdot s\{1 + B_l^I(s)\cdot s\}\,. \tag{4.5}$$

Here the $B_l^I(s)$ may depend at most logarithmically on s. Using Eq. (4.1), we obtain for $\cot\delta_l^I$

$$\cot\delta_l^I(s) = \frac{1}{A_l^I s} - \frac{B_l^I}{A_l^I}\,. \tag{4.6}$$

The resonance is located at $s = M^2$ where $\cot\delta_l^I$ vanishes;

$$\cot\delta_l^I(s = M^2) = 0\,. \tag{4.7}$$

The tree amplitude, given by Eq. (3.6), determines the terms linear in s and for A_l^I we find

$$A_0^0 = \frac{1}{16\pi F_\pi^2},$$
$$A_1^1 = \frac{1}{96\pi F_\pi^2},$$
$$A_0^2 = \frac{-1}{32\pi F_\pi^2}. \tag{4.8}$$

The $B_l^I(s)$ terms (thus the s^2 terms) may be obtained by calculating the one-loop amplitude. Then besides the direct $\pi\pi$ interactions (see Fig. 2a), πN interactions contribute as well (see Fig. 2b).

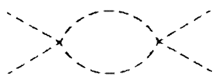


Fig. 2a. Pion loop contributing to the $\pi\pi$ one-loop amplitude.

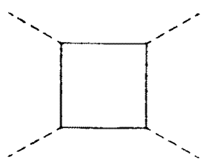


Fig. 2b. Nucleon loop contributing to the $\pi\pi$ one-loop amplitude.

4.2 The Brown-Goble analysis

Brown and Goble [20] derive the s^2 terms by considering the tree amplitude together with the bootstrap analysis and thus without any knowledge about the πN interactions. They obtained

$$t_l^I = A_l^I s \left\{1 - \frac{s}{M_I^2} + \frac{A_l^I s}{\pi} \cdot \left(\ln\left(\frac{s}{M_I^2}\right) - i\pi\right)\right\}^{-1}, \tag{4.9}$$

where the A_l^I, as derived from the tree amplitude, are given by Eq. (4.8). Thus three unknown parameters M_I have been introduced, such that the resonance is located at $s = M_I^2$. The values for M_I^2 were subsequently explicitly calculated by using subtracted forward dispersion relations. The obtained results were found to be in reasonable agreement with the experimental values for M_I^2;

$$M_0^2 = m_\sigma^2,$$
$$M_1^2 = m_\rho^2. \tag{4.10}$$

Furthermore, in the $I = 2$ channel as in agreement with experiment, they found no resonant value for M_2^2.

Expanded at $s = 0$, Eq. (4.9) reads

$$t_0^0 = \frac{s}{16\pi F_\pi^2}\left\{1 + \frac{s}{m_\sigma^2} - \frac{s}{(4\pi F_\pi)^2}\ln\frac{s}{m_\sigma^2}\right\},$$

$$t_1^1 = \frac{s}{96\pi F_\pi^2}\left\{1 + \frac{s}{m_\rho^2} - \frac{1}{6}\frac{s}{(4\pi F_\pi)^2}\ln\frac{s}{m_\rho^2}\right\},$$

$$t_0^2 = \frac{-s}{32\pi F_\pi^2}\left\{1 - \frac{s}{|m_2^2|} + \frac{1}{2}\frac{s}{(4\pi F_\pi)^2}\ln\frac{s}{|m_2^2|}\right\}. \tag{4.11}$$

Note that all the expressions for t_l^I contain a logarithmic dependence on s.

4.3 The Lehmann analysis

Let us now turn to the analysis as given in Ref. [21]. Through the use of dispersion relations the expression for the one-loop amplitude is derived up to two unknown parameters β_1 and β_2;

$$A(\pi^a\pi^b \to \pi^c\pi^d) = \delta_{ab}\delta_{cd}F(s,t,u) + \delta_{ac}\delta_{bd}F(u,t,s) + \delta_{ad}\delta_{bc}F(t,s,u), \tag{4.12}$$

where

$$F(s,t,u) = \frac{s}{F_\pi^2}$$

$$-\frac{1}{96\pi^2F_\pi^4}\{3s^2(\ln s - \beta_1) + t(t-u)(\ln t - \beta_2) + u(u-t)(\ln u - \beta_2)\}. \tag{4.13}$$

Note that two arbitrary parameters have been introduced. Only one subtraction is needed to make the occurring integral convergent, thereby introducing the first unknown parameter. A second finite subtraction has been made, thus introducing the second arbitrary parameter.

Using Eqs (4.2) and (4.3) we may derive the expressions for the isospin amplitudes $T^{(I)}$, where for example the $T^{(1)}$ amplitude is given by

$$T^{(1)} = \frac{(t-u)}{F_\pi^2} - \frac{(t-u)}{96\pi^2F_\pi^4}\{s\ln s + t\ln t + u\ln u - 3s\beta\}, \tag{4.14}$$

where

$$\beta = \beta_2 - \beta_1. \tag{4.15}$$

Using Eqs (4.4)–(4.6) we then proceed by deriving $\cot\delta_l^I(s)$. Expressed in terms of the unknown parameters α and β we have

$$\cot\delta_0^0(s) = \frac{16\pi F_\pi^2}{s} + \frac{25}{18\pi}\ln s - \frac{1}{36\pi}\left(\alpha + \frac{11}{3}\right), \tag{4.16}$$

$$\cot \delta_1^1(s) = \frac{96\pi F_\pi^2}{s} - \frac{3}{\pi}\left(\beta - \frac{1}{9}\right), \tag{4.17}$$

$$\cot \delta_0^2(s) = \frac{-32\pi F_\pi^2}{s} + \frac{20}{9\pi}\ln s - \frac{2}{9\pi}\left(\frac{\alpha}{5} + \frac{18\beta}{5} + \frac{1}{3}\right) - \frac{7}{18\pi}. \tag{4.18}$$

As before $\beta = \beta_2 - \beta_1$. Furthermore, $\alpha = 33\beta_1 + 17\beta_2$. The two unknown parameters β_1 and β_2, or equivalently α and β, may be fixed experimentally such that the resonance in the $I = 0$ channel is located at $s = m_\sigma^2$ and in the $I = 1$ channel at $s = m_\rho^2$. For example for the $I = 1$ channel we derive the following value for β;

$$\cot \delta_1^1(s = m_\rho^2) = 0 \Rightarrow \beta = \frac{32\pi^2 F_\pi^2}{m_\rho^2} + \frac{1}{9} \simeq 5. \tag{4.19}$$

We note that, after having fixed α and β in this way, no resonance can occur in the $I = 2$ channel at any value for s. Furthermore, from Eqs (4.16) and (4.17) we see that two parameters are indeed necessary in order to be able to reproduce both, the σ resonance and the ρ resonance. One parameter would have been insufficient.

It is now a straightforward matter to derive t_l^I, expressed in terms of m_σ and m_ρ;

$$t_0^0 = \frac{s}{16\pi F_\pi^2}\left\{1 + \frac{s}{m_\sigma^2} - \frac{25}{18}\frac{s}{(4\pi F_\pi)^2}\ln\frac{s}{m_\sigma^2}\right\},$$

$$t_1^1 = \frac{s}{96\pi F_\pi^2}\left\{1 + \frac{s}{m_\rho^2}\right\},$$

$$t_0^2 = \frac{-s}{32\pi F_\pi^2}\left\{1 - \frac{4}{5}\left(\frac{s}{m_\rho^2} + \frac{s}{m_\sigma^2}\right) - \frac{7}{36}\frac{s}{(4\pi F_\pi)^2} + \frac{10}{9}\frac{s}{(4\pi F_\pi)^2}\ln\frac{s}{m_\sigma^2}\right\} \tag{4.20}$$

When comparing Eq. (4.20) and Eq. (4.11) we see that they do not agree. For example for t_1^1 the $\ln s$ term appearing in Eq. (4.11) should not even be there! Recently, this has also been noted by Dobado, Herrero and Truong [19, 22]. We now realize that the experimental success of the analysis by Brown and Goble is no more that a coincidence.

Finally, to conclude the analysis of Ref. [21], the values for β_1 and β_2 were explicitly calculated by evaluating the one-loop amplitude for $\pi\pi$ scattering from perturbation theory. $\pi\pi$ and πN interactions need to be considered and diagrams of the type of Figs 2a and 2b were evaluated. The results obtained were in reasonable agreement with the experimental values for β_1 and β_2.

5. High energy $W_L W_L$ scattering

In this Section we discuss $W_L W_L$ scattering in the TeV region in the absence of a Higgs particle. We assume that the physics in the energy region below 1 TeV is described by the effective Lagrangian of the SU(2) model in the large Higgs mass limit, where the one-loop correction is still a reasonable approximation. In order to examine $W_L W_L$ scattering in the TeV region we consider the partial wave analysis, just as it was done in the previous Section for low energy $\pi\pi$ scattering. As it has been made clear from the analysis discussed in Section 4.3, we not only need to derive the tree amplitude but also the one-loop amplitude, both evaluated in the energy region

$$M^2 \ll s, t, u \ll m^2 . \tag{5.1}$$

In this energy region we may apply the equivalence theorem of Eq. (1.1), which is valid in all orders of perturbation theory. The leading energy term of the tree and one-loop amplitude for $W_L W_L$ scattering may thus be found by calculating the leading energy term of the tree and one-loop amplitude for $\phi\phi$ scattering.

Consider now the amplitude for $\phi\phi$ scattering as derived from the Lagrangian of Eq. (A.6), where the large Higgs mass limit has been taken in the tree level Lagrangian. This thus corresponds to the non-linear σ-model. The leading energy term of the tree amplitude has already been evaluated in Section 3 and is given by Eq. (3.4). In order to find the leading energy term of the one-loop amplitude, we only need to consider one diagram. This diagram is precisely the diagram shown in Fig. 2a if we replace F_π by v. While in low energy $\pi\pi$ scattering the πN interaction leads to the contribution of the diagram shown in Fig. 2b, such a diagram is absent in high energy $\phi\phi$ scattering. The SM Lagrangian does not contain such a heavy Nucleon contribution. The evaluation of the leading energy term for the one-loop amplitude, using the dimensional regularization scheme, is given in Appendix B. Together with the tree amplitude, the result may be written as

$$A(W_L^a W_L^b \to W_L^c W_L^d) = \delta_{ab}\delta_{cd}F(s,t,u) + \delta_{ac}\delta_{bd}F(u,t,s) + \delta_{ad}\delta_{bc}F(t,s,u) , \tag{5.2}$$

where

$$F(s,t,u) = \frac{s}{v^2}$$

$$-\frac{1}{96\pi^2 v^4}\{3s^2(\ln s - \beta_1) + t(t-u)(\ln t - \beta_2) + u(u-t)(\ln u - \beta_2)\} \tag{5.3}$$

and

$$\beta_1 = \Delta + \tfrac{11}{6} ,$$

$$\beta_2 = \Delta + \tfrac{13}{6}. \tag{5.4}$$

Here Δ is the pole term in dimensional regularization and is defined in Eq. (B.3). This divergent term is cancelled by adding a new term to the Lagrangian, which contains an arbitrary finite part. When comparing Eq. (5.3) to the result obtained in Section 4.3 (see Eq. (4.13)) we see that up to the two known parameters β_1 and β_2, there is complete agreement if we replace F_π by v, thereby confirming the analysis of Ref. [21]. Thus the expressions for $\cot\delta_l^I$ as given by Eqs (4.16)–(4.18) may be used here, except that we need to replace F_π by v and with β_1 and β_2 as given in Eq. (5.4).

Consider now the $I = 1$ channel. The $I = 1$ amplitude is given by (see Eq. (4.1)):

$$T(1) = 32\pi \sum_{l=1}^{\infty}(2l+1)P_l(\cos\theta)\cdot t_l^1(s),$$
$$t_l^1(s) = (\cot\delta_l^1(s) - i)^{-1}, \tag{5.5}$$

where the expression for $\cot\delta_1^1$ is given by

$$\cot\delta_1^1(s) = \frac{96\pi v^2}{s} - \frac{3}{\pi}\Big(\beta - \frac{1}{9}\Big). \tag{5.6}$$

As usual $\beta = \beta_2 - \beta_1$. In low energy $\pi\pi$ scattering the ρ resonance is located at around 0.77 GeV and corresponds to a value for β of 5. Therefore, a resonance located at

$$m = m_\rho \cdot \Big(\frac{v}{F_\pi}\Big) = 2\ \mathrm{TeV}, \tag{5.7}$$

also corresponds to $\beta = 5$ (see Eq. (4.19)).

Let us now consider the value for β as derived from the non-linear model. From Eq. (5.4) we derive

$$\beta = \tfrac{1}{3}. \tag{5.8}$$

We see that β does not depend on the pole term Δ and seems to be a well-defined quantity. This value for β is much too small to account for a resonance at around 2 TeV. From the discussion preceding Eq. (5.2) we, therefore, could argue that for low energy $\pi\pi$ scattering, the large value for β is due to the diagram containing the Nucleon loop (see Fig. 2b).

Concerning the $I = 0$ channel, the occurrence of a resonance depends on α which is given by

$$\alpha = 33\beta_1 + 17\beta_2 = 50\Delta + 292/3. \tag{5.9}$$

We conclude that due to the dependence of α on the pole term Δ, α is an arbitrary parameter and needs to be fixed by experiment.

The value for β may, of course, also be derived by considering the SU(2) model Lagrangian of Eq. (A.2), which contains the linear σ-model. In the evaluation of the one-loop amplitude for $W_L W_L$ scattering only the leading Higgs mass terms are kept. This calculation has in fact already been done by Veltman and Yndurain [9] and independently also by Dawson and Willenbrock [10] and Passarino [6]. The resulting amplitude is again given by Eq. (5.2), with $F(s,t,u)$ as in Eq. (5.3), however, the expressions for β_1 and β_2 are in this case

$$\beta_1 = \ln(m^2) + \frac{4}{3} + 9\left(\frac{\pi}{\sqrt{3}} - 2\right),$$
$$\beta_2 = \ln(m^2) - \frac{2}{3}. \tag{5.10}$$

Here β_1 and β_2 both contain the divergent term $\ln(m^2)$, which corresponds to Δ of the non-linear model. From Eq. (5.10) we derive for β

$$\beta = -2 - 9\left(\frac{\pi}{\sqrt{3}} - 2\right) = -0.32. \tag{5.11}$$

Just like in non-linear case, we find that β is independent of the divergent term. Observing that this value for β has an extra minus sign, we find that it can never produce a resonance at any energy. However, when comparing Eq. (5.11) to Eq. (5.8), we find that the numerical values for β are different and that thus the result seems to depend in which way the large Higgs mass limit is taken.

It is may be interesting to observe that in low energy $\pi\pi$ scattering a finite subtraction has been made, thereby introducing a second arbitrary parameter. Here we seem to have a similar situation; there is one arbitrary parameter corresponding to $\ln(m^2)$ in the linear model, or equivalently Δ in the non-linear model, but at the same time there is a finite arbitrariness. This would then correspond to the second arbitrary parameter. This immediately raises the following question: if we known of a third way that we can take the large Higgs mass limit, does this mean that we could obtain a value for β of say 5? In the next Section we investigate this problem somewhat further.

6. Arbitrariness of the large Higgs mass limit and the U-particle

The fact that when evaluating the amplitude for some process, the result depends on how the limit of a large Higgs mass has been taken is well-known. For example in the calculation of the two-loop correction to the ρ parameter (J. van der Bij, M. Veltman in Ref. [2]), the result obtained by keeping the leading quadratic Higgs mass terms differs from the result when calculating the quadratic divergencies as obtained from the non-linear model.

As we have seen in Section 5, this arbitrariness already shows up at the one-loop level for $W_L W_L$ scattering. Part of the problem is due to the self interaction of the Higgs particle. For example the diagram of Fig. 3 is responsible for the $\sqrt{3}$ term, which appears in the value for β as derived from the linear model. The non-linear model does not give such a term. In order to understand this result, we first derive this $\sqrt{3}$ term. We then consider the U particle [9], which may be interpreted as a tool to examine what the diagram of Fig. 3 precisely does. We find that the U particle explicitly demonstrates the arbitrariness of the large Higgs mass limit. In Section 6.3 we briefly discuss the arbitrariness of β as taken from the point of view of the non-linear model.

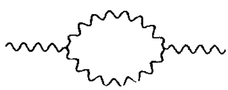


Fig. 3. Higgs self energy diagram.

6.1 Derivation of the $\sqrt{3}$ term in β from the linear model

The tree diagram for $W_L W_L$ scattering, containing the Higgs propagator, is proportional to $1/m^2$ and, therefore, vanishes in the large Higgs mass limit. However, at the one-loop level this is not the case. The diagram of Fig. 3 does contribute, since the Higgs tree point vertex is proportional to m^2 and thus cancels the $1/m^2$ term coming from the propagator.

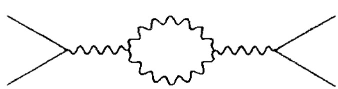


Fig. 4. One-loop Higgs exchange diagram.

The one-loop diagram is shown in Fig. 4 and the corresponding expression is given by

$$A^1 = \epsilon^\mu(p_1)\epsilon^\mu(p_2)\epsilon^\nu(p_3)\epsilon^\nu(p_4)\delta_{ab}\delta_{cd}\frac{g^4}{16\pi^2}\frac{9}{8}B_0(k,m,m)\,. \qquad (6.1)$$

Note that for the sake of the argument we consider only the $\delta_{ab}\delta_{cd}$ piece. The one-loop integral $B_0(k,m,m)$ is defined as

$$B_0(k,m,m) = \frac{1}{i\pi^2}\int d_n q \frac{1}{(q^2+m^2)\{(q+k)^2+m^2\}}, \tag{6.2}$$

with $k = p_1 + p_2$. In the limit $M^2 \ll s,t,u$, we may write for the polarization vector $\epsilon^\alpha(p_i)$ of the external longitudinally polarized vector boson

$$\epsilon^\alpha(p_i) \simeq \frac{p_i^\alpha}{M}. \tag{6.3}$$

We remark that although the above approximation is good enough here, this is not always the case [11, 13]. Substituting Eq. (6.3) into Eq. (6.1) we arrive at

$$A^1 = \frac{1}{96\pi^2 v^4}\delta_{ab}\delta_{cd}M^2 g^2 \cdot (3s^2) \cdot 9 B_0(k,m,m), \tag{6.4}$$

where $v = 2M/g$. We see that due to the fact that the longitudinal polarization vectors are proportional to their momenta in the limit of a large energy, the one-loop diagram of Fig. 4 contributes to the s^2 term.

We now need to take the large Higgs mass limit in the one-loop integral $B_0(k,m,m)$. The result of the non-linear model is obtained by taking the limit $m \to \infty$ *before* evaluating the integral of Eq. (6.2). The result is obviously zero and the diagram of Fig. 4 will not contribute in this case. The result of the linear model is obtained by taking the limit $m \to \infty$ *after* evaluating the integral, keeping only the leading Higgs mass terms. The result will not be zero. Of course, when we evaluate the contribution of the diagram of Fig. 4 in the linear model, we need to renormalize and introduce a counter term. This counter term may be fixed for example by requiring that the Higgs mass m is located at the pole of the propagator. This then amounts to the following contribution to the one-loop amplitude for $\mathrm{W_L W_L}$ scattering:

$$A^1_{\rm ren} = \frac{1}{96\pi^2 v^4}\delta_{ab}\delta_{cd} \cdot (3s^2) \cdot 9\left\{B_0(k,m,m) - B_0(k,m,m)|_{k^2=-m^2}\right\}. \tag{6.5}$$

Using dimensional regularization the result for B_0 in the limit $-k^2 \ll m^2$ is

$$B_0(k,m,m) = \Delta - \ln(m^2) + O\left(\frac{-k^2}{m^2}\right) \tag{6.6}$$

and evaluated at $k^2 = -m^2$ we find

$$B_0(k,m,m)|_{k^2=-m^2} = \Delta - \ln(m^2) - \left(\frac{\pi}{\sqrt{3}} - 2\right). \tag{6.7}$$

We see that the $\sqrt{3}$ term is derived from the counter term when we evaluate the one-loop integral at $k^2 = -m^2$. Substituting Eqs (6.6) and (6.7) into Eq. (6.5), we obtain for the renormalized one-loop amplitude

$$A^1_{\text{ren}} = \frac{1}{96\pi^2 v^4}\delta_{ab}\delta_{cd}(3s^2)\cdot 9\left(\frac{\pi}{\sqrt{3}} - 2\right). \tag{6.8}$$

Now compare this result to Eq. (5.3), then we find that the diagram of Fig. 4 gives the following contribution to β_1 and β_2:

$$\begin{aligned} \beta_1 &= 9\left(\frac{\pi}{\sqrt{3}} - 2\right) \\ \beta_2 &= 0 \end{aligned} \tag{6.9}$$

and the contribution to β is

$$\beta = \beta_2 - \beta_1 = -9\left(\frac{\pi}{\sqrt{3}} - 2\right). \tag{6.10}$$

6.2 The U-particle

By introducing the U-particle [9], we will now examine the Higgs self-energy in the large Higgs mass limit, taken according to the linear model. The U-particle is coupled to the Higgs with a strength m^2. Furthermore the U-particle is not coupled to the W. The corresponding Lagrangian is given by

$$\mathcal{L}(U) = -\tfrac{1}{2}(\partial_\mu U)^2 - \tfrac{1}{2}m_U^2 U^2 - g g_U \alpha M U^2 H - \tfrac{1}{4}g^2 g_U \alpha U^2(H^2 + \phi^2), \tag{6.11}$$

with $\alpha = m^2/4M^2$ and g_U is the parameter associated with the U-particle. Now if $m_U = m \to \infty$, then the U-particle becomes completely invisible and thus in principle the result for the one-loop amplitude for $W_L W_L$ scattering should remain unchanged.

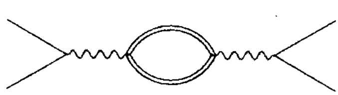


Fig. 5. Contribution of the U-particle to the one-loop amplitude.

It is very easy to see what the influence of the U-particle will be. At one-loop there is only one diagram. See Fig. 5. Following the analysis of Section 6.1, the corresponding expression can immediately be written down;

$$A^1_{\rm ren}(U) = g_U^2 \frac{1}{96\pi^2 v^4}(3s^2) \cdot \{B_0(k, m_U, m_U) - B_0(k, m_U, m_U)|_{k^2=-m^2}\} \,. \tag{6.12}$$

Let us now take the limit $m_U = m \to \infty$. With the help of Eqs (6.6) and (6.7) we find for the one-loop amplitude

$$A^1_{\rm ren}(U) = g_U^2 \frac{1}{96\pi^2 v^4}(3s^2) \cdot \left(\frac{\pi}{\sqrt{3}} - 2\right) . \tag{6.13}$$

We see that in the limit $m_U = m \to \infty$, a finite piece proportional to g_U^2 remains. This shows clearly the arbitrariness of the large Higgs limit, since we can take for g_U any value that we want. If we add the contribution of the U-particle to β of Eq. (5.10), we obtain

$$\beta(U) = -2 - (9 + g_U^2)\left(\frac{\pi}{\sqrt{3}} - 2\right) . \tag{6.14}$$

If we take $g_U^2 = 3.5$, then we have $\beta = 1/3$. If we take $g_U^2 = 28.6$, then $\beta = 5$.

Thus through the U-particle, we now have β as a function of a finite arbitrary parameter. Note that had the limit of a large Higgs mass been unique, then β would have been independent of g_U.

6.3 Arbitrariness of β and the non-linear model

Through the U-particle we have demonstrated in the previous section the arbitrariness of β as taken from the point of view of the linear model. If we now take the point of view of the non-linear model, then it may be very easy to see that β is indeed an unknown parameter.

The amplitude for $W_L W_L$ scattering may be written as follows (compare with Eq. (5.3))

$$A = c_0 p^2 + c_1 p^4[\Delta - \ln(p^2)] + c_2 p^4 + \ldots \tag{6.15}$$

Here p denotes a characteristic momentum and Δ is defined in Eq. (B.3), which corresponds to an unknown parameter to be fixed by experiment. For clarity, we have not written down the isospin indices explicitly. According to the studies of the non-linear model, it is well known that the coefficients c_0 and c_1 are uniquely determined, however, the coefficient c_2 is not [23]. The parameter β is precisely derived from the c_2 term and we must therefore conclude that β is not uniquely determined.

7. β as a function of the cross-sections

Consider once again the amplitude for the process $W_L^a W_L^b \to W_L^c W_L^d$, including the one-loop correction:

$$A(W_L^a W_L^b \to W_L^c W_L^d) = \delta_{ab}\delta_{cd}F(s,t,u) + \delta_{ac}\delta_{bd}F(u,t,s) + \delta_{ad}\delta_{bc}F(t,s,u), \tag{7.1}$$

where

$$F(s,t,u) = \frac{s}{v^2} - \frac{1}{96\pi^2 v^4}\{3s^2(\ln s - \beta_1) + t(t-u)(\ln t - \beta_2) + u(u-t)(\ln u - \beta_2)\}. \tag{7.2}$$

As explained in the previous chapters, the location of a resonance in the $I = 1$ channel in the TeV region depends on the value for $\beta = \beta_2 - \beta_1$, which may be measured below 1 TeV. For the isospin $I = 1$ amplitude of Eq. (5.5) we have plotted in Fig. 6 the absolute value of $t_1^1(s)$ as a function of the center of mass energy $\sqrt{s}$ for various values of β. Fig. 7, showing more clearly the sensitivity of the amplitude to β in the perturbative energy region, is the part of Fig. 6, where $\sqrt{s} < 1$ TeV.

In order to be able to measure β, we need to derive a relationship between β and cross sections. We first need to derive from Eq. (7.1) the amplitudes for the various processes involving the $W^\pm$ and W^0. We have

$$W^+ = \frac{1}{\sqrt{2}}(W^1 - iW^2), \qquad W^- = \frac{1}{\sqrt{2}}(W^1 + iW^2), \qquad W^0 = W^3. \tag{7.3}$$

For the amplitude A we so obtain

$$\begin{aligned}
A_1 &= A(W_L^+ W_L^- \to W_L^+ W_L^-) = F(s,t,u) + F(t,s,u)\,,\\
A_2 &= A(W_L^+ W_L^- \to W_L^0 W_L^0) = F(s,t,u)\,,\\
A_3 &= A(W_L^+ W_L^0 \to W_L^+ W_L^0) = F(u,t,s)\,,\\
& A(W_L^- W_L^0 \to W_L^- W_L^0) = F(u,t,s)\,,\\
A_4 &= A(W_L^+ W_L^+ \to W_L^+ W_L^+) = F(u,t,s) + F(t,s,u)\,,\\
& A(W_L^- W_L^- \to W_L^- W_L^-) = F(u,t,s) + F(t,s,u)\,,\\
& A(W_L^0 W_L^0 \to W_L^0 W_L^0) = F(s,t,u) + F(u,t,s) + F(t,s,u)\,.
\end{aligned} \tag{7.4}$$

The cross-section is defined by

$$\sigma_i = \int_{-1}^{1} d\cos\theta \frac{|A_i|^2}{32\pi s}\,. \tag{7.5}$$

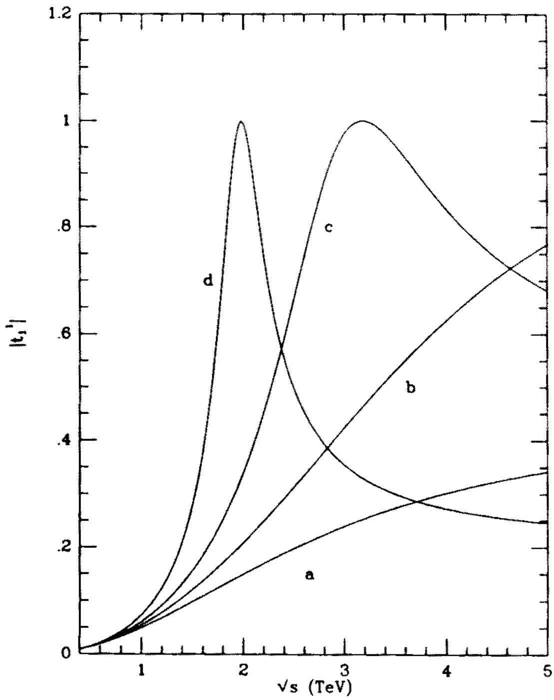


Fig. 6. Absolute value of $t_1^1(s)$ of Eq. (5.5) as a function of $\sqrt{s}$, the center of mass energy, for the following values of β: line a: $\beta = -2$; line b: $\beta = 0$; line c: $\beta = 2$; line d: $\beta = 5$.

A factor 1/2 needs to be included if we have identical particles in the final state. The Mandelstam variables u and t may be expressed in terms of the scattering angle θ as follows

$$t = -\frac{s}{2}(1 - \cos\theta),$$
$$u = -\frac{s}{2}(1 + \cos\theta). \qquad (7.6)$$

In order to determine β, the differential cross-section needs to be measured at two different values of the scattering angle. As an example consider the process $W_L^+W_L^- \to W_L^0W_L^0$. The differential cross-section is given by

$$\frac{d\sigma_2}{d\cos\theta} = C_1\left\{\frac{1}{2} - C_2\left(3(\ln s - \beta_1) + \frac{t(t-u)}{s^2}(\ln t - \beta_2) + \frac{u(u-t)}{s^2}(\ln u - \beta_2)\right)\right\}, \qquad (7.7)$$

where C_1 and C_2 are defined by

$$C_1 = \frac{s}{32\pi v^4},$$

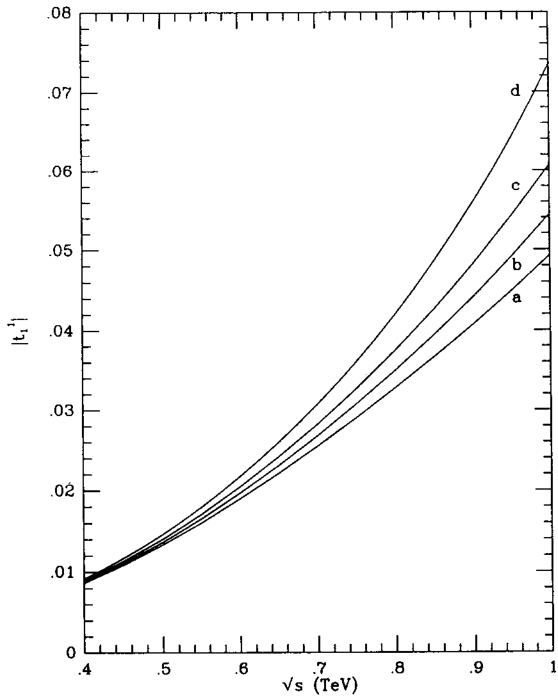


Fig. 7. Same as Fig. 6, but here $\sqrt{s} < 1$ TeV.

$$C_2 = \frac{s}{96\pi^2 v^2}\,. \tag{7.8}$$

If we take $\cos\theta = 1$ and $\cos\theta = 0$ as our two measuring points, then β is given by

$$\beta = \frac{1}{C_1 C_2}\left\{\frac{d\sigma_2(\cos\theta = 1)}{d\cos\theta} - \frac{4}{3}\,\frac{d\sigma_2(\cos\theta = 0)}{d\cos\theta}\right\} + \frac{1}{6C_2}\,. \tag{7.9}$$

Let us now turn to the total cross-sections. Performing the scattering angle integration, we find

$$\sigma_1 = C_1\left\{\frac{2}{3} + C_2\Big(\frac{7}{6} - 7(\ln s - \beta_1) - \frac{19}{3}(\ln s - \beta_2)\Big)\right\}, \tag{7.10}$$

$$\sigma_2 = C_1\left\{1 + C_2\Big(-\frac{1}{9} - 6(\ln s - \beta_1) - \frac{2}{3}(\ln s - \beta_2)\Big)\right\}, \tag{7.11}$$

$$\sigma_3 = C_1\left\{\frac{2}{3} + C_2\Big(-\frac{3}{2} + 3(\ln s - \beta_1) + \frac{11}{3}(\ln s - \beta_2)\Big)\right\}, \tag{7.12}$$

$$\sigma_4 = C_1\left\{1 + C_2\Big(-\frac{25}{9} + 4(\ln s - \beta_1) + \frac{28}{3}(\ln s - \beta_2)\Big)\right\}, \tag{7.13}$$

$$\sigma(\mathrm{W_L^0 W_L^0} \to \mathrm{W_L^0 W_L^0}) = 0\,. \tag{7.14}$$

To determine β, only two cross-sections need to be measured. For example from Eqs (7.12) and (7.13) we derive

$$\beta = \frac{1}{C_1 C_2}\left(\sigma_3 - \frac{1}{2}\sigma_4\right) - \frac{1}{6C_2} + \frac{1}{9}. \tag{7.15}$$

Let us now consider all four cross-sections. Due to the relation

$$\sigma_1 + \sigma_4 = \sigma_2 + \sigma_3 , \tag{7.16}$$

β may be expressed in terms of just three cross-sections. For example

$$\begin{aligned}\beta &= \frac{1}{13C_1C_2}(5\sigma_1 - 6\sigma_2 + 4\sigma_3) - \frac{1}{26} \\ &\simeq \frac{1}{13C_1C_2}(5\sigma_1 - 6\sigma_2 + 4\sigma_3).\end{aligned} \tag{7.17}$$

8. Concluding remarks

We have shown that within the framework of the partial wave analysis, the occurrence of a resonance in the $I = 1$ channel for $\mathrm{W_L W_L}$ scattering far above threshold depends on the value for β, which is measured near threshold. Similarly, the occurrence of a resonance in the $I = 0$ channel depends on the value for α. α and β are thus parameters of great phenomenological importance. When $\mathrm{W_L W_L}$ scattering is observed, it will also be at relatively low energy. If at this point the parameters α and β can be measured, then we are able to predict at what energy a resonance will occur (if at all) in each of these two isospin channels. If, for example, we measure that $\beta = 5$, then we know that a resonance will occur in the $I = 1$ channel at around 2 TeV.

Since these values for α and β are derived from the one-loop amplitude for $\mathrm{W_L W_L}$ scattering at low energy, the various models that are currently being considered may be tested by what they predict for α and β. In this paper we have considered the SM Lagrangian in the large Higgs mass limit. We found that in the $I = 0$ channel the parameter α depends on the divergent term $\ln(m^2)$ as derived from the linear model (or equivalently Δ as derived from the non-linear model). Therefore, although an experimental measurement of α may tell us what will happen at higher energies, it does not test our model. The situation concerning the $I = 1$ channel is far more interesting, since the parameter β is independent of the divergent term. We are thus able to calculate an actual number and may therefore, in principle, put a more stringent test on the model considered. However, as has been demonstrated by the U particle in the case of the SM Lagrangian in the large Higgs mass limit, β is an arbitrary parameter. Nevertheless, it is an interesting fact that of the two known standard ways that we know that we can take the large Higgs mass limit, no resonance will occur at around 2 TeV in either case. For example, from the non-linear model we derived, using dimensional regularization, that $\beta = 1/3$. This would correspond to a resonance at around 9 TeV!

What our analysis has shown, is that the occurrence of a resonance in the $I = 1$ channel at around or below 2 TeV is by no means guaranteed and that it only emphasizes the importance of an experimental measurement of β.

We wish to thank Prof. H. Lehmann for the many interesting discussions and for a critical reading of the manuscript.

APPENDIX A

The SU(2) Lagrangian

In this Appendix we give the expression for the SU(2) Lagrangian. We then show briefly how the non-linear model is obtained and state the resulting Lagrangian.

The simple SU(2) Lagrangian is obtained by taking the weak mixing angle θ_w equal to zero in the Standard Model Lagrangian and thus disregard the U(1) contribution. In the SU(2) model the Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\text{inv}} - \tfrac{1}{2}C^2 + \mathcal{L}_{\text{FP}}\,, \tag{A.1}$$

where $\mathcal{L}_{\text{inv}}$ is the part of the Lagrangian that is invariant for the gauge transformations for the fields. $\mathcal{L}_{\text{inv}}$ contains the linear σ-model as the Higgs sector. $-C^2/2$ is the gauge fixing term and $\mathcal{L}_{\text{FP}}$ is the corresponding Faddeev-Popov ghost Lagrangian. The explicit form of the invariant part of the Lagrangian is given by

$$\begin{aligned}\mathcal{L}_{\text{inv}} = {}& \tfrac{1}{4}G_a^{\mu\nu}G_a^{\mu\nu} - \tfrac{1}{2}M^2W^2 - \tfrac{1}{2}(\partial^\mu H)^2 - \tfrac{1}{2}m^2H^2 - \tfrac{1}{2}(\partial^\mu\phi_a)(D_a^\mu\phi) \\ & + \tfrac{1}{2}gW_a^\mu(H\partial^\mu\phi_a - \phi_a\partial^\mu H) - \tfrac{1}{8}g^2W^2(\phi^2 + H^2) - \tfrac{1}{2}gMW^2H \\ & - \tfrac{1}{2}\lambda vH(\phi^2 + H^2) - \frac{1}{8}\lambda(\phi^2 + H^2)^2 - M\phi_a\partial^\mu W_a^\mu \\ & - \gamma\left\{\frac{1}{2}(H^2 + \phi^2) + \frac{2M}{g}H\right\}.\end{aligned} \tag{A.2}$$

Here

$$\begin{aligned}W^2 &= W_a^\mu W_a^\mu\,, \quad \phi^2 = \phi^a\phi^a\,, \\ G_a^{\mu\nu} &= \partial^\mu W_a^\nu - \partial^\nu W_a^\mu + g\epsilon_{abc}W_b^\mu W_c^\nu\,, \\ D_a^\mu\phi &= \partial^\mu\phi_a + g\epsilon_{abc}W_b^\mu\phi_c\,.\end{aligned} \tag{A.3}$$

Furthermore, λ is the Higgs self coupling, $\lambda = m^2/v^2$. v is the vacuum expectation value (v.e.v.), with $M = gv/2$. γ is a constant and is fixed such that the total tadpole contribution is zero. In the lowest order $\gamma = 0$. The Lagrangian containing the non-linear σ-model is obtained by taking the large Higgs mass limit in the tree level Lagrangian of Eq. (A.2). The Higgs is removed by requiring that that part of the Lagrangian which is proportional to the Higgs mass m is zero. Thus

$$\phi^2 + H^2 + 2vH = 0\,. \tag{A.4}$$

For the expression of the Higgs field H we find

$$H = -v + v\sqrt{1 - \frac{\phi^2}{v^2}}\,. \tag{A.5}$$

Substituting this result into the Lagrangian of Eq. (A.2) we find

$$\mathcal{L}_{\rm inv} = \tfrac{1}{4}G^{\mu\nu}_a G^{\mu\nu}_a - \tfrac{1}{2}M^2W^2 - \tfrac{1}{2}(\partial^\mu\phi_a)(D^\mu_a\phi) - M\phi_a\partial^\mu W^\mu_a$$
$$-\frac{1}{8v^2}(\partial^\mu\phi^2)^2\left\{1 + \frac{\phi^2}{v^2} + \ldots\right\} + \frac{1}{v^2}MW^\mu_a(\phi_a\partial^\mu\phi^2 - \phi^2\partial^\mu\phi_a + \ldots)\,. \tag{A.6}$$

APPENDIX B

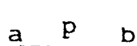
One-loop calculation for $\phi\phi$ scattering in the non-linear σ-model

Here we will derive the one-loop amplitude for $\phi\phi$ scattering in the $I = 1$ channel, using the dimensional regularization scheme. The reason for giving the calculation is that only one diagram needs to be evaluated to find the leading energy terms (terms of order s^2). The corresponding one-loop

Fig. 8. $\phi\phi$ one-loop amplitude in the non-linear model.

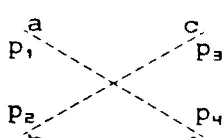
diagram is given in Fig. 8. The Feynman rules (needed to derive the expression for the diagram) are found from the Lagrangian of Eq. (A.6). They are

ϕ-propagator:



$$\frac{\delta_{ab}}{p^2 + M^2}$$

ϕ-four vertex:



$$-\frac{1}{v^2}\{\delta_{ab}\delta_{cd}(p_1 + p_2)^2 + \delta_{ac}\delta_{bd}(p_1 + p_3)^2 + \delta_{ad}\delta_{bc}(p_2 + p_3)^2\}$$

For the one-loop amplitude A we have (5.2)

$$A = \delta_{ab}\delta_{cd}A(s,t,u) + \delta_{ac}\delta_{bd}A(u,t,s) + \delta_{ad}\delta_{bc}A(t,s,u)\,,$$

where

$$\begin{aligned}A = {} & \frac{1}{(2\pi)^4 i}\,\frac{1}{2v^4}\int d_n q \frac{1}{(q^2+M^2)\{(q+p_1+p_2)^2+M^2\}} \\ & \times \{\delta_{ab}\delta_{cd}\{3s^2 - s(q-p_3)^2 - s(q-p_4)^2\} \\ & + \delta_{ac}\delta_{bd}\{(p_1+q)^2(q-p_3)^2 + (q+p_2)^2(q-p_4)^2\} \\ & + \delta_{ad}\delta_{bc}\{(q+p_1)^2(q-p_4)^2 + (q+p_2)^2(q-p_3)^2\}\} \\ & + (b \leftrightarrow c\,,\ p_2 \leftrightarrow p_3) + (d \leftrightarrow b\,,\ p_4 \leftrightarrow p_2)\,. \end{aligned} \tag{B.1}$$

In the dimensional regularization scheme, we have, for example, for the scalar two-point function in the limit of small vector boson mass M

$$\begin{aligned} B_0(p,M,M) &= \int d_n q \frac{1}{(q^2+M^2)\{(q+p)^2+M^2\}} \\ &= i\pi^2\{\Delta - \ln(-p^2) + 2\}\,, \end{aligned} \tag{B.2}$$

where

$$\Delta = -\frac{2}{n-4}\,. \tag{B.3}$$

It is now a straightforward matter to evaluate the expression for the amplitude A, keeping only the terms of the order s^2 and neglecting the vector boson mass M. Consider the $\delta_{ab}\delta_{cd}$ piece;

$$\begin{aligned} A(s,t,u) = {} & -\frac{1}{96\pi^2 v^4}\Big\{-4\Delta(u^2+ut+t^2) - \frac{20}{3}ut - \frac{23}{3}(u^2+t^2) \\ & + 3s^2\ln s + t(t-u)\ln t + u(u-t)\ln u\Big\}\,. \end{aligned} \tag{B.4}$$

As usual s, t and u are the Mandelstam variables. We have

$$s = -(p_1+p_2)^2\,,\ u = -(p_1+p_3)^2\,,\ t = -(p_1+p_4)^2 \tag{B.5}$$

and the one-loop amplitude is evaluated in the energy domain

$$M^2 \ll s,t,u\,.$$

Compare now expression (B.4) with Eq. (5.3);

$$A(s,t,u) = -\frac{1}{96\pi^2 v^4}\cdot\{3s^2(\ln s - \beta_1) + t(t-u)(\ln t - \beta_2) + u(u-t)(\ln u - \beta_2)\}\,. \tag{B.6}$$

We then find the following expressions for β_1 and β_2:

$$\begin{aligned} \beta_1 &= \Delta + \frac{11}{6} \\ \beta_2 &= \Delta + \frac{13}{6}, \end{aligned} \tag{B.7}$$

and for β, the difference of β_1 and β_2, we find

$$\beta = \frac{1}{3}. \tag{B.8}$$

The amplitude $T(1)$ in the $I = 1$ channel is given by the $\delta_{ad}\delta_{bc} - \delta_{ac}\delta_{bd}$ piece given by Eq. (4.14);

$$\begin{aligned} T(1) &= A(t,s,u) - A(u,t,s) \\ &= -\frac{t-u}{96\pi^2 v^4}\{s\ln s + t\ln t + u\ln u - 3s\beta\}. \end{aligned} \tag{B.9}$$

If we had evaluated the one-loop amplitude for $\phi\phi$ scattering using the Standard Model Lagrangian, keeping only the leading Higgs mass terms, we would have obtained

$$\begin{aligned} \beta_1 &= \ln(m^2) + \frac{4}{3} + 9\Big(\frac{\pi}{\sqrt{3}} - 2\Big), \\ \beta_2 &= \ln(m^2) - \frac{2}{3}, \end{aligned} \tag{B.10}$$

with

$$\beta = -2 - 9\Big(\frac{\pi}{\sqrt{3}} - 2\Big).$$

Thus the $\ln(m^2)$ terms in the linear model correspond to the Δ terms in the non-linear model. However, the remainder of the leading energy terms do not agree.

Note added in proof: After completion of this work, we received a paper by R.S. Willey (PITT-91-06), that covers much of the same subject. In particular, in there it is also emphasized that the ρ resonance is related to the Nucleon contribution.

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