

# RADIATIVE EFFECTS IN ELLIS-JAFFE SUM RULE AND STRUCTURE FUNCTION $g_1$

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For additional analysis of the experimentally observed quantities exact formulae for electromagnetic effects and leading log formulae for radiative electroweak and double bremsstrahlung effects have been applied to the results of a polarized experiment of EMC group. Their influence on the value of Ellis-Jaffe sum rule and polarized structure function  $g_1(x)$  has been discussed.

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## 1. Introduction

In the recent years high-energy experiments with deep inelastic scattering of polarized particles have attracted considerable attention. These experiments provide additional important information about the spin structure of the nucleon. A recent measurement of the polarized proton structure function by the EMC group [1, 2] has stimulated renewed interest in this subject. It appears that the spin distribution of a proton into its constituents is not as easy as was previously thought. This effect has led to a great deal of speculations. A considerable amount of attention has been paid to the role of axial anomaly, the gluon contribution to proton spin, the structure of sum rule for angular momenta and the description of this phenomenon from the standpoint of various models. In addition, in a number of articles experimentally measured quantities have been calculated. Forthcoming

investigation of these quantities can confirm or refute a great number of existing hypotheses (for an overview see Ref. [3] and references therein).

However, before studying the effects indicated, the investigation of which is connected with various theoretical uncertainties we must be certain that the physical effects whose nature is better understood have been taken into account correctly. It is the calculation of such effects, *i.e.* the radiative and electroweak effects, that is the object of the present paper. Radiative and electroweak effects, as is well-known, have corrections relative to the basic contribution of nearly 10–20% in the range of the EMC experiment. In the experiment [1, 2] these calculations have been carried out in the following way:

1. Electroweak, radiative electroweak and double bremsstrahlung effects have been entirely disregarded in the range of the experiment.
2. The major contribution (the electromagnetic radiative corrections) has been calculated by means of the Mo and Tsai formulae [4]. It was noted in Ref. [2], that these formulae are strictly valid only for the spin-averaged cross section, but according to the assertion of the authors of Ref. [2] the results obtained on the basis of the Mo and Tsai formulae for the polarized particles are very similar to those of the more exact treatment of Kukhto and Shumeiko [5].

The cross section including these effects can be represented as follows

$$\sigma_{\text{obs}} = \sigma_0 + \sigma_{\text{ws}} + \sigma_{\gamma} + \sigma_{\text{rws}} + \sigma_{\text{db}} + \sigma_{\text{N}}, \quad (1)$$

where  $\sigma_{\text{obs}}$  is the observed cross section of the scattering of polarized particles,  $\sigma_0$  — the electromagnetic Born cross section,  $\sigma_{\text{ws}}$  — the Born cross section caused by  $Z^0$ -boson exchange,  $\sigma_{\text{rws}}$  — the radiative electroweak effect (only  $Z^0$ -exchange effects),  $\sigma_{\text{db}}$  — the double bremsstrahlung correction and  $\sigma_{\text{N}}$  — the nuclear corrections. We would like now to formulate the task of our investigation more precisely. We shall examine  $\sigma_{\text{ws}}$ ,  $\sigma_{\text{rws}}$ ,  $\sigma_{\text{db}}$  as corrections to the EMC data and provide a more exact calculation of  $\sigma_{\gamma}$ .

Corresponding corrections can be obtained for polarization asymmetry.

$$A_{\text{obs}} = \frac{(\sigma_{\text{obs}}(\uparrow\uparrow) - \sigma_{\text{obs}}(\uparrow\downarrow))}{(\sigma_{\text{obs}}(\uparrow\uparrow) + \sigma_{\text{obs}}(\uparrow\downarrow))} \stackrel{\text{def}}{=} \frac{\sigma_{\text{obs}}^-}{\sigma_{\text{obs}}^+}, \quad (2)$$

where  $A_{\text{obs}}$  is the measured polarization asymmetry and the arrows show the polarization of lepton and nucleon, respectively.

After expanding the denominator in (2) in perturbative series we obtain a correction to the data presented by the EMC group.

$$\delta A = A_{\text{EMC}} \frac{\delta_{\text{p,EMC}} - \delta_{\text{p}}}{1 + \delta_{\text{p}}} - (A_{\text{w}} + A_{\text{r}} + A_{\text{d}}) \frac{1 + \delta_{\text{u}}}{(1 + \delta_{\text{p}})D}, \quad (3)$$

where  $A_{\text{EMC}}$  is the EMC data for Born spin asymmetry,  $\delta_{\text{p,EMC}}$  is the polarized correction obtained by means of Mo and Tsai formulae,  $\delta_{\text{u}}$  — the unpolarized correction,  $\delta_{\text{p}}$  — the polarized correction, calculated by Kukhto and Shumeiko [5],  $A_{\text{w}}$ ,  $A_{\text{r}}$ ,  $A_{\text{d}}$  are the one-nucleon electroweak, radiative electroweak and the double bremsstrahlung corrections, respectively, and  $D$  is the kinematic factor.

The electromagnetic correction is the basic contribution to the overall correction. In the range of the EMC experiment it can reach 15%, therefore, any more precise calculation is very important for reducing the magnitude of the systematic error. In Section 2 we discuss the difference between the calculation of the electromagnetic correction in terms of the Mo and Tsai formulae and those according to the exact covariant expressions in Ref. [5] (the first item in (3)). By "exact" we understand the calculation of electromagnetic correction in next-to-Born order of the perturbation theory without any approximation. For this purpose we need the structure functions  $g_1(x)$ , which can be extracted from EMC experimental data with the help of the formula

$$A_{\text{EMC}} + \delta A(g_1) = \frac{2xg_1(x)(1+R)}{F_2(x, Q^2)}. \quad (4)$$

But  $\delta A$ , in turn, also depends on  $g_1(x)$  from which it follows that an iteration procedure is required. In this case for the fit of  $g_1(x)$  we use the form suggested in Refs [1,2]

$$g_1(x) = Ax^B(1 - \exp(-Cx)) \frac{F_2(x, Q^2)}{2x(1+R)}, \quad (5)$$

where  $A, B, C$  are variables which are specified in each step of the iteration procedure. This procedure converges within 4-5 steps. As a result, we obtain a fit with the new variables  $A, B, C$ , which takes into account the correction more exactly in the sense of the above given definition. Thus, we shall say that we have obtained a "corrected" fit.

In Section 3 we investigate the influence of the effects of  $Z^0$ -boson exchange in the standard electroweak theory since the following corrections are taken into account: electroweak interference, pure weak and radiative electroweak effects. The first and second corrections are calculated according to the formulae in Ref. [6] and the last calculation employing formula (10), which we obtained in the improved leading log approximation. In this case the organization of the iteration procedure demands the choice of a model for polarized parton distributions. It has been determined that the SU(6) relation between polarized and unpolarized parton distributions is more convenient for this aim. As in the previous Section, the corrections to physically measured quantities are calculated.

Continuing this outline in Section 4 we investigate the double bremsstrahlung process by using the formulae obtained in the aforementioned approximation. The iteration procedure and the calculation of the correction, to the polarized structure function  $g_1$  and the Ellis-Jaffe sum rule are carried out analogously to calculations in Section 2.

In the last Section the obtained results are analyzed and compared with each other, as well as with the systematic error of the EMC experiment caused by radiative corrections [2].

We also take care of the difference in the results, which is connected with the choice of unpolarized structure functions (see also Refs [2,7]). We present the results for different values of  $R$ : for  $R = R_{\text{QCD}}$  as in Refs [1,2] and for  $R = 0$ . The latter corresponds to the choice of the fit for  $F_2(x, Q^2)$  from Ref. [8], used by the authors of Refs [1,2] for analysis of experimental data.

## 2. The calculation of the electromagnetic correction by exact formulae

In order to obtain the polarized structure function  $g_1(x)$  from the Born spin asymmetry according to the formula

$$A_1 = 2xg_1(x) \frac{(1+R)}{F_2(x, Q^2)} \quad (6)$$

it is necessary to extract this spin asymmetry from measured asymmetry by taking into account such radiative effects, which cannot be neglected. According to Ref. [2]

$$A_m \sim \frac{\sigma_0^- + \sigma_\gamma^-}{\sigma_0^+ + \sigma_\gamma^+} = \frac{\sigma_0^-}{\sigma_0^+} \frac{1 + \sigma_\gamma^-/\sigma_0^-}{1 + \sigma_\gamma^+/\sigma_0^+} \stackrel{\text{def}}{=} D A_1 \frac{1 + \delta_p}{1 + \delta_u}, \quad (7)$$

where  $A_m$  is the measured polarized asymmetry,  $A_1$  is the Born spin asymmetry,  $\delta_p$  is the polarized relative electromagnetic correction,  $D$  is a kinematic factor,  $\delta_u$  is the unpolarized relative electromagnetic correction.  $\delta_p$  and  $\delta_u$  contain (each of them) the proton correction and the neutron correction. We have employed exact formulae [5] for exact calculation of  $\delta_p$ , where only the polarized proton correction was considered. The neutron polarized correction is small and can be obtained on the basis of the formulae for the one-nucleon correction. The data for  $\delta_u$  was taken from Ref. [2] inasmuch as the Mo and Tsai formulae are valid in this case with sufficient accuracy.

For  $R = R_{\text{QCD}}$  the initial value of the first moment  $g_1$  is  $M_1 = \int_0^1 dx g_1(x) = \{Q^2 = 10.7 \text{ GeV}^2\} = 0.114$ . The iteration procedure yields

the value  $M_1 = 0.113$ . The correction  $\delta_p$  and the corrections from Ref. [2] are presented in Fig. 1. The correction to spin asymmetry  $\Delta A$  is proportional to the difference between  $\delta_u$  and  $\delta_p$

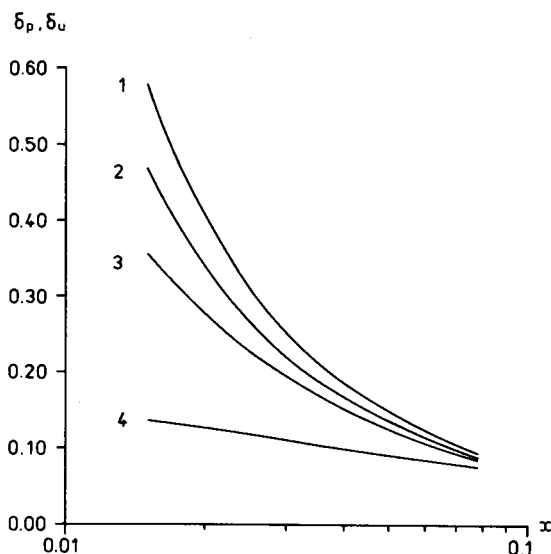


Fig. 1. The polarized and unpolarized corrections to the spin asymmetry. Curves 3, 4 were copied from Fig. 4 of Ref. [2] (3 —  $\delta_u$ , 4 —  $\delta_p$ ). Curves 1 and 2 are our results for  $\delta_p$  (1 —  $R = R_{\text{QCD}}$ , 2 —  $R = 0$ ).

$$\frac{\Delta A}{A_B} = \frac{\delta_u - \delta_p}{1 + \delta_u}. \quad (8)$$

As one can see in Fig. 1 the exact calculated correction (8) has even an opposite sign in comparison with that obtained on the basis of the Mo and Tsai formulae. Both results are listed in Table I. As a result of the iteration procedure we arrive at the “corrected” fit of the  $g_1(x)$

$$g_1(x) = 0.963 x^{0.222} (1 - \exp(-3.80 x)) \frac{F_2(x, Q^2)}{2x(1 + R)}, \quad (9)$$

and thus inserting this fit in the (6) we get the “corrected” asymmetry.

As was already mentioned the choice of  $F_2(x, Q^2)$  corresponds to  $R = 0$ . The results of the iteration procedure in this case are the following:  $\delta_p$  — presented in Fig. 1,  $M_1 = 0.119$  and the fit of  $g_1(x)$  becomes

$$g_1(x) = 0.969 x^{0.204} (1 - \exp(-3.61 x)) \frac{F_2(x, Q^2)}{2x}. \quad (10)$$

TABLE I

$\delta A$  calculated by Mo and Tsai formulae ( $\delta A_1$ ) and by the exact covariant formulae of Ref. [5] ( $\delta A_2$ )

x	$Q^2 \text{ GeV}^2$	$\delta A_1$	$\delta A_2$
0.015	3.5	0.005	-0.002
0.025	4.5	0.005	-0.001
0.035	6.0	0.005	-0.001
0.050	8.0	0.005	-0.001
0.078	10.3	0.004	0.000
0.124	12.9	0.004	0.002
0.175	15.2	0.005	0.003
0.248	18.0	0.007	0.005
0.344	22.5	0.011	0.007
0.466	29.5	0.017	0.012

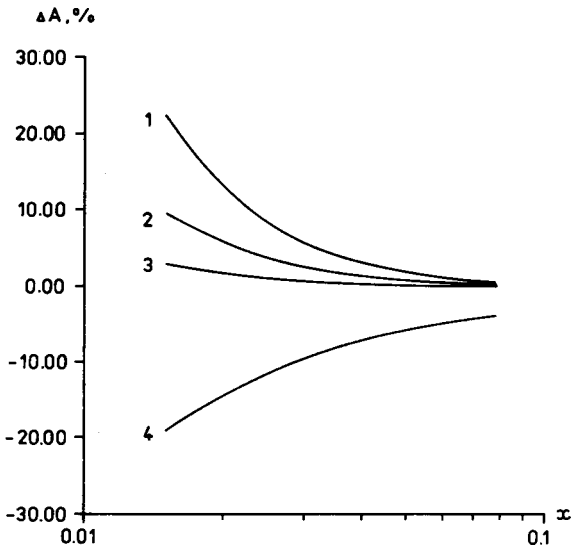


Fig. 2. One-nucleon corrections (curves 1-3) show the results of the iteration procedure in three initial sets of the modified EMC data : 1 —  $A_1^{(0)} = A_1^{\text{EMC}} - \sigma_{\text{sys}}^{\text{EMC}}$ , 2 —  $A_1^{(0)} = A_1^{\text{EMC}}$  and 3 —  $A_1^{(0)} = A_1^{\text{EMC}} + \sigma_{\text{sys}}^{\text{EMC}}$ . Curve 4 describes the one-nucleon correction in the Carlitz-Kaur model

We notice that the sign and the magnitude of the one-nucleon electro-magnetic correction depend essentially on the value of  $g_1(x)$  and, there-

fore, of  $A_1$ . This dependence is shown in Fig. 2. The analysis establishes the following: if the Carlitz–Kaur model is extrapolated to the range  $0.01 \leq x \leq 0.05$ , then  $A_1$  grows up to 10–15%. It was found that the one-nucleon correction shows a qualitatively different behaviour (see Fig. 2).

### 3. Electroweak, weak and radiative electroweak effects

As we have said earlier in the Introduction all effects connected with the  $Z^0$ -exchange were disregarded when processing the EMC experimental data. But as we shall show the contribution of these effects is considerable and must be taken into account.

There are several ways to study the influence of the electroweak effects on the fit of  $g_1(x)$  and the sum rules. If we had more abundant experimental data, for example, experimental data for electroweak asymmetries defined in Ref. [9], we could organize the iteration procedure for extracting the polarized parton distribution. That would allow us to estimate the quantities

$$\Delta q = \int_0^1 dx (q^\uparrow(x) - q^\downarrow(x) + \bar{q}^\uparrow(x) - \bar{q}^\downarrow(x)) \text{ and to answer the question about}$$

parton contributions to the spin of the proton. Nevertheless, for such precise procedure it is necessary to have formulae with the help of which one could calculate the radiative effects exactly.

One more method of the calculation of the electroweak effects is the modification of the Ellis–Jaffe sum rule. That was already done by Fayyazuddin and Riazuddin in Ref. [10]. They have found that

$$\int_0^1 dx (4\tilde{g}_1^{\text{eP}}(x, Q^2) - c_w g_1^{\text{eP}}(x, Q^2)) = \frac{1}{6}(F - D) \left(1 - \frac{\alpha_s(Q^2)}{\pi}\right), \quad (11)$$

where  $g_1^{\text{eP}}$  is the usual spin dependent structure function for electroproduction;  $\tilde{g}_1^{\text{eP}}$  is the spin-dependent structure function arising from the interference of the electromagnetic and weak neutral current. This function can be extracted from the data of the scattering of unpolarized leptons,  $c_w = 3 - 8 \sin^2 \theta_w$ . Extracting  $A_1$  in the iteration procedure from experimental  $A_m$  allows one to calculate the structure functions and to verify the sum rule. This sum rule is obtained in the limit  $m \rightarrow 0$ , therefore radiative effects can be calculated according to approximate formulae.

In this paper we do not deal with any additional sets of experimental data. We obtain the radiative electroweak correction to  $g_1(x)$  and the sum rule using only the data of the EMC experiment. In this case it is necessary to use a model for polarized parton distribution. We choose SU(6)-relation

between polarized and unpolarized parton distribution for the organization of the iteration procedure.

$$\begin{aligned}\Delta u_v(x) &= \cos \theta (u_v(x) - 2/3 d_v(x)), \\ \Delta d_v(x) &= -1/3 \cos \theta d_v(x),\end{aligned}\quad (12)$$

where  $u_v$ ,  $d_v$  — unpolarized valence distributions,  $\theta$  — valence spin-dilution angle. If  $x \rightarrow 1$  then  $\cos \theta$  also will tend to 1. We note that the basic models, often used nowadays, are constructed on the base of the SU(6) relation (see Refs [11-13]). The iteration procedure gives the best fit for  $\cos \theta$  on each step, meanwhile the unpolarized distributions are considered as correct. Feynman diagrams of considered effects are presented in Fig. 3.

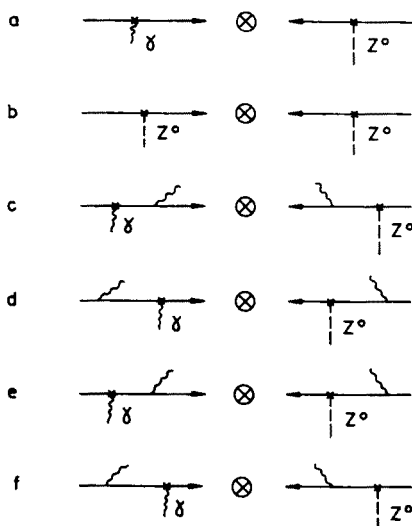


Fig. 3. Feynman diagrams of the electroweak and the radiative electroweak effects

For their calculation we present our formula in the improved leading log approximation

$$\begin{aligned}A_{\text{rws}} &= \frac{\alpha}{2\pi} \ln \left( f \frac{Q^2}{m^2} \right) \sum_{i=1}^2 \int_{z_i}^1 dz \frac{1+z^2}{z(1-z)} \frac{x_i}{x} \left\{ \frac{\sigma_u(x_i, y_i, E_i)}{\sigma_u(x, y, E)} \right. \\ &\quad \left. \times (A_1(x_i, y_i, E_i) - A_1(x, y, E)) \right\} - A_{\text{em}},\end{aligned}\quad (13)$$

where  $x_2 = xy/(z-1+y)$ ,  $x_1 = zx_2$ ,  $y_1 = y_2 = (z-1+y)/z$ ,  $z_1 = 1-y+xy$ ,  $E_2 = E$ ,  $E_1 = zE$ ,  $z_2 = (1-y)/(1-xy)$ ,  $m$  — muon mass,



$\sigma_u = d^2\sigma_u/dxdy$  — Born cross section for scattering of the unpolarized particles. The diagrams, presented in Fig. 3, are described by this formula (the item with  $i = 1$  corresponds to diagram c;  $i = 2$  — d). There are no leading log contributions from the interference diagram. Function  $f = f(x, y)$  is chosen in such a way that the electromagnetic correction calculated by means of formula (13) in experimental points scheduled in Refs [1,2] coincides with those calculated by exact formulae. If  $f \equiv 1$ , then we get the standard leading log approximation. In this case the approximation is quite satisfactory [14].

Using (1) we obtain the "corrected" fit of  $g_1$

$$g_1(x) = 0.962 x^{0.205} (1 - \exp(-3.64 x)) \frac{F_2(x, Q^2)}{2x}, \quad (14)$$

and the correction to the Ellis-Jaffe sum rule  $\Delta M = 0.0003$ . For large  $x$  electroweak effects reach 2%, so one may expect an essential influence of these effects on the value of integral in the region  $x \geq 0.7$ . For the estimation of its value the authors of Refs [1,2] extrapolate their fit in the region  $x \geq 0.7$  and they obtain

$$\int_{0.7}^1 dx g_1(x) = 0.001. \quad (15)$$

However, our calculations show that taking into account electroweak effects do not change this value.

#### 4. Double hard photon radiation

The next significant contribution to the considered observables is the radiation of two hard photons from the lepton line. The preliminary estimate shows that this effect can contribute to polarized asymmetry up to 5% for  $x \sim 0.01-0.02$ . The six Feynman diagrams, presented in Fig. 4 should be examined. Due to the large error of the experiment it is reasonable to investigate only electromagnetic effect, notwithstanding the presented formulae are model-independent and allow one to calculate the electroweak effect too.

For calculation the improved leading log approximation is also used (see also Ref. [15]). In this case we obtain the formula for the correction to the asymmetry which does not contain infrared divergence terms.

$$A_D = \left( \frac{\alpha}{2\pi} \ln \left( \frac{fQ^2}{m^2} \right) \right)^2 \sum_{i=3}^6 \int_{z_i}^1 dz_1 \int_{z'_i}^1 dz_2 F_i(z_1, z_2) + \frac{G_i(z_1, z_2)}{(1-z_1)(1-z_2)}. \quad (16)$$

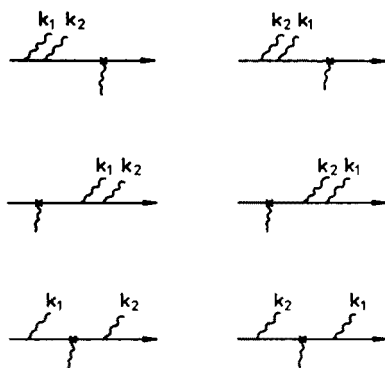


Fig. 4. Feynman diagrams of the double bremsstrahlung

The notation used in formula and the explicit form of functions  $F_i$  and  $G_i$  are gathered in the Appendix. Four items ( $i = 3, 4, 5, 6$ ) correspond to different momentum configuration in collinear kinematics ( $i = 3 - k_1 \| k_2 \| p_1$ ;  $i = 4 - k_1 \| k_2 \| p_2$ ;  $i = 5 - k_1 \| p_1, k_2 \| p_2$ ;  $i = 6 - k_1 \| p_2, k_2 \| p_1$ ). In leading log approximation only collinear photons are considered. However, noncollinear photons are also of importance. They also factorize in the similar way to (16). The presence of these photons is taken into account by a corresponding choice of the function  $f$  which improves the approximation. In our case this function is chosen as follows. For  $k_i \rightarrow 0$  formula for calculation of double bremsstrahlung must be splitted up in two factors. One of them corresponds to the soft radiation ( $k_i \approx 0$ ), and the second one — to the hard radiation of another photon. The formula for double bremsstrahlung will be improved if the second factor coincides with the single photon leading log contribution (13).

Let us repeat that the integrand in (16) does not contain the infrared divergences if  $z_{1,2} \rightarrow 1$ , because of  $G_i(1, z_2) = G_i(z_1, 1) = 0$  (see Appendix). The result (16) contains all items of the order  $\alpha^2$  obtained after expanding the denominator in (2). Only in this case it is possible to cancel correctly the infrared divergences. The expression (16) includes contributions from the single photon radiation, which were already taken into account exactly in Section 2. For this reason for numerical analysis we extract from (16) only the double bremsstrahlung effect.

Using for  $g_1^P(x)$  the fit (5) we obtain the result of the iteration procedure

$$g_1(x) = 0.957 x^{0.220} (1 - \exp(-3.82x)) \frac{F_2 x, Q^2}{2x} \quad (17)$$

and  $\Delta M = -0.0005$ .

We note that both, the exact calculation of the single photon radiation and the calculation of double bremsstrahlung, change essentially the values

of  $A_1$  for small  $x$  in comparison with these presented in Ref. [2]. It leads to

$$\int_0^{0.01} dx g_1(x) = 0.0018 \quad (18)$$

instead of 0.0025 as in Ref. [2]. When calculating (18) we do not change the form of asymptotic of  $g_1^P(x)$  for  $x \rightarrow 0$ .

## 5. Conclusions

We have considered various effects, which can have influence on the measured quantities. It is important for the investigation of the physical phenomena in deep inelastic scattering of polarized particles (real photon radiation, electroweak physics) as well as for reducing the systematic errors in the forthcoming experiments. The authors of Ref. [2] estimate the error caused by radiative corrections as 1% of the measured value of spin asymmetry and conclude that radiative corrections are an insignificant source of error for  $A_1$ . In this article we have considered effects disregarded in Refs [1,2]. The first of them has a mathematical origin. The correction to the EMC experimental data was achieved by treating more exactly the electromagnetic effects. All the other effects have genuine physical origin. They are contributions of effects, which have been disregarded in the analysis of experimental data in Refs [1,2]. In the present paper the electroweak and double bremsstrahlung effects are taken into account as a correction to the EMC data. All discussed corrections, in comparison with the level of systematic error caused by radiative corrections ( $\sigma_{sys}$ ), are plotted in Fig. 5.

It should be noticed that the authors of Ref. [2] have evidently underestimated their influence on  $\sigma_{sys}$ . In the region  $x < 0.1$  the electromagnetic correction can exceed the level of the  $\sigma_{sys}$  even by 25 times and the double bremsstrahlung contribution — 5 times (see Fig. 5). In this region all corrections have the same sign and, therefore, the resulting effect increases. In the opposite region of the EMC range the electroweak interferences and electromagnetic correction can exceed the level of the  $\sigma_{sys}$  1.5-2 times. But they have there opposite signs and are mutually compensated when calculating the first moment of the  $g_1(x)$ , thereby the correction to the moment  $M_1 = \int_0^1 dx g_1(x)$  is small.

The notion of stability can be introduced for the structure function  $g_1(x)$  i.e.: how the observables change if experimental points are included or not into the calculation? For example, if the third point of the EMC data is excluded, the value of  $M_1$  is increased by 0.006. It is clear that

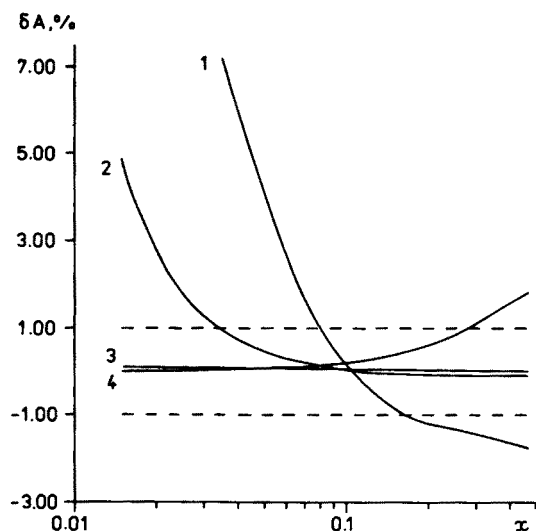
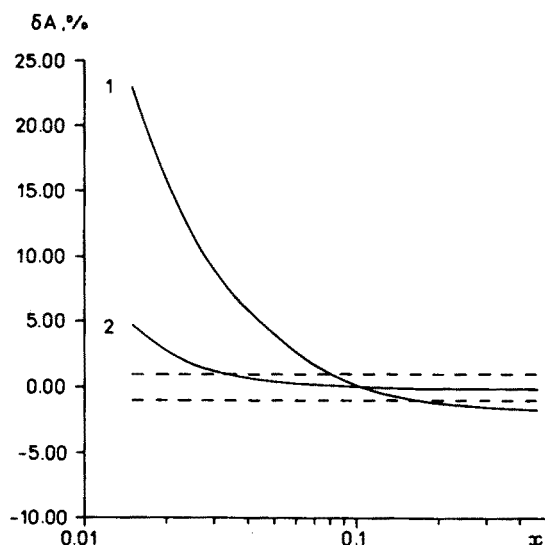


Fig. 5a,b. The corrections to the EMC data. 1 shows the correction due to more precise calculation of the electromagnetic effect. 2,3,4 demonstrate the double bremsstrahlung, the radiative electroweak and the electroweak effects, respectively. The dashed line is the average level of the systematic error due to the radiative effects from Ref. [2].

the appearance of new experimental data in the case of scattered polarized leptons and nucleons can change the situation in this area.

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## APPENDIX

We present the notation and explicit form of functions in expression (16).

$$\begin{aligned}
 F_3(z_1, z_2) &= \frac{1}{4} \left( \frac{3 + 5P_3^2}{z_1 z_2} - P_3(z_1^{-2} + z_2^{-2} + 2z_1^{-1} + 2z_2^{-1}) \right) W_3 \theta(z_2 - \bar{z}_3), \\
 F_4(z_1, z_2) &= \frac{1}{4} \left( \frac{3 + 5P_4^2}{z_1 z_2} - P_4(z_1^2 + z_2^2 + 2z_1 + 2z_2) \right) W_4 \theta(z_2 - \bar{z}_4), \\
 F_5(z_1, z_2) &= \frac{1}{2} \left( 2 - P_5^2 - \frac{z_1}{z_2} \right) W_5 \theta(z_2 - \bar{z}_5), \\
 F_6(z_1, z_2) &= \frac{1}{2} \left( 2 - P_5^2 - \frac{z_2}{z_1} \right) W_6 \theta(z_2 - \bar{z}_6), \\
 G_3(z_1, z_2) &= P_3 \frac{1 + P_3^2}{z_1 z_2} W_3 \theta(z_2 - \bar{z}_3) - \frac{1}{2} W_{11}, \\
 G_4(z_1, z_2) &= P_4(1 + P_4^2) z_1^2 z_2^2 W_4 \theta(z_2 - \bar{z}_4) - \frac{1}{2} W_{22}, \\
 G_5(z_1, z_2) &= z_2(1 + P_5^2) W_5 \theta(z_2 - \bar{z}_5) - \frac{1}{2} W_{12}, \\
 G_6(z_1, z_2) &= z_1(1 + P_6^2) W_6 \theta(z_2 - \bar{z}_6) - \frac{1}{2} W_{21}, \tag{A1}
 \end{aligned}$$

$$W_i = \frac{x_i}{x R_i} \frac{\sigma_u(x_i, y_i, E_i)}{\sigma_u(x, y, E)} [A_1(x_i, y_i, E_i) - A_1(x, y, E)],$$

$$(i = 3, 4, 5, 6),$$

$$W_{ij} = (1 + z_1^2)(1 + z_2^2) \frac{x_{i1} x_{j2}}{x^2 z_1 z_2} \frac{\sigma_{ui1} \sigma_{uj2}}{\sigma_u(x, y, E)} [A_{i1} + A_{j2} - 2A_1(x, y, E)],$$

$$(i, j = 1, 2).$$

In the latter expression the following notations are introduced:  $\sigma_{uij} = \sigma_u(x_{ij}, y_{ij}, E_{ij})$ ,  $A_{ij} = A_1(x_{ij}, y_{ij}, E_{ij})$ ,  $x_{ij} = x_i(z_j)$ ,  $y_{ij} = y_i(z_j)$ ,  $E_{ij} = E_i(z_j)$ .  $A_1$  is the Born spin asymmetry and  $\sigma_u$  stands for the cross section for scattering of unpolarized particles. Quantities  $P_i$ ,  $R_i$ ,  $x_i$ ,  $y_i$ ,  $E_i$ ,  $\bar{z}_i$ , are

listed in Table II.  $x_{1,2}(z)$ ,  $y_{1,2}(z)$ ,  $E_{1,2}(z)$  are defined in Section 3. The limits of integration are:

$$\begin{aligned} z_3 &= z_5 = z'_3 = z'_6 = (1-y)/(1-xy) \\ z_4 &= z_6 = z'_4 = z'_5 = 1-y+xy. \end{aligned} \quad (\text{A2})$$

TABLE II

Quantities used in formulae (A.1)

	$i = 3$	$i = 4$	$i = 5$	$i = 6$
$P_i$	$z_1 + z_2 - 1$	$\frac{1}{z_1} + \frac{1}{z_2} - 1$	$z_1 + \frac{1}{z_2} - 1$	$z_2 + \frac{1}{z_1} - 1$
$R_i$	$z_1 + z_2 - 1$	$(z_1 + z_2 - z_1 z_2)^2$	$z_1$	$z_2$
$\bar{z}_i$	$1 + z_3 - z$	$\left(1 + \frac{1}{z_4} - \frac{1}{z}\right)^{-1}$	$\frac{1-y}{z} + xy$	$\frac{1-y}{z-xy}$
$y_i$	$1 - \frac{1-y}{P_3}$	$1 - (1-y)P_4$	$1 - \frac{1-y}{z_1 z_2}$	$1 - \frac{1-y}{z_1 z_2}$
$x_i$	$\frac{xy}{y_3}$	$xy \frac{P_4}{y_4}$	$\frac{xy}{y_5 z_2}$	$\frac{xy}{y_6 z_1}$
$E_i$	$P_3 E$	$E$	$z_1 E$	$z_2 E$

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