THERMODYNAMIC PROPERTIES OF OPEN NONCRITICAL STRING IN EXTERNAL ELECTROMAGNETIC FIELD

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We investigate the thermodynamics of open noncritical string (charged and neutral) in an external constant magnetic field. The free energy and Hagedorn temperature are calculated. It is shown that Hagedorn temperature is the same as in the absence of constant magnetic field. We present also the expressions for the free energy and Hagedorn temperature of the neutral open noncritical string in an external constant electromagnetic field. In this case Hagedorn temperature depends on the external electric field.

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1. Introduction

At present, the popular point of view is that we do not understand string theory (for review see [1]) at very small distances where the non-point string structure should be important. It is quite possible that the novel description of strings is necessary at distances comparable with fundamental string length.

If this point is correct then it is natural to expect the manifestations of new string physics at extremal conditions. That is why, recently the attempts to study the strings in strong gauge fields [2–5], high energy strings scattering [6] high and string behaviour at high temperature and high energy density [7–10] have been presented. Some interesting results have been obtained in Refs [2–10].

In particular, it has been found that open string is stable in a constant magnetic field and is unstable in a constant electric field. Moreover, there are two types of instability for open string in a constant electric field. One instability consists of string pair production and has the analogue in field theory. The other instability has no analogue in field theory. It arises when background field becomes larger than a critical value, E_c. In the quantum field theory this is manifested by the unboundedness of mass operator from below and above [2].

The study of string thermodynamics [7–10] has explained the role of the Hagedorn temperature as critical temperature of the system.

In Refs [3,5] the investigation of the open string thermodynamics in an external electromagnetic field has been started. In particular, it has been shown that the Hagedorn temperature depends on the external electric field.

2. Open string in a constant magnetic field

Let us give some details of derivation of mass operator for the critical open string in a constant magnetic field. We follow paper [3].

The action of the open critical string in external gauge field is given as

$$\begin{aligned} S &= S_0 + S_1 , \\ S_0 &= -\frac{1}{4\pi\alpha'} \int d^2\sigma \partial_a X^\mu \partial^a X_\mu , \\ S_1 &= -\frac{g_0}{2\pi\alpha'} \int d\tau A_\mu(X)\delta(\sigma) X^\mu - \frac{g_\pi}{2\pi\alpha'} \int d\tau A_\mu(X)\delta(\sigma-\pi)\dot{X}^\mu . \end{aligned} \tag{2.1}$$

Here A_μ is an external gauge field, g_0 and g_π are coupling constants at each end of a string. We consider two cases: neutral and charged string. For neutral string $g_0 + g_\pi = 0$.

We decompose the string field into the classical X_μ and quantum part ξ_μ and take (as in [2–5]) $F_{\mu\nu}$ as

$$F_{\mu\nu} = \begin{bmatrix} 0 & -f_0 & & & \\ f_0 & 0 & & & \\ & & 0 & f_1 & \\ & & f_1 & 0 & \\ & & & & \ddots \end{bmatrix} , \tag{2.2}$$

where f_i are real (for details, see [3]). Now one can obtain the following equations for quantum fields [3].

$$\frac{\partial}{\partial\sigma}\xi^{\pm}_m \pm i g_0 f_m \frac{\partial}{\partial\tau}\xi^{\pm}_m = 0 \quad \text{for } \sigma = 0\,, \tag{2.3}$$

$$\frac{\partial}{\partial\sigma}\xi^{\pm}_m \mp i g_\pi f_m \frac{\partial}{\partial\tau}\xi^{\pm}_m = 0 \quad \text{for } \sigma = \pi\,. \tag{2.4}$$

Here $\xi^{\pm}_m = \frac{1}{\sqrt{2}}(\xi_j \pm i\xi_{j+1})$ for $j = 2m$, m is a positive integer, and $\xi_{0\pm} = \frac{1}{\sqrt{2}}(\xi_0 \pm i\xi_1)$ for 0-th and 1-st components. We chose $f_0 = 0$, *i.e.*, we consider the system in a constant magnetic field where the system is stable.

The solutions of (2.3), (2.4) are [3] (we omit the subscript m)

$$\xi^+ = \xi^+_0 + i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left[a_n\psi_n(\tau,\sigma) - b^\dagger_n\psi_{-n}(\tau,\sigma)\right] \tag{2.5}$$

for charged string and

$$\xi^+ = \xi^+_0 + 2\alpha'\frac{\left[\tau - g_0 f(\sigma - \frac{\pi}{2})\right]p^-}{1 + g_0^2 f^2} + i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}} \times \left[a_n\psi_n(\tau,\sigma) - b^\dagger_n\psi_{-n}(\tau,\sigma)\right] \tag{2.6}$$

for neutral string. ξ^- is complex conjugate of (2.5), (2.6). Here ψ_n and ψ_{-n} are the solutions of (2.3), (2.4) and definition of τ independent inner product is

$$(\psi_m, \psi_n) = \frac{1}{\pi}\int_0^\pi d\sigma\left[i\psi^*_m \overleftrightarrow{\partial}_\tau \psi_n + g_0 f\psi^*_m\psi_n\delta(\sigma) + g_\pi f\psi^*_m\psi_n\delta(\sigma - \pi)\right],$$

$$\psi^*_m \overleftrightarrow{\partial}_\tau \psi_n = \psi^*_m\partial_\tau\psi_n - \partial_\tau\psi^*_m\psi_n\,. \tag{2.7}$$

Using (2.5)-(2.7) we get mass operator for charged string [3]

$$\alpha' m^2 = \sum_{i=1}^{24-2d}\sum_{n=1}^{\infty} nC^{i\dagger}_n C^i_n + \sum_{i=1}^{d}\Big[\epsilon_i b^{i\dagger}_0 b^i_0 + \frac{1}{2}\epsilon_i(1-\epsilon_i) + \sum_{n=1}^{\infty}(n-\epsilon_i)(a^{i\dagger}_n a^i_n + b^{i\dagger}_n b_n)\Big] - 1\,, \tag{2.8}$$

and for neutral string [3]

$$\alpha' m^2 = \sum_{i=1}^{24-2d} \sum_{n=1}^{\infty} n C_n^{i\dagger} C_n^i - \alpha' \sum_{j=1}^{2d} \frac{p_j^2 g_j^2 f_j^2}{1+g_j^2 f_j^2} + \sum_{i=1}^{d} \sum_{n=1}^{\infty} n(a_n^{i\dagger} a_n^i + b_n^{i\dagger} b_n^i) - 1 . \tag{2.9}$$

Here $\tan\gamma_{01} = g_0 f_i$, $\tan\gamma_{\pi i} = -g_\pi f_i$, $\epsilon_i = \gamma_{01} + \gamma_{\pi i}$, d is the number of nonzero 2×2 matrices in stress tensor $F_{\mu\nu}$, C and C^+ are creation and annihilation operators in the absence of external field and g_j is coupling constant for external field in j-th dimension.

Now we consider (in frames of the canonical ensemble) the open non-interacting critical string gas in external magnetic field. The free energy of such a gas in one-loop approximation is given by

$$F(\beta) = \frac{1}{\beta} \mathrm{tr}\, \ln\left(1 - e^{-\beta\omega}\right), \tag{2.10}$$

where tr includes the momentum integration and trace over string states, β is inverse temperature, and

$$\omega^2 = p_1^2 + \sum_{i=1}^{24-2d} \left[p_i^2 + \frac{1}{\alpha'} \sum_{n=1}^{\infty} n C_n^{i\dagger} C_n^i \right] - \frac{1}{\alpha'} + \frac{1}{\alpha'} \sum_{i=1}^{d} \left[\epsilon_i b_0^{i\dagger} b_0^i + \frac{1}{2}\epsilon_i (1-\epsilon_i) + \sum_{n=1}^{\infty} (n-\epsilon_i)(a_n^{i\dagger} a_n^i + b_n^{i\dagger} b_n^i) \right] \tag{2.11}$$

for charged string,

$$\omega^2 = p_1^2 + \sum_{i=1}^{24-2d} \left[p_i^2 + \frac{1}{\alpha'} \sum_{n=1}^{\infty} n C_n^{i\dagger} C_n^i \right] + \sum_{j=1}^{2d} \frac{p_j^2}{1+g_j^2 f_j^2} - \frac{1}{\alpha'} + \frac{1}{\alpha'} \sum_{i=1}^{d} \sum_{n=1}^{\infty} n\left(a_n^{i\dagger} a_n^i + b_n^{i\dagger} b_n^i\right) \tag{2.12}$$

for neutral string.

It is easy to rewrite (2.10) as the following

$$F(\beta) = -\int_0^\infty \frac{d\tau}{\tau} (2\pi\tau)^{-\frac{1}{2}} \mathrm{tr} \exp\left(-\frac{\omega^2 \tau}{2}\right) \sum_{r=1}^{\infty} \exp\left(-\frac{r^2\beta^2}{2\tau}\right). \tag{2.13}$$

Using the definition of θ_3-function

$$\sum_{n=-\infty}^{\infty} q^{n^2} \equiv \theta_3(0,q),$$

taking the trace over string states and calculating the momentum integrals we get:

a. Charged string

$$F(\beta) = -\int_0^\infty \frac{d\tau}{2\tau}(2\pi\tau)^{-(26-2d)/2}\left[\theta_3\left(0, \exp\left(-\frac{\beta^2}{2\tau}\right)\right) - 1\right] I(\tau), \quad (2.14)$$

where

$$I(\tau) = I_1(\tau) I_2(\tau) I_3(\tau),$$

$$I_1(\tau) = \exp\left(\frac{\tau}{2\alpha'}\right) \prod_{i=1}^{d} \frac{\exp\left(-\frac{1}{4\alpha'}\epsilon_i(1-\epsilon_i)\tau\right)}{1-\exp\left(\frac{-\epsilon_i\tau}{2\alpha'}\right)}, \quad (2.15)$$

$$I_2(\tau) = \prod_{n=1}^{\infty}\left(1 - \exp\left(\frac{-\tau n}{2\alpha'}\right)\right)^{2d-24}, \quad (2.16)$$

$$I_3(\tau) = \prod_{i=1}^{d}\prod_{n=1}^{\infty}\left(1 - \exp\left(\frac{-\tau(n-\epsilon_i)}{2\alpha'}\right)\right)^2. \quad (2.17)$$

b. Neutral string

$$F(\beta) = -\left[\prod_{j=1}^{2d}(1+g_j^2 f_j^2)^{\frac{1}{2}}\right] \times \int_0^\infty \frac{d\tau}{2\tau}(2\pi\tau)^{-13}\eta^{-24}\left[\frac{i\tau}{4\pi\alpha'}\right]\left[\theta_3\left(0, \exp\left(\frac{-\beta^2}{2\tau}\right)\right) - 1\right], \quad (2.18)$$

where $\eta(\tau) = e^{(i\pi\tau)/12} \prod_{n=1}^{\infty} \left(1 - e^{2\pi i n\tau}\right)$. Let us note that expression (2.14) is given for the first time, (2.18) has been obtained earlier in [5].

It is interesting to find the asymptotic of the free energy when $\tau \to 0$. This gives the possibility to define the Hagedorn temperature. We have

$$I(\tau) \xrightarrow[\tau\to 0]{} \frac{2\alpha'}{\tau\prod_{i=1}^{d}\epsilon_i} \exp\left(\frac{8\pi^2\alpha'}{\tau}\right) \exp\left(-2\left(1+e^2-\frac{\pi^2}{6}\right)\sum_{i=1}^{d}\right)\epsilon_i,$$

$$\theta_3\left(0, \exp\left(\frac{-\beta^2}{2\tau}\right)\right) - 1 \xrightarrow[\tau\to 0]{} 2\exp\left(\frac{-\beta^2}{2\tau}\right).$$

As a result we get the following expression for the Hagedorn temperature β_c of charged string

$$\beta_c = 4\pi\sqrt{\alpha'}.$$

If $\beta > \beta_c$ the integrand of (2.14) is finite when $\tau \to 0$.

In the same way, we get for neutral string

$$\eta^{-24}\left[\frac{i\tau}{4\pi\alpha'}\right] \xrightarrow[\tau\to 0]{} \left(\frac{\tau}{4\pi\alpha'}\right)^{12} \exp\left(\frac{8\pi^2\alpha'}{\tau}\right)$$

and $\beta_c = 4\pi\sqrt{\alpha'}$.

Thus, the critical temperature β_c of open string in a constant magnetic field does not depend on magnetic field.

3. Noncritical open string in a constant magnetic field

Let us consider the non-interacting open noncritical string gas in an external magnetic field. The open-loop free energy can be written as follows (see [12]):

$$F(\beta) = \frac{1}{\beta L}\,\mathrm{Tr}\ln(1 - \exp(-\beta\omega)), \tag{3.1}$$

where for charged string

$$\omega^2 = \sum_{i=1}^{D-1-2d}\left[p_i^2 + \frac{1}{\alpha'}\sum_{n=1}^{\infty} nC_n^{i\dagger}C_n^i\right] + \frac{1}{\alpha'}\sum_{i=1}^{d}\left\{\epsilon_i b_0^{i\dagger}b_0^i + \frac{1}{2}\epsilon_i(1-\epsilon_i)\right.$$
$$\left. + \sum_{n=1}^{\infty}(n-\epsilon_i)(a_n^{i\dagger}a_n^i + b_n^{i\dagger}b_n^i)\right\} - \frac{D-1}{24\alpha'} + \frac{d_0^2}{2\alpha'} \tag{3.2}$$

and for neutral string

$$\omega^2 = \sum_{i=1}^{D-1-2d}\left[p_i^2 + \frac{1}{\alpha'}\sum_{n=1}^{\infty} nC_n^{i\dagger}C_n^i\right] + \sum_{j=1}^{2d}\frac{p_j^2}{1+g_j^2 f_j^2}$$
$$+ \frac{1}{\alpha'}\sum_{i=1}^{d}\sum_{n=1}^{\infty} n(a_n^{i\dagger}a_n^i + b_n^{i\dagger}b_n^i) - \frac{D-1}{24\alpha'} + \frac{d_0^2}{2\alpha'} \tag{3.3}$$

the Weyl mode is compactified in a box with a period length L: $d_0 = \frac{\sqrt{2\pi}k}{L}$, $k \in Z$, Tr contains the momentum integration, the discrete sum over the Weyl mode and the sum over the string states.

As in Section 2 we get for charged string

$$F(\beta) = -\int_0^\infty \frac{d\tau}{2\tau}(2\pi\tau)^{-(D-2d)/2} I(D,\tau)\left[\theta_3\left(0, \exp\left(-\frac{\beta^2}{2\tau}\right)\right) - 1\right]$$
$$\times \frac{1}{L}\sum_{k=-\infty}^{\infty} \exp\left(-\frac{\pi k^2 \tau}{2L^2\alpha'}\right), \tag{3.4}$$

where $I(D,\tau) = I_1(D,\tau)I_2(D,\tau)I_3(D,\tau)$

$$I_1(D,\tau) = \exp\left(\frac{\tau(D-1)}{48\alpha'}\right)\prod_{i=1}^{d} \frac{\exp\left(-\frac{1}{4\alpha'}\epsilon_i(1-\epsilon_i)\tau\right)}{1-\exp\left(-\frac{\epsilon_i\tau}{2\alpha'}\right)} \tag{3.5}$$

$$I_2(D,\tau) = \prod_{n=1}^{\infty}\left(1-\exp\left(-\frac{\tau n}{2\alpha'}\right)\right)^{2d-D+1} \tag{3.6}$$

$$I_3(D,\tau) = \prod_{i=1}^{d}\prod_{n=1}^{\infty}\left(1-\exp\left(-\frac{\tau}{2\alpha'}(n-\epsilon_i)\right)\right)^{-2} \tag{3.7}$$

For neutral string

$$F(\beta) = -\left[\prod_{j=1}^{2d}(1+g_j^2 f_j^2)^{\frac{1}{2}}\right]\int_0^\infty \frac{d\tau}{2\tau}(2\pi\tau)^{-\frac{D}{2}}\left[\theta_3\left(0,\exp\left(-\frac{\beta^2}{2\tau}\right)\right)-1\right]$$
$$\times\, \eta^{-(D-1)}\left[\frac{i\tau}{4\pi\alpha'}\right]\frac{1}{L}\sum_{k=-\infty}^{\infty}\exp\left(-\frac{\pi k^2\tau}{2L^2\alpha'}\right) \tag{3.8}$$

Summing over k we obtain:

a. Charged string

$$F(\beta) = -\int_0^\infty \frac{d\tau}{2\tau}(2\pi\tau)^{(-D-2d)/2}\sqrt{\frac{2\alpha'}{\tau}}I(D,\tau)$$
$$\times\left[\theta_3\left(0,\exp\left(-\frac{\beta^2}{2\tau}\right)\right)-1\right]\theta_3\left(0,\exp\left(-\frac{2\pi\alpha' L^2}{\tau}\right)\right) \tag{3.9}$$

b. Neutral string

$$F(\beta) = -\left[\prod_{j=1}^{2d}(1+g_j^2 f_j^2)^{\frac{1}{2}}\right]\int_0^\infty \frac{d\tau}{2\tau}(2\pi\tau)^{-\frac{D}{2}}\sqrt{\frac{2\alpha'}{\tau}}\left[\theta_3\left(0,\exp\left(-\frac{\beta^2}{2\tau}\right)\right)-1\right]$$
$$\times\,\eta^{-(D-1)}\left[\frac{i\tau}{4\pi\alpha'}\right]\theta_3\left(0,\exp\left(-\frac{2\pi\alpha' L^2}{\tau}\right)\right). \tag{3.10}$$

Calculating the asymptotic of integrand in (3.9), (3.10) when $\tau \to 0$, we get that the Hagedorn temperature for charged or open noncritical string is the same:

$$\beta_c = 4\pi\Big(\frac{D-1}{24}\alpha'\Big)^{\frac{1}{2}} . \tag{3.11}$$

If $D = 25$, β_c in (3.11) is equal to β_c found in Section 2. As in Section 2, β_c does not depend on magnetic field.

If $D = 1$, $\beta_c = 0$. This fact for noncritical strings in the absence of background field has been mentioned in Refs [14, 15]. It is also interesting that for $D = 1$ closed noncritical string [14, 15] the free energy has a very simple form.

4. Noncritical open string in a constant electromagnetic field

In this section we discuss the thermodynamics of open neutral noncritical string in a constant electromagnetic field.

Mass operator has the form

$$\alpha' m^2 = -\alpha' \sum_{i=1}^{\frac{D-1}{2}} \frac{e^2 + h_i^2}{1 + h_i^2}\left[(p_{2i}^2)^2 + (p_{2i+1})^2\right] + \frac{d_0^2}{2}$$
$$+ (1 - e^2)\Big[\sum_{n=0}^{\infty} \sum_{j=1}^{\frac{D-1}{2}} n\big(a_n^{2j\dagger} a_n^{2j} + a^{2j+1\dagger} a_n^{2j+1}\big) - \frac{D-1}{24}\Big], \tag{4.1}$$

where $g_j = g$, $g f_j = h_j$ and $-i f_0 = e$.

The one-loop free energy can be calculated in the form

$$F(\beta) = -\left[\prod_{i=1}^{\frac{D-1}{2}} \frac{1 + h_i^2}{1 - e^2}\right] \int_0^\infty \frac{d\tau}{2\tau}(2\pi\tau)^{-\frac{D}{2}} \sqrt{\frac{2\alpha'}{\tau}}\left[\theta_3\Big(0, \exp\Big(-\frac{\beta^2}{2\tau}\Big)\Big) - 1\right]$$
$$\times \eta^{-(D-1)}\left[\frac{i\tau(1 - e^2)}{4\pi\alpha'}\right]\theta_3\Big(0, \exp\Big(-\frac{2\pi\alpha' L^2}{\tau}\Big)\Big) . \tag{4.2}$$

Introduce $t = \tau(1 - e^2)$, then

$$F(\beta) = -\det(1 + F_{\mu\nu}) \int_0^\infty \frac{dt}{2t}(2\pi t)^{-D/2} \sqrt{\frac{2\alpha'}{t}}\, \eta^{-(D-1)}\Big[\frac{it}{4\pi\alpha'}\Big]$$
$$\times \left[\theta_3\Big(0, \exp\Big(-\frac{\beta^2(1 - e^2)}{2t}\Big)\Big) - 1\right]\theta_3\Big(0, \exp\Big(-\frac{2\pi\alpha' L^2(1 - e^2)}{t}\Big)\Big) . \tag{4.3}$$

Here $(1-e^2)\prod_{i=1}^{D-1/2}(1+h_i^2)=\det(1+F_{\mu\nu})$ is the square of the Born-Infeld action.

The asymptotics of integrand in (4.3) when $t\to 0$ are

$$\theta_3\left(0,\exp\left(-\frac{\beta^2(1-e^2)}{2t}\right)\right)-1\longrightarrow 2\exp\left(-\frac{\beta^2(1-e^2)}{2t}\right),$$

$$\eta^{-(D-1)}\left[\frac{it}{4\pi\alpha'}\right]\longrightarrow\left(\frac{t}{4\pi\alpha'}\right)^{D-1/2}\exp\left(\frac{\alpha'\pi^2(D-1)}{3t}\right).$$

Using these asymptotics and (4.3) one gets

$$\beta_c=4\pi\left[\frac{(D-1)\alpha'}{24(1-e^2)}\right]^{\frac{1}{2}}. \tag{4.4}$$

We see that the Hagedorn temperature depends on the electric field. The larger electric field, the lower the Hagedorn temperature is. For $D=25$ (4.4) coincides with the expression found in Ref. [5].

For $D=1$, $\beta_c=0$. It is interesting that in this case $F(\beta)$ (4.3) can be written in very simple form:

$$F(\beta)=-\sqrt{\frac{\alpha'}{\pi}}\sum_{r=1}^{\infty}\sum_{n=-\infty}^{\infty}\left[\frac{\beta^2r^2}{2}+2\pi\alpha'L^2n^2\right]^{-1}. \tag{4.5}$$

5. Conclusion

We have investigated the thermodynamics of noncritical open strings in an external electromagnetic field. We have found some interesting features of noncritical strings in an external electromagnetic field.

Let us give another remark connected with noncritical strings. It is well known that an observer at constant acceleration in a Minkowski vacuum will feel the existence of a heat bath with a temperature $T=a/2\pi$ [16]. Let us imagine that an observer accelerates in a vacuum of free noncritical strings (in absence of electromagnetic field). Then there is a critical acceleration $a_c=2\pi T_c=2\pi/\beta_c$, where $\beta_c=4\pi\sqrt{\frac{D-1}{24}\alpha'}$. What would happen if try to accelerate the detector at an acceleration $a>a_c$. The limiting nature of T_c would tend to indicate that it is impossible to do it. However, for $D=1$, $\beta_c=0$ and $a_c\to\infty$. Thus, we see that critical acceleration depends on D (we consider D-dimensional space-time). For $D=1$ critical acceleration is not limited.

In conclusion we note that it would be of interest to address these problems for the theory in the background gravitational field.

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