IS THERE A MASS MATRIX COMMON FOR LEPTONS AND QUARKS?*

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An algebraic model of mass matrix common for leptons and quarks is described. It predicts $m_{\nu_\mu} = m_{\nu_\tau} = m_{\nu_e} \simeq 0$, $m_\tau = 1783.47$ MeV (with the input of experimental $m_{\nu_e} \simeq 0, m_e, m_\mu$) and $m_\mu \simeq 10 \div 12$ MeV, $m_d \simeq 17 \div 18$ MeV, $m_s \simeq 128 \div 142$ MeV, $m_t \simeq 149 \div 172$ GeV (with the input of $m_c \simeq 1.3 \div 1.5$ GeV, $m_b \simeq 4.5 \div 5$ GeV). The moduli of all elements of CKM matrix are predicted within their experimental limits (with the input of $|V_{us}| = 0.217 \div 0.223$). In particular, $|V_{ub}| = 0.0036 \div 0.0037$ and $|V_{cb}| = 0.056 \div 0.057$. The phasing-invariant CP-violating phase $\delta = \arg(V_{us} V_{cb} V_{ub}^* V_{cs}^*)$ comes out equal to $45° \div 55°$. Together, the model describes sixteen masses and independent mixing parameters in terms of six free parameters, so it predicts ten independent quantities (with the input of six). Nontrivial interrelations between the lepton sector and quark sector seem to appear.

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As our understanding of the spectrum of fundamental fermions is very preliminary, the study of phenomenological models for their mass matrices [1] seems to be reasonable. In this paper we describe a particular algebraic model of mass matrix common for leptons and quarks. The matrix is parameterized by six real constants and is intended to produce sixteen experimental masses and independent mixing parameters of all fundamental fermions of three generations. We have come to this model through some studies of separate mass matrices for charged leptons, up quarks and down quarks [2]. These studies were motivated by our previous formal results concerning the existence of three replicas of the Dirac particle, conjectured to correspond physically to three observed fermion generations [3].

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The conjecture of Ref. [3] may be expressed in a figurative language by saying that the fundamental fermions of three generations consist of algebraic "intrinsic partons" with spin 1/2, namely of one "visible parton" (coupled to external gauge fields of the standard model) and $n = 0, 1, 2$ pairs of "hidden partons" for the $1^{\rm st}$, $2^{\rm nd}$ and $3^{\rm rd}$ generation, respectively. In the case of $n = 1$ and $n = 2$ the hidden partons form relativistic scalar states (one in each case) due to the interplay of theory of relativity (if extended to the hidden partons) and "Pauli hidden principle" (that provides for hidden partons, only totally antisymmetric states in Dirac hidden indices) [3]. Then, there exists one and only one Dirac particle of a given flavor and color in each of three fermion generations.

In this picture, it is convenient to introduce the (restricted) annihilation and creation operators for pairs of hidden partons, defined by the 3×3 matrices

$$\hat{a} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{a}^+ = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}, \tag{1}$$

Then

$$\hat{n} = \hat{a}^+ \hat{a} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad \hat{a}^3 = 0 = \hat{a}^{+^3} \tag{2}$$

and

$$[\hat{a}, \hat{n}] = \hat{a}, \quad [\hat{a}^+, \hat{n}] = -\hat{a}^+ . \tag{3}$$

Thus, $\hat{a}$ and $\hat{a}^+$ satisfy the annihilation- and creation-operator relations (3), though $[\hat{a}, \hat{a}^+] \neq \hat{1}$ and $\{\hat{a}, \hat{a}^+\} \neq \hat{1}$. Here, $\hat{n}$ denotes the operator of the number of hidden-parton pairs, while

$$\widehat{N} = \hat{1} + 2\hat{a}^+ \hat{a} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} \tag{4}$$

is the operator of the number of all intrinsic partons. Of course, all functions of $\hat{a}$ and $\hat{a}^+$ can be expressed as linear combinations of nine Gell-Mann matrices $\hat{\lambda}_1, \ldots, \hat{\lambda}_8$ and $\hat{1}$ generating a global U(3) integration algebra.

In terms of the (restricted) annihilation and creation operators $\hat{a}$ and $\hat{a}^+$ our algebraic model for the fermion mass matrix

$$\widehat{M} = \mathrm{diag}\left(\widehat{M}^{(\nu)}, \widehat{M}^{(\mathrm{e})}, \widehat{M}^{(\mathrm{u})}, \widehat{M}^{(\mathrm{d})}\right) \tag{5}$$

can be formulated as follows:

$$\widehat{M}^{(i)} = \hat{\rho} \hat{f}^{(i)} \hat{\rho} \qquad (i = \nu, \mathrm{e}, \mathrm{u}, \mathrm{d}), \tag{6}$$

where

$$\hat{\rho} = \frac{1}{\sqrt{29}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{4} & 0 \\ 0 & 0 & \sqrt{24} \end{pmatrix} \tag{7}$$

denotes the generation-weighting matrix following from the argument in Refs [3,2], whilst the Higgs-coupling strength matrix is postulated in the form

$$\hat{f}^{(i)} = C^{(i)} \Big(\lambda^{(i)2} \widehat{N}_{\text{eff}}^{(i)2} - \frac{\lambda^{(i)2}+1}{\widehat{N}_{\text{eff}}^{(i)2}} + 4\eta^{(i)} \Big(\hat{a} \exp\big(i\varphi^{(i)}\big) + \hat{a}^{+} \exp\big(-i\varphi^{(i)}\big) \Big) \Big) \tag{8}$$

with

$$\widehat{N}_{\text{eff}}^{(i)} = \widehat{N} + \xi^{(i)} \hat{a}^{+^2} \hat{a}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5+2\xi^{(i)} \end{pmatrix} . \tag{9}$$

Here, the coefficients $\lambda^{(i)}$, $\xi^{(i)}$ and $\eta^{(i)}$ are taken as given by the universal formulae for $i = \nu, \mathrm{e}, \mathrm{u}, \mathrm{d}$ parameterized by $\lambda^{(\mathrm{e})}$:

$$\lambda^{(i)} = \lambda^{(\mathrm{e})} N_{\mathrm{c}} Q^{(i)^2}, \quad \xi^{(i)} = \big(3Q^{(i)}\big)^2 (3B)^2, \quad \eta^{(i)} = \lambda^{(i)} (3B)^2 \tag{10}$$

with

$$Q^{(i)} = \begin{cases} 0, -1 & \text{for } \nu,\ \mathrm{e} \\ 2/3, -1/3 & \text{for u,d} \end{cases} ,$$

$$N_{\mathrm{c}} = \begin{cases} 1 & \text{for } \nu,\ \mathrm{e} \\ 3 & \text{for u, d} \end{cases} ,$$

$$B = \begin{cases} 0 & \text{for } \nu,\ \mathrm{e} \\ 1/3 & \text{for u, d} \end{cases} . \tag{11}$$

Note that $\xi^{(\nu)} = \xi^{(\mathrm{e})} = 0$, $\xi^{(\mathrm{u})} = 4$, $\xi^{(\mathrm{d})} = 1$, $\eta^{(\nu)} = \eta^{(\mathrm{e})} = 0$, $\eta^{(\mathrm{u})} = \lambda^{(\mathrm{u})}$, $\eta^{(\mathrm{d})} = \lambda^{(\mathrm{d})}$ and $\lambda^{(\nu)} = 0$, $\lambda^{(\mathrm{e})} : \lambda^{(\mathrm{u})} : \lambda^{(\mathrm{d})} = 3:4:1$.

We can see that the model contains six free parameters

$$C^{(\nu)}, \quad C^{(\mathrm{e})}, \quad C^{(\mathrm{u})}, \quad C^{(\mathrm{d})}, \quad \lambda^{(\mathrm{e})}, \quad \varphi^{(\mathrm{u})} - \varphi^{(\mathrm{d})} \tag{12}$$

that can be used to reproduce sixteen experimental values of the masses and independent mixing parameters

$$m_{\nu_e},\quad m_{\nu_\mu},\quad m_{\nu_\tau},\quad m_e,\quad m_\mu,\quad m_\tau$$

and

$$m_u,\quad m_c,\quad m_t,\quad m_d,\quad m_s,\quad m_b,$$

$$|V_{us}|,\ |V_{ub}|,\ |V_{cb}|,\ \delta = \arg\left(V_{us}V_{cb}V_{ub}^*V_{cs}^*\right). \tag{13}$$

Thus, it predicts ten of these quantities, whereas the six remaining, say m_{ν_e}, m_e, m_μ, m_c, m_b, $|V_{us}|$, are taken from experiment as the input.

In the lepton sector, where $\widehat{M}^{(\nu)}$ and $\widehat{M}^{(e)}$ are diagonal, our model gives

$$m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = \frac{C^{(\nu)}}{29} \tag{14}$$

and

$$m_e = \frac{C^{(e)}}{29},\quad m_\mu = m_e\tfrac{4}{9}\left(80\lambda^{(e)^2} - 1\right),$$
$$m_\tau = m_e\tfrac{24}{25}\left(624\lambda^{(e)^2} - 1\right), \tag{15}$$

where all Dirac masses are defined as nonnegative. Eliminating $\lambda^{(e)^2}$ from Eq. (15) we get the mass rule for charged leptons:

$$\tfrac{6}{125}\left(136m_e + 351m_\mu\right) = m_\tau \tag{16}$$

which is satisfied excellently by experimental masses (indeed, its lhs and rhs is 1783.47 MeV and $1784^{+2.7}_{-3.6}$ MeV, respectively). The free parameter $\lambda^{(e)^2}$, when evaluated from Eq. (15) with experimental m_e and m_μ as the input, is given by

$$\frac{1}{\lambda^{(e)^2}} = \frac{320m_e}{4m_e + 9m_\mu} = 0.171590 = \tan^2\frac{\pi}{7.99965}, \tag{17}$$

where rhs approximates closely $\tan^2\pi/8$. Hence, $\lambda^{(e)} = 2.414093$. Making use of this value of $\lambda^{(e)}$ we obtain from Eq. (10)

$$\lambda^{(u)} = 3.218791,\qquad \lambda^{(d)} = 0.804698. \tag{18}$$

We will apply these values to the quark sector.

In contrast to $\widehat{M}^{(\nu)}$ and $\widehat{M}^{(e)}$, the quark mass matrices $\widehat{M}^{(u)}$ and $\widehat{M}^{(d)}$ are nondiagonal. Carrying out the numerical diagonalization of these matrices and using as the input the experimental estimates $m_c \simeq 1.3 \div 1.5$ GeV and $m_b \simeq 4.5 \div 5$ GeV we calculate

$$m_{\rm u} = 0.271104 \frac{C^{(\rm u)}}{29} \simeq 10 \div 12\,\text{MeV}, \quad m_{\rm t} \simeq 149 \div 172\,\text{GeV} \tag{19}$$

and

$$m_{\rm d} = 4.352301 \frac{C^{(\rm d)}}{29} \simeq 17 \div 18\,\text{MeV}, \quad m_{\rm s} \simeq 128 \div 142\,\text{MeV}, \tag{20}$$

where all Dirac masses are defined as nonnegative. Further, taking as the input the experimental value $|V_{\rm us}| = 0.217 \div 0.223$ we evaluate the phase difference

$$\varphi^{(\rm u)} - \varphi^{(\rm d)} = 31.5^\circ \div 38.6^\circ \tag{21}$$

and then the ub- and cb- mixing parameters

$$|V_{\rm ub}| = 0.00361 \div 0.00367, \qquad |V_{\rm cb}| = 0.0564 \div 0.0567\,. \tag{22}$$

In general, we evaluate in this way all elements $V_{kl} = |V_{kl}|\exp(i\alpha_{kl})$ of the Cabibbo-Kobayashi-Maskawa matrix $\widehat{V} = \widehat{U}^{(\rm u)+}\widehat{U}^{(\rm d)}$ where $\widehat{U}^{(\rm u,d)}$ are diagonalizing unitary matrices (so that $\widehat{U}^{(\rm u,d)}\widehat{M}^{(\rm u,d)}\widehat{U}^{(\rm u,d)+}$ are diagonal). It turns out that the resulting phases α_{kl} satisfy the relations

$$\alpha_{\rm ud} + \alpha_{\rm cs} + \alpha_{\rm tb} = 0 \tag{23}$$

and

$$\begin{aligned} \alpha_{\rm us} + \alpha_{\rm cd} + \alpha_{\rm tb} &= (0.0406^\circ \div 0.0467^\circ) - 180^\circ\,, \\ \alpha_{\rm cb} + \alpha_{\rm ts} + \alpha_{\rm ud} &= (0.562^\circ \div 0.673^\circ) - 180^\circ\,. \end{aligned} \tag{24}$$

Then, performing in addition the rephasing of up and down quark states

$$u_k \to u_k \exp\left(i\varphi_k^{(\rm u)}\right) \quad (k = \rm u, c, t), \qquad d_k \to d_k \exp\left(i\varphi_k^{(\rm d)}\right) \quad (k = \rm d, s, b) \tag{25}$$

such that

$$\sum_k \varphi_k^{(\rm u)} = \sum_k \varphi_k^{(\rm d)}, \qquad \varphi_k^{(\rm u)} - \varphi_k^{(\rm d)} = \alpha_{kk} \tag{26}$$

and

$$\varphi_u^{(\rm u)} - \varphi_s^{(\rm d)} = \alpha_{us}\,, \qquad \varphi_c^{(\rm u)} - \varphi_b^{(\rm d)} = \alpha_{cb}\,, \tag{27}$$

we are able to eliminate from $\widehat{V}$ the phases of its elements $V_{\rm ud}$, $V_{\rm cs}$, $V_{\rm tb}$ and $V_{\rm us}$, $V_{\rm cb}$, since then $\alpha_{kl} \to \alpha_{kl} - \varphi_k^{(\rm u)} + \varphi_l^{(\rm d)}$. The resulting $\widehat{V}$, rephased in this way, reads

$$\widehat{V} = \begin{pmatrix} 0.976 & 0.217 & 0.00361{\rm e}^{-i45.0^\circ} \\ -0.216{\rm e}^{i0.0406^\circ} & 0.975 & 0.0564 \\ 0.0146{\rm e}^{-i9.28^\circ} & -0.0542{\rm e}^{i0.562^\circ} & 0.998 \end{pmatrix}$$
$$\div \begin{pmatrix} 0.975 & 0.223 & 0.00366{\rm e}^{-i54.5^\circ} \\ -0.223{\rm e}^{i0.0467^\circ} & 0.973 & 0.0567 \\ 0.0150{\rm e}^{-i10.8^\circ} & -0.0545{\rm e}^{i0.673^\circ} & 0.998 \end{pmatrix}. \tag{28}$$

In all these calculations in the quark sector we used, of course, the value of $\lambda^{(e)}$ as determined in the lepton sector with the input of experimental m_e and m_μ.

Concluding, we can see that the moduli of all elements of CKM matrix $\widehat{V}$, predicted by our model (with the input of $|V_{us}| = 0.217 \div 0.223$ and the use of $\lambda^{(e)}$ as determined in the lepton sector) are consistent with their experimental limits [4]. The rephasing-invariant CP-violating phase

$$\delta = \alpha_{us} + \alpha_{cb} - \alpha_{ub} - \alpha_{cs} = -\left(\alpha_{ub} - \varphi_u^{(u)} + \varphi_b^{(d)}\right) \tag{29}$$

(that is still unknown experimentally) is predicted to be $\delta = 45.0° \div 54.5°$. As far as the lepton masses are concerned, the model predicts $m_\tau = 1783.47$ MeV in a very satisfactory agreement with experiment and $m_{\nu_\mu} = m_{\nu_\tau} = m_{\nu_e} \simeq 0$ consistently with the simplest possibility of massless neutrinos (here, the input of experimental m_e, m_μ and $m_{\nu_e} \simeq 0$ is made). The predictions for the quark masses (with the input of $m_c \simeq 1.3 \div 1.5$ GeV and $m_b \simeq 4.5 \div 5$ GeV and the use of $\lambda^{(e)}$ as determined in the lepton sector) are generally reasonable, though $m_u \simeq 10 \div 12$ MeV and $m_d \simeq 17 \div 18$ MeV (and even $m_s \simeq 128 \div 142$ MeV) may be a bit too large [5], while the large mass for top quark, $m_t \simeq 149 \div 172$ GeV, is not inconsistent with the present semiexperimental expectations as *e.g.*, $m_t = 144^{+23+19}_{-26-21}$ GeV [6].

The reasonable agreement of our model predictions with the experimental mass spectrum of fundamental fermions (as it is presently seen) and their mixing parameters seems to indicate that there are nontrivial interrelations between the lepton sector and quark sector, as those expressed (in terms of our model) by the formulae (10).

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