THE EFFECT OF $\Sigma\Lambda$ CONVERSION ON THE Σ HYPERNUCLEAR STATES

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The effect of the $\Sigma\Lambda$ coupling on the bound Σ hypernuclear states is investigated in the case of the Σ^- hypernucleus produced in the (K^-, π^+) reaction on a target nucleus with double closed shells. In both Σ and Λ channels, the nuclear core is represented by a system of nucleons. The calculation of the width and energy shift of the bound Σ states, in which the analytical expression for the two-particle Green's function is applied, leads to the conclusion that the $\Sigma\Lambda$ coupling increases the binding of these states.

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1. Introduction

The problem of the Σ hypernuclear states has been discussed in recent reviews [1–3]. The existing experimental data on these states produced in (K, π) reactions are rather scarce. Nevertheless, it appears to be established that the states are surprisingly narrow and their energy is positive (the only exception is the observation of the bound ground state of ${}^4_\Sigma$He reported in [4]).

What distinguishes the Σ hypernuclear states is the strong $\Sigma\Lambda$ conversion process $\Sigma\mathcal{N} \to \Lambda\mathcal{N}'$ (we use the notation $\mathcal{N}$ for nucleons, P for protons, and N for neutrons). Once it was believed that this process should lead to very short lived Σ states with the corresponding width $\Gamma \gtrsim 30$ MeV. However, it was realized later that states much narrower, with $\Gamma < 10$ MeV, should be expected because of the following factors: *(i)* reduced overlap of Σ and nuclear wave functions for small Σ binding, *(ii)* Pauli blocking and dispersive (binding) effects [5–9], *(iii)* spin-isospin selection rules [10–11]. (Other possible factors like quenching of the one-pion-exchange (OPE) component of the $\Sigma\Lambda$ conversion in nuclear matter [2,12] and SU(3) symmetry [13] have been discussed also.) It should be stressed that factors *(i)* and *(ii)* reconcile the data on Σ hypernuclear and Σ^- atomic widths [6].

The conversion process not only produces the width but also affects the energy of the Σ hypernuclear states. In most of the published papers on the production of Σ hypernuclei in the (K,π) reactions, the $\Sigma\Lambda$ conversion was taken into account by adding an absorptive part iW_Σ to the s.p. potential V_Σ in the Schrödinger equation for the Σ hypernuclear wave function ψ_Σ. If V_Σ alone leads to a bound state with the (negative) energy $\varepsilon_{\Sigma 0}$, then adding iW_Σ leads to a decaying state with the energy $\varepsilon_\Sigma > \varepsilon_{\Sigma 0}$. This decrease in the binding introduced by the absorptive potential iW_Σ may be understood by noticing that absorption reduces the magnitude of ψ_Σ and thus acts similarly as repulsion. However, this way of treating the coupling to the Λ channel ignores the effect of the coupling on the real part V_Σ of the Σ s.p. potential.

To treat the $\Sigma\Lambda$ conversion in a more satisfactory way, one should include the Λ channel explicitly into the description of the Σ hypernuclear states. The first step in this direction has been the extreme s.p. approach in which both Σ and Λ move in s.p. potentials V_Σ and V_Λ, and both channels are coupled by a s.p. potential V_x. The s.p. potentials are supposed to result from folding the two-body interactions responsible respectively for the $\Sigma\mathcal{N} \to \Sigma\mathcal{N}$, $\Lambda\mathcal{N} \to \Lambda\mathcal{N}$, and $\Sigma\mathcal{N} \to \Lambda\mathcal{N}$ processes into the nucleon density of the nuclear core. In this approach applied in [13] (see also [14]), one solves the coupled-channel Schrödinger equations for the s.p. hyperon wave functions ψ_Σ and ψ_Λ, and adjusts the three potentials to the experimental (K,π) spectra. A simplified version [15] of this approach (restriction to Σ bound states and applying the scheme used in [16] in the theory of autoionization of atoms) leads to simple expressions for the width and energy shift of the Σ bound states caused by the $\Sigma\Lambda$ coupling.

The same radial shape of V_Σ, V_Λ, and V_x assumed in [13–15] may be justified by the folding procedure when both Σ and Λ attach to the same nuclear core. As pointed out by Gal (see, *e.g.*, [17]), this situation does not occur in Σ^- hypernuclei: *e.g.*, the $^{15}\mathrm{N} \otimes \Sigma^-$ system (produced in the (K^-,π^+) reaction on the $^{16}\mathrm{O}$ target) is coupled by the $\Sigma^- P \to \Lambda N$ conversion process with the $^{15}\mathrm{O} \otimes \Lambda$ system, *i.e.*, the nuclear cores are different in the Σ and Λ channels. The situation may occur in Σ^0 hypernuclei, but also here because of the large momentum release in the $\Sigma\mathcal{N} \to \Lambda\mathcal{N}'$ process the emerging nucleon $\mathcal{N}'$ most likely leaves the hypernucleus. In other words, the nuclear core in the Λ channel is most likely in an excited state. It means that only part of the energy released in the $\Sigma\Lambda$ conversion process is available to Λ, which has been simulated in [13] and [14] by replacing the Λ mass M_Λ by a bigger mass $\widetilde{M}_\Lambda$. In this effective Λ channel method one attempts to overcome the inadequacy of the rigid nuclear core model by introducing an additional parameter $\widetilde{M}_\Lambda$ whose precise value is hard to estimate. As shown in [15], the results for the width and especially for the energy shift (including its sign) of Σ hypernuclear states are very sensitive to the magnitude of $\widetilde{M}_\Lambda$.

In this paper, we present the calculation of the effect of the $\Sigma\Lambda$ conversion on the bound Σ hypernuclear states, in which the nuclear core is represented by a system of nucleons. Thus our calculation is not burdened by the untenable assumption of the rigid nuclear core or by the ambiguity of the value of $\widetilde{M}_\Lambda$. We consider a simple model for Σ^- hypernuclei produced in (K^-, π^+) reactions (with the underlying elementary process $K^-P \to \pi^+\Sigma^-$) on closed shell nuclei. We assume that nucleons and Σ^- are bound by s.p. potentials $V_{\mathcal{N}}$ and V_Σ. As the result of the conversion process $\Sigma^-P \to \Lambda N$, caused by the two-body interaction V, unbound Λ and N emerge. The coupling between Σ^- hypernuclear states and the ΛN continuum is described within the scheme of [16]. The unbound Λ and N are represented by plane waves. Our present procedure follows closely the procedure of [15] except that the Green's function for one free particle (Λ) is now replaced by the Green's function for two free particles (Λ and N), for which the known analytical expression is applied. Results of our calculation for a simplified model of the ^{16}O target show that the $\Sigma\Lambda$ coupling leads to a negative energy shift of the bound Σ hypernuclear states, *i.e.*, the states become more bound (and acquire a width). The novel element of the present approach is the explicit use of the two-particle Green's function.

The paper is organized as follows. In Section 2, our theoretical scheme is described. In Section 3, the calculations for the simplified model of the ^{16}O target and their results are presented, discussed and compared with the effective Λ channel method. In Appendix A, we outline the derivation of the analytical formulae for the width and the energy shift of Σ^- bound states. In Appendix B, we derive in the plane-wave impulse approximation (PWIA) expressions for the cross section for the (K^-, π^+) in-flight reactions.

2. The theoretical scheme

We apply for the target nucleus the simplest shell model of independent nucleons moving in a fixed central spin-independent potential. (Including the *ls* coupling would not change the essential scheme of the present procedure.) We shall consider the case of two closed proton shells (the generalization to more shells is trivial) with the orbital quantum numbers $l_0 = 0$ and $l_1 = 1$ (our model of the ^{16}O nucleus). There are $Z_i = 2(2l_i + 1)$ protons in the l_i shell ($i = 1, 2$). Neutrons, which also are assumed to fill closed shells, are not directly involved in the $\Sigma^-P \to \Lambda N$ conversion process. They play only a passive role of blocking all the bound states for the neutrons produced in the conversion process.

The s.p. wave functions of protons moving in the s.p. potential $V_{\rm P}(r)$ in the l_i shell (with the s.p. energy $\varepsilon_{\rm P}^i$) are:

$$\psi^{\rm P}_{\lambda^i}(\vec{r}\xi) = \psi^{\rm P}_{l_i m}(\vec{r})\zeta_\nu(\xi) = Y_{l_i m}(\hat{r})R^{\rm P}_{l_i}(r)\zeta_\nu(\xi)\,, \tag{2.1}$$

where m and ν are the orbital and spin magnetic numbers, ζ_ν is the spin wave function, and $\lambda^i = l_i m\nu$.

The simple shell model is also applied to the Σ^- hypernucleus produced in the (K^-,π^+) reaction. In the absence of the $\Sigma\Lambda$ conversion, the Σ^- hyperon moves in a fixed central spin-independent potential $V_\Sigma(r)$. Its wave functions in the l_i shell (with the s.p. energy ε^i_Σ) are:

$$\psi^{\Sigma}_{\lambda^i}(\vec{r}\xi) = \psi^{\Sigma}_{l_i m}(\vec{r})\zeta_\nu(\xi) = Y_{l_i m}(\hat{r})R^{\Sigma}_{l_i}(r)\zeta_\nu(\xi)\,. \tag{2.2}$$

Here and later on, we use simply the notation Σ for the Σ^- hyperon.

We want to investigate the effect of the $\Sigma\Lambda$ conversion on the substitutional states $\left(l_i^{-1}l_i\right)_{\mathrm{P}\Sigma}$ and on the hypernuclear ground state (g.s.). In the absence of the $\Sigma\Lambda$ conversion, the normalized substitutional states are:

$$\begin{aligned} |\Psi^i_{0\Sigma}\rangle &= (Z_i)^{-1/2}\sum\{m\nu\}a^\dagger_{\Sigma l_i m\nu}a^{}_{\mathrm{P}l_i m\nu}|0\rangle \\ &= (Z_i)^{-1/2}\sum\{\lambda^i\}a^\dagger_{\Sigma\lambda^i}a_{\mathrm{P}\lambda^i}|0\rangle\,, \end{aligned} \tag{2.3}$$

where $|0\rangle$ is the g.s. of the target nucleus, the operator $a_{\mathrm{P}\lambda}$ annihilates a proton in the λ state, and the operator $a^\dagger_{\Sigma\lambda}$ creates a Σ^- hyperon in the λ state. The notation $\Sigma\{\lambda^i\}$ indicates summation over m and ν with the fixed value of $l = l_i$.

The unperturbed (by $\Sigma\Lambda$ coupling) energy of the state (2.3) is:

$$E^i_0 = \varepsilon^i_\Sigma + (Z_i-1)\,\varepsilon^i_{\rm P} + Z_j\varepsilon^j_{\rm P}\,. \tag{2.4}$$

(Here and hereafter, we use the convention $j \neq i$, *i.e.*, if $i = 0(1)$, then $j = 1(0)$. To indicate any value (either 0 or 1), we use the index k.) This is the energy of Σ and all the protons, without counting the energy of the neutrons in the target nucleus, which do not participate in the $\Sigma\Lambda$ conversion (and in the (K^-,π^+) reaction).

The total angular momentum L of the l_i substitutional state, Eq. (2.3) is $L = 0$, and the total spin $S = 0$. With our central spin-independent shell model potentials, and with the two-body $\Sigma\Lambda$ conversion interaction which we shall assume to be of pure Wigner type, the spin S of the substitutional state is irrelevant. However, in the (K^-, π^+) reaction which we shall describe in the PWIA with a simple spin-independent elementary interaction (see Appendix B), the total spin S is conserved, and is equal to the total spin of the target nucleus, which is zero. Thus in the (K^-, π^+) reaction only the substitutional states with $S = 0$ are excited. We shall simply call them the l_i substitutional states (or equivalently the s and p substitutional states for $l_i = l_0 = 0$ and $l_i = l_1 = 1$ respectively).

The normalized unperturbed hypernuclear g.s. $\left(l_1^{-1} l_0\right)_{\mathrm{P}\Sigma}$ is:

$$|\Psi_{0\Sigma}^{\mathrm{gs}(m_1)}\rangle = 2^{-1/2} \sum \{\nu\} a^\dagger_{\Sigma l_0 m_0 \nu} a_{\mathrm{P} l_1 m_1 \nu} |0\rangle \,, \tag{2.5}$$

and its unperturbed energy (without the energy of neutrons) is:

$$E_0^{\mathrm{gs}} = \varepsilon_\Sigma^0 + Z_0 \varepsilon_\mathrm{P}^0 + (Z_1 - 1)\, \varepsilon_\mathrm{P}^1 \,. \tag{2.6}$$

The total angular momentum of the g.s. (m_1) is $L = l_1$, its z-component $L_z = -m_1$, and the total spin $S = 0$. The reason for considering the g.s. with $S = 0$ is the same as in the case of the substitutional states.

When we switch on the ΣΛ coupling, the hyperon may be either Σ or Λ. Thus the state $|\Psi\rangle$ of our system has two components, the Hamiltonian H becomes a 2×2 matrix,

$$|\Psi\rangle = \begin{pmatrix} |\Psi_\Sigma\rangle \\ |\Psi_\Lambda\rangle \end{pmatrix} , \qquad H = \begin{pmatrix} h_\Sigma & , & V_{\Sigma\Lambda} \\ V_{\Lambda\Sigma} & , & h_\Lambda \end{pmatrix} , \tag{2.7}$$

and the Schrödinger equation for the state $|\Psi_E\rangle$ with energy E is:

$$H|\Psi_E\rangle = \begin{pmatrix} E & , & 0 \\ 0 & , & E + \Delta \end{pmatrix} |\Psi_E\rangle \,, \tag{2.8}$$

where $\Delta = (M_{\Sigma^-} + M_\mathrm{P} - M_\Lambda - M_\mathrm{N})\, c^2$.

By h_Σ and h_Λ we denote the s.p. Hamiltonians in the Σ and Λ channels. They consist of the nuclear shell model Hamiltonians and the s.p. Hamiltonians of $\Sigma\,(T_\Sigma + V_\Sigma)$ and $\Lambda\,(T_\Lambda + V_\Lambda)$ respectively. Since $\Delta = 80.45$ MeV, the ΣΛ conversion leads to an energetic Λ, not much affected by V_Λ which we neglect in our model (also in [13] the influence of V_Λ was found to be only marginal).

By $V_{\Lambda\Sigma} \equiv V$ we denote the coupling potential responsible for the $\Sigma \to \Lambda$ conversion. Similarly $V_{\Sigma\Lambda} = V^\dagger$ denotes the coupling potential responsible for the $\Lambda \to \Sigma$ conversion. We have:

$$V = \sum \{\alpha\beta\gamma\delta\} \left(\psi_\beta^\Lambda \psi_\delta^\mathrm{N} | v | \psi_\alpha^\Sigma \psi_\gamma^\mathrm{P}\right) a^\dagger_{\Lambda\beta} a^\dagger_{\mathrm{N}\delta} a_{\mathrm{P}\gamma} a_{\Sigma\alpha} \,, \tag{2.9}$$

where v is the two-body conversion potential and, *e.g.*, $a^\dagger_{\mathrm{N}\delta}$ is the creation operator of a neutron in the state described by the s.p. wave function ψ^{N}_δ. It may be a scattering state (normalized in unit volume) with the asymptotic wave vector $\vec{k}_{\mathrm{N}}$ and spin projection ν_{N} ($\delta = \underline{k}_{\mathrm{N}} = \vec{k}_{\mathrm{N}}, \nu_{\mathrm{N}}$) and with energy $\varepsilon_{\mathrm{N}}(k_{\mathrm{N}}) = \hbar^2 k_{\mathrm{N}}^2/2M_{\mathrm{N}}$, or one of the bound states ($\delta = \lambda^i$) which, however, are blocked by the target neutrons.

We present our procedure of solving (2.8) when the unperturbed state $|\Psi_0\rangle$ (*i.e.* in the absence of V) is the substitutional l_i state: $|\Psi_{0\Sigma}\rangle = |\Psi^i_{0\Sigma}\rangle$, $|\Psi_{0\Lambda}\rangle = 0$. In the case of the hypernuclear g.s., the procedure is essentially the same, and we shall present only the final results.

To investigate the effect of the $\Sigma\Lambda$ coupling on the unperturbed substitutional l_i state, we make for $|\Psi^i_{E\Sigma}\rangle$ (the Σ component of the state $|\Psi^i_E\rangle$ which evolves from the unperturbed substitutional l_i state when the coupling is switched on) the Ansatz:

$$|\Psi^i_{E\Sigma}\rangle = A^i(E)|\Psi^i_{0\Sigma}\rangle\,. \tag{2.10}$$

Our Ansatz for $|\Psi^i_{E\Lambda}\rangle$ (the Λ component of $|\Psi^i_E\rangle$) consists of all states $|\Phi\rangle$ directly connected with $|\Psi^i_{0\Sigma}\rangle$ by the coupling V. To determine these states, we act with V, Eq. (2.9), on $|\Psi^i_{0\Sigma}\rangle$, Eq. (2.3), and get (with the notation $\lambda^i_{a(b)} = l_i m_{a(b)}\nu_{a(b)}$):

$$\begin{aligned}
V|\Psi^i_{0\Sigma}\rangle &= (Z_i)^{-1/2}\sum\{\underline{k}_\Lambda \underline{k}_{\mathrm{N}}\lambda^i_a\} \\
&\times\Big[\tfrac{1}{2}\sum\{\lambda^i_b\}\big(\psi^\Lambda_{\underline{k}_\Lambda}\psi^{\mathrm{N}}_{\underline{k}_{\mathrm{N}}}|v|\psi^\Sigma_{\lambda^i_a}\psi^{\mathrm{P}}_{\lambda^i_b} - \psi^\Sigma_{\lambda^i_b}\psi^{\mathrm{P}}_{\lambda^i_a}\big)|\Phi(\underline{k}_\Lambda\underline{k}_{\mathrm{N}}\lambda^i_b\lambda^i_a)\rangle \\
&+\sum\{\lambda^j_b\}\big(\psi^\Lambda_{\underline{k}_\Lambda}\psi^{\mathrm{N}}_{\underline{k}_{\mathrm{N}}}|v|\psi^\Sigma_{\lambda^i_a}\psi^{\mathrm{P}}_{\lambda^j_b}\big)|\Phi(\underline{k}_\Lambda\underline{k}_{\mathrm{N}}\lambda^j_b\lambda^i_a)\rangle\Big]\,,
\end{aligned} \tag{2.11}$$

where

$$|\Phi(\underline{k}_\Lambda\underline{k}_{\mathrm{N}}\lambda^k_b\lambda^i_a)\rangle = a^\dagger_{\Lambda\underline{k}_\Lambda}a^\dagger_{\mathrm{N}\underline{k}_{\mathrm{N}}}a_{\mathrm{P}\lambda^k_b}a_{\mathrm{P}\lambda^i_a}|0\rangle\,. \tag{2.12}$$

Notice that both neutron and Λ are produced in the conversion process in the continuum, because all neutron bound states are occupied in $|0\rangle$, and because in our model there are no Λ bound states ($V_\Lambda = 0$).

Our Ansatz for $|\Psi^i_{E\Lambda}\rangle$ is thus:

$$|\Psi^i_{E\Lambda}\rangle = \sum_{k=i,j}\sum\{\underline{k}_\Lambda\underline{k}_{\mathrm{N}}\lambda^i_a\lambda^k_b\}B^i(E,\underline{k}_\Lambda\underline{k}_{\mathrm{N}}\lambda^i_a\lambda^k_b)|\Phi(\underline{k}_\Lambda\underline{k}_{\mathrm{N}}\lambda^k_b\lambda^i_a)\rangle\,, \tag{2.13}$$

with

$$B^i\left(E,\underline{k}_\Lambda\underline{k}_{\mathrm{N}}\lambda^i_a\lambda^i_b\right) = -B^i\left(E,\underline{k}_\Lambda\underline{k}_{\mathrm{N}}\lambda^i_b\lambda^i_a\right)\,.$$

Notice that

$$|\Phi(\underline{k}_\Lambda \underline{k}_N \lambda_b^i \lambda_a^i)\rangle = -|\Phi(\underline{k}_\Lambda \underline{k}_N \lambda_a^i \lambda_b^i)\rangle.$$

To determine the unknown quantities A and B, we insert (2.10) and (2.13) into Schrödinger equation (2.8), and use the ortho-normalization properties of the states $|\Phi\rangle$ (for $k = i, j$ and $k' = i, j$):

$$\begin{aligned}&\langle \Phi(\underline{k}'_\Lambda \underline{k}'_N \lambda_{b'}^{k'} \lambda_{a'}^i)|\Phi(\underline{k}_\Lambda \underline{k}_N \lambda_b^k \lambda_a^i)\rangle \\ &= \delta\left(\underline{k}'_\Lambda, \underline{k}_\Lambda\right)\delta\left(\underline{k}'_N, \underline{k}_N\right)\left[\delta\left(\lambda_{b'}^{k'}, \lambda_b^k\right)\delta\left(\lambda_{a'}^i, \lambda_a^i\right) - \delta\left(\lambda_{b'}^{k'}, \lambda_a^i\right)\delta\left(\lambda_{a'}^i, \lambda_b^k\right)\right],\end{aligned} \tag{2.14}$$

where, *e.g.*, $\delta\left(\lambda_{b'}^{k'}, \lambda_b^k\right) = \delta_{l_{k'} l_k}\delta_{m'_b m_b}\delta_{\nu'_b \nu_b}$. In this way, we get the following system of equations for A and B:

$$\begin{aligned}&\left[E_\Sigma - \varepsilon_\Sigma^i\right] A^i(E) \\ &= \sum_{k=i,j} \sum \{\underline{k}_\Lambda \underline{k}_N \lambda_b^k \lambda_a^i\} \langle \Psi_{0\Sigma}^i | V^\dagger | \Phi(\underline{k}_\Lambda \underline{k}_N \lambda_b^k \lambda_a^i)\rangle B^i(E, \underline{k}_\Lambda \underline{k}_N \lambda_a^i \lambda_b^k),\end{aligned} \tag{2.15}$$

$$\begin{aligned}&\left[E_\Sigma + \Delta + \varepsilon_P^i - \varepsilon_\Lambda(k_\Lambda) - \varepsilon_N(k_N)\right] 2B^i(E, \underline{k}_\Lambda \underline{k}_N \lambda_a^i \lambda_b^i) \\ &\qquad = \langle \Phi(\underline{k}_\Lambda \underline{k}_N \lambda_b^i \lambda_a^i)|V|\Psi_{0\Sigma}^i\rangle A^i(E),\end{aligned} \tag{2.16}$$

$$\begin{aligned}&\left[E_\Sigma + \Delta + \varepsilon_P^j - \varepsilon_\Lambda(k_\Lambda) - \varepsilon_N(k_N)\right] B^i(E, \underline{k}_\Lambda \underline{k}_N \lambda_a^i \lambda_b^j) \\ &\qquad = \langle \Phi(\underline{k}_\Lambda \underline{k}_N \lambda_b^j \lambda_a^i)|V|\Psi_{0\Sigma}^i\rangle A^i(E),\end{aligned} \tag{2.17}$$

where

$$E_\Sigma = E - (Z_i - 1)\varepsilon_P^i - Z_j \varepsilon_P^j, \tag{2.18}$$

is the energy E minus the unperturbed energy of the proton core in the substitutional l_i state. Here E is the energy (without rest masses) of the system in the Σ channel without counting the passive neutrons. In the absence of the ΣΛ coupling $E = E_0^i$ and $E_\Sigma = \varepsilon_\Sigma^i$ (see Eq. (2.4)).

In solving Eqs (2.15–17) for A and B, we follow closely Fano [16]. Eqs (2.16–17) are satisfied by:

$$\begin{aligned}&B^i\left(E, \underline{k}_\Lambda \underline{k}_N \lambda_a^i \lambda_b^k\right) \\ &\quad = c_{ik}^{-1}\{\mathcal{P}/e^k + z_i(E)\delta(e^k)\}\langle \Phi(\underline{k}_\Lambda \underline{k}_N \lambda_b^k \lambda_a^i)|V|\Psi_{0\Sigma}^i\rangle A^i(E),\end{aligned} \tag{2.19}$$

where $e^k = E_\Sigma + \Delta + \varepsilon_{\rm P}^k - \varepsilon_\Lambda(k_\Lambda) - \varepsilon_{\rm N}(k_{\rm N})$, and $c_{ik} = 1 + \delta_{ik}$. To determine $z_i(E)$, we insert expression (2.19) into Eq. (2.15), and get:

$$z_i(E) = 2\pi \left[E_\Sigma - \varepsilon_\Sigma^i - F^i(E_\Sigma)\right] / \Gamma^i(E_\Sigma) , \tag{2.20}$$

where

$$F^i = \sum\{k = i, j\} Z_k F^{i(k)} , \quad \Gamma^i = \sum\{k = i, j\} Z_k \Gamma^{i(k)} , \tag{2.21}$$

$$\left.\begin{matrix} Z_k F^{i(k)} \\ Z_k \Gamma^{i(k)} \end{matrix}\right\} = c_{ik}^{-1} \sum\{\underline{k}_\Lambda \underline{k}_{\rm N} \lambda_b^k \lambda_a^i\} \times \left|\langle \Phi(\underline{k}_\Lambda \underline{k}_{\rm N} \lambda_b^k \lambda_a^i) | V | \Psi_{0\Sigma}^i \rangle\right|^2 \times \begin{cases} \mathcal{P}/e^k , \\ \pi\delta\left(e^k\right) . \end{cases} \tag{2.22}$$

To determine A^i, we use the ortho-normalization condition:

$$\langle \psi_{E'}^i | \Psi_E^i \rangle = A^i\left(E'\right)^* A^i(E) + \sum_{k=i,j} \times \sum\{\underline{k}_\Lambda \underline{k}_{\rm N} \lambda_a^i \lambda_b^k\} c_{ik} B^i\left(E', \underline{k}_\Lambda \underline{k}_{\rm N} \lambda_a^i \lambda_b^k\right)^* B^i\left(E, \underline{k}_\Lambda \underline{k}_{\rm N} \lambda_a^i \lambda_b^k\right) = \delta\left(E' - E\right) . \tag{2.23}$$

After inserting into this condition expression (2.19), and using Eq. (2.20), we obtain our final result for $A^i(E)$:

$$|A^i(E)|^2 = (2\pi)^{-1} \frac{\Gamma^i(E_\Sigma)}{\left(\left[E_\Sigma - \varepsilon_\Sigma^i - F^i(E_\Sigma)\right]^2 + \left[\Gamma^i(E_\Sigma)/2\right]^2\right)} . \tag{2.24}$$

As long as F^i and Γ^i are slowly varying with E_Σ, they represent the energy shift and the width of the l_i substitutional state, implied by the $\Sigma\Lambda$ coupling.

With the help of the identity $1/(x + i\eta) = \mathcal{P}/x - i\pi\delta(x)$, we may write expressions (2.22) for $F^{i(k)}$ and $\Gamma^{i(k)}$ as a single expression for $\mathcal{F}^{i(k)} \equiv F^{i(k)} - i\Gamma^{i(k)}/2$:

$$\mathcal{F}^{i(k)} = (c_{ik} Z_k)^{-1} \times \sum\{\underline{k}_\Lambda \underline{k}_{\rm N} \lambda_b^k \lambda_a^i\} \left|\langle \Phi(\underline{k}_\Lambda \underline{k}_{\rm N} \lambda_b^k \lambda_a^i) | V | \Psi_{0\Sigma}^i \rangle\right|^2 / \left(e_k + i\eta\right) , \tag{2.25}$$

and Eqs (2.21) as:

$$\mathcal{F}^i \equiv F^i - i\Gamma^i/2 = Z_i \mathcal{F}^{i(i)} + Z_j \mathcal{F}^{i(j)} . \tag{2.26}$$

From Eq. (2.11), with ortho-normalization properties (2.14) of the $|\Phi\rangle$ states, we have:

$$\begin{aligned}&\langle \Phi(\underline{k}_\Lambda \underline{k}_\mathrm{N} \lambda_b^k \lambda_a^i) | V | \Psi_{0\Sigma}^i \rangle \\ &\qquad = Z_i^{-1/2} \left(\psi_{\underline{k}_\Lambda}^\Lambda \psi_{\underline{k}_\mathrm{N}}^\mathrm{N} \middle| v \middle| \psi_{\lambda_a^i}^\Sigma \psi_{\lambda_b^k}^\mathrm{P} - \delta_{ki} \psi_{\lambda_b^i}^\Sigma \psi_{\lambda_b^i}^\mathrm{P} \right) . \end{aligned} \tag{2.27}$$

Since we have neglected V_Λ in our model, the Λ scattering states $\psi_{\underline{k}_\mathrm{N}}^\Lambda$ are simply plane waves. Similarly, in our calculation of $\mathcal{F}^i$, we approximate the neutron scattering states $\psi_{\underline{k}_\mathrm{N}}^\mathrm{N}$ by plane waves (the energetic N produced in the $\Sigma\Lambda$ conversion should not be much affected by V_N). Thus we have:

$$\begin{aligned}\psi_{\underline{k}_\Lambda}^\Lambda (\vec{r}_1 \xi_1) &= \varphi_{\vec{k}_\Lambda} (\vec{r}_1) \zeta_{\nu_\Lambda} (\xi_1) , \\ \psi_{\underline{k}_\mathrm{N}}^\mathrm{N} (\vec{r}_2 \xi_2) &= \varphi_{\vec{k}_\mathrm{N}} (\vec{r}_2) \zeta_{\nu_\mathrm{N}} (\xi_2) , \end{aligned} \tag{2.28}$$

where $\varphi_{\vec{k}}(\vec{r}) = \exp(i\vec{k}\vec{r})$.

Using expression (2.27) and approximation (2.28), we may write (2.25) in the form:

$$\begin{aligned}\mathcal{F}^{i(k)} &= (Z_i Z_k)^{-1} \sum \{\lambda_a^i \lambda_b^k\} \int d(1)d(2)d(1')d(2') \psi_{\lambda_a^i}^{\Sigma^*}(1) \psi_{\lambda_b^k}^{\mathrm{P}^*}(2) v(1,2) \\ &\quad \times \widetilde{\mathcal{G}}_{E_\Sigma + \Delta + \epsilon_\mathrm{P}^k} (12; 1'2') v(1'2') \left[\psi_{\lambda_a^i}^\Sigma (1') \psi_{\lambda_b^k}^\mathrm{P} (2') - \delta_{ik} \psi_{\lambda_b^i}^\Sigma (1') \psi_{\lambda_a^i}^\mathrm{P} (2') \right] \\ &\equiv (Z_i Z_k)^{-1} \sum \{\lambda_a^i \lambda_b^k\} \langle \psi_{\lambda_a^i}^\Sigma \psi_{\lambda_b^k}^\mathrm{P} | v \widetilde{\mathcal{G}}_{E_\Sigma + \Delta + \epsilon_\mathrm{P}^k} v | \psi_{\lambda_a^i}^\Sigma \psi_{\lambda_b^k}^\mathrm{P} - \delta_{ik} \psi_{\lambda_b^i}^\Sigma \psi_{\lambda_a^i}^\mathrm{P} \rangle , \end{aligned} \tag{2.29}$$

where $1 = \vec{r}_1 \xi_1, \ldots, 2' = \vec{r}_2{}' \xi_2'$, and $\widetilde{\mathcal{G}}$ is the Green's function of two non-interacting particles, Λ and N,

$$\widetilde{\mathcal{G}}_\mathcal{E}(12; 1'2') = \delta(\xi_1, \xi_1') \, \delta(\xi_2, \xi_2') \, \mathcal{G}_\mathcal{E} (\vec{r}_1 \vec{r}_2; \vec{r}_1{}' \vec{r}_2{}') , \tag{2.30}$$

$$\mathcal{G}_\mathcal{E} (\vec{r}_1 \vec{r}_2; \vec{r}_1{}' \vec{r}_2{}') = (2\pi)^{-6} \frac{\int d\vec{k}_\Lambda d\vec{k}_\mathrm{N} \varphi_{\vec{k}_\Lambda} (\vec{r}_1) \varphi_{\vec{k}_\mathrm{N}} (\vec{r}_2) \varphi_{\vec{k}_\Lambda}^* (\vec{r}_1{}') \varphi_{\vec{k}_\mathrm{N}}^* (\vec{r}_2{}')}{[\mathcal{E} - \varepsilon_\Lambda (k_\Lambda) - \varepsilon_\mathrm{N} (k_\mathrm{N}) + i\eta]} . \tag{2.31}$$

If the coupling potential is central and spin-independent (which we shall assume in our model calculation),

$$v(1,2) = v(r_{12}) \,, \tag{2.32}$$

then we may carry out the spin summations, and we get:

$$\mathcal{F}^{i(k)}(E_\Sigma) = \mathcal{F}_D^{i(k)}(E_\Sigma) - \tfrac{1}{2}\delta_{ik}\mathcal{F}_{Ex}^{i(i)}(E_\Sigma) \,, \tag{2.33}$$

where

$$\begin{aligned}
\mathcal{F}_D^{i(k)}(E_\Sigma) &= \left[(2l_i+1)(2l_k+1)\right]^{-1}\sum\{m_a m_b\} \\
&\times \int d\vec{r}_1 d\vec{r}_2 d\vec{r}_1{}' d\vec{r}_2{}' \psi^{\Sigma^*}_{l_i m_a}(\vec{r}_1)\,\psi^{\mathrm{P}^*}_{l_k m_b}(\vec{r}_2) \\
&\times v(r_{12})\,\mathcal{G}_{E_\Sigma+\Delta+\varepsilon^k_{\mathrm{P}}}(\vec{r}_1\vec{r}_2;\vec{r}_1{}'\vec{r}_2{}')\, v(r'_{12})\,\psi^{\Sigma}_{l_i m_a}(\vec{r}_1{}')\,\psi^{\mathrm{P}}_{l_k m_b}(\vec{r}_2{}') \\
&\equiv \left[(2l_i+1)(2l_k+1)\right]^{-1}\sum\{m_a m_b\} \\
&\times \left\langle \psi^{\Sigma^*}_{l_i m_a}\psi^{\mathrm{P}^*}_{l_k m_b}\right| v\mathcal{G}_{E_\Sigma+\Delta+\varepsilon^k_{\mathrm{P}}}(\vec{r}_1\vec{r}_2;\vec{r}_1{}'\vec{r}_2{}')\, v \left|\psi^{\Sigma}_{l_i m_a}\psi^{\mathrm{P}}_{l_k m_b}\right\rangle ,
\end{aligned} \tag{2.34}$$

$$\begin{aligned}
\mathcal{F}_{Ex}^{i(i)}(E_\Sigma) &= (2l_i+1)^{-2}\sum\{m_a m_b\} \\
&\times \left\langle \psi^{\Sigma^*}_{l_i m_a}\psi^{\mathrm{P}^*}_{l_i m_b}\right| v\mathcal{G}_{E_\Sigma+\Delta+\varepsilon^i_{\mathrm{P}}}(\vec{r}_1\vec{r}_2;\vec{r}_1{}'\vec{r}_2{}')\, v \left|\psi^{\Sigma}_{l_i m_b}\psi^{\mathrm{P}}_{l_i m_a}\right\rangle .
\end{aligned} \tag{2.35}$$

Notice that for $i = 0$, *i.e.*, for $l_i = l_0 = 0$, $\mathcal{F}_{Ex}^{0(0)} = \mathcal{F}_D^{0(0)}$, and thus $\mathcal{F}^{0(0)} = \frac{1}{2}\mathcal{F}_D^{0(0)}$.

A similar procedure in the case of the hypernuclear g.s. (m_1), Eq. (2.5), leads (for central spin-independent v) to the result:

$$|A^{\mathrm{gs}}(E)|^2 = (2\pi)^{-1}\frac{\Gamma^{\mathrm{gs}}(E_\Sigma)}{\left(\left[E_\Sigma - \varepsilon^0_\Sigma - F^{\mathrm{gs}}(E_\Sigma)\right]^2 + \left[\Gamma^{\mathrm{gs}}(E_\Sigma)/2\right]^2\right)} \,, \tag{2.36}$$

where

$$\mathcal{F}^{\mathrm{gs}} \equiv F^{\mathrm{gs}}(E_\Sigma) - i\Gamma^{\mathrm{gs}}(E_\Sigma)/2 = Z_0\mathcal{F}_D^{0(0)}(E_\Sigma) + (Z_1-1)\mathcal{F}_D^{0(1)}(E_\Sigma) \,, \tag{2.37}$$

and the connection between E and E_Σ is given by:

$$E_\Sigma = E - Z_0\varepsilon^0_{\mathrm{P}} - (Z_1-1)\,\varepsilon^1_{\mathrm{P}} \,. \tag{2.38}$$

Notice that A^{gs} does not depend on m_1. In the absence of the $\Sigma\Lambda$ coupling $E = E^{\mathrm{gs}}_0$ and $E_\Sigma = \varepsilon^0_\Sigma$ (see Eq. (2.6)). The connection between E and E_Σ

is for the g.s. the same as for substitutional p state (Eq. (2.18) for $i = 1$), but the ranges of E_Σ relevant for A^{gs} and A^1 differ by about $\varepsilon_\Sigma^1 - \varepsilon_\Sigma^0$.

By using the known analytical formula for the two-particle Green's function [18–21], one obtains for $\mathcal{F}_D^{i(k)}$ and $\mathcal{F}_{Ex}^{i(i)}$ expressions involving threefold integrations. These expressions are derived in Appendix A.

3. Results and discussion

For the $\Sigma\Lambda$ coupling potential v responsible for the $\Sigma^-\mathrm{P} \to \Lambda\mathrm{N}$ process, we assume the central spin-independent Yukawa form:

$$v(r_{12}) = \chi \exp\left(-\mu r_{12}\right)/\mu r_{12}\,, \tag{3.1}$$

and thus in expansion (A.10), we have

$$v_L(r_1 r_2) = j_L(i\mu r_<)\, h_L^{(1)}(i\mu r_>)\,, \tag{3.2}$$

where j_L is the spherical Bessel function, $h_L^{(1)}$ the spherical Hankel function of first kind, $r_< = \mathrm{Min}(r_1 r_2)$ and $r_> = \mathrm{Max}(r_1 r_2)$.

For the range parameter μ, we use the OPE value: $\mu = m_{\pi^+}c/\hbar = 0.7\ \mathrm{fm}^{-1}$, and for strength parameter χ the value: $\chi = 28.6$ MeV. With these parameters, the total cross section σ for $\Sigma^-\mathrm{P} \to \Lambda\mathrm{N}$, calculated in Born approximation, coincides at $p_\Sigma(\mathrm{lab}) = 300$ MeV/c with the nuclear total cross section of 4.87 mb calculated with model D of the Nijmegen hyperon-hyperon interaction [22]. Also at higher momenta p_Σ, where the Born approximation is more justified, our Born results agree satisfactorily with the model D results. This is shown in Fig. 1, taken from [22] with added Born results (broken curve) obtained with our v.

We assume $V_\mathrm{P}(r)$ and $V_\Sigma(r)$ to be square well potentials of radius $R = 3$ fm, which should simulate the ^{16}O target. For the depth of $V_\mathrm{P}(r)$ we use $V_\mathrm{P}^0 = 60$ MeV, which leads to the s.p. energies $\varepsilon_\mathrm{P}^0 \equiv \varepsilon_\mathrm{P}^s = -44.3$ MeV and $\varepsilon_\mathrm{P}^1 \equiv \varepsilon_\mathrm{P}^p = -28.5$ MeV. The corresponding empirical proton s.p. energies in ^{16}O are: -44 MeV in the $s_{1/2}$ state and $-19(-12.5)$ MeV in the $p_{3/2}(p_{1/2})$ state (see [13]). For the depth of $V_\Sigma(r)$ we use $V_\Sigma^0 = 20$ MeV, which is compatible with the estimates based on model D of the Nijmegen interaction (see [1]). The corresponding s.p. Σ energies are: $\varepsilon_\Sigma^0 \equiv \varepsilon_\Sigma^s = -10.0$ MeV and $\varepsilon_\Sigma^1 \equiv \varepsilon_\Sigma^p = -1.0$ MeV. The Coulomb interaction is disregarded in our calculations.

In calculating the functions $\mathcal{F}_D^{i(k)}$ and $\mathcal{F}_{Ex}^{i(i)}$, we applied the formulae collected in Appendix A. First, we calculated and tabulated the functions $A_{nl_1l_2L}^{i(k)}(r)$ by performing the single integration over α, Eq. (A.17). Next,

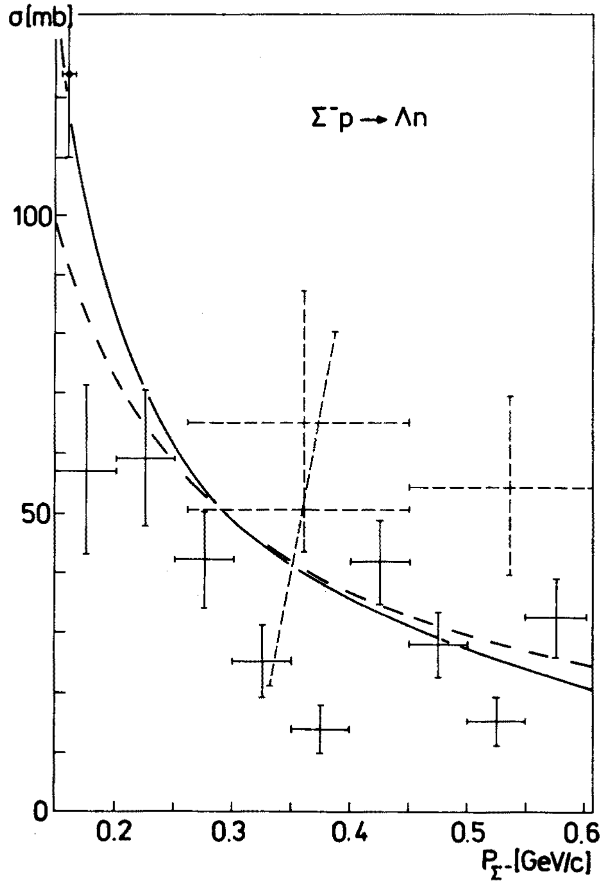

Fig. 1. The D model results (solid curve) and our Born results (broken curve) for the total cross section $\sigma(\Sigma^- \mathrm{P} \to \Lambda\mathcal{N})$ compared with the data of the Massachusetts group [23]. The datum with the black square is from the Heidelber group [24]. The dashed data have been calculated in [22] from the results for $\Lambda\mathrm{P} \to \Sigma^0\mathrm{P}$ [25].

we calculated the functions $f^{i(k)}(nl_1l_2, LL'; \kappa)$ by carrying out the twofold integration in (A.16). The Simpson rule was used in all integrations. The summation over the orbital quantum numbers in (A.12) and in the analogical expression for $\mathcal{F}^{i(i)}_{Ex}$ (severely restricted by the triangular conditions and parity rules obeyed by the Wigner 3-j and 6-j symbols in Eqs (A.18–19)) was performed for each value of n within limits assuring a sufficient accuracy. The sum over n in (A.15) was restricted to $n = 0, 1, 2$ (in the case of $\widetilde{\Delta} = 4\Delta$ terms with $n = 4, 5$ were also included), which assured the overall

5–10% accuracy of our results. In general, the accuracy of our results for the imaginary part of $\mathcal{F}^i$ (hence for Γ^i) is better than that for the real part (hence for F^i). Furthermore, the accuracy of our results for $\mathcal{F}^0$ (hence for F^0, Γ^0, and F^{gs}, Γ^{gs}) is better than that for $\mathcal{F}^1$ (hence for F^1, Γ^1).

After inserting into expressions (2.26) and (2.37) the calculated values of $\mathcal{F}_D^{i(k)}(E_\Sigma)$ and $\mathcal{F}_{Ex}^{i(i)}(E_\Sigma)$, we get for F^i, Γ^i, and F^{gs}, Γ^{gs} the following results (in MeV):

$$\begin{aligned} F^s\left(\varepsilon_\Sigma^s\right) &= -1.6\,, \quad F^p\left(\varepsilon_\Sigma^p\right) = -0.5\,, \quad F^{gs}\left(\varepsilon_\Sigma^s\right) = -1.5\,, \\ \Gamma^s\left(\varepsilon_\Sigma^s\right) &= 5.1\,, \quad \Gamma^p\left(\varepsilon_\Sigma^p\right) = 3.4\,, \quad \Gamma^{gs}\left(\varepsilon_\Sigma^s\right) = 4.8\,, \end{aligned} \tag{3.3}$$

where we use the notation s for $i = 0$ and p for $p = 1$.

Our results for $|A(E)|^2$ for the substitutional p state and for the hypernuclear g.s., Eqs (2.24) and (2.36), are plotted in Fig. 2 as functions of $E_\Sigma = E - 2\varepsilon_P^s - 5\varepsilon_P^p$. Results for the substitutional s state are similar to those for the g.s. They are shown in Fig. 3 (here $E_\Sigma = E - \varepsilon_P^s - 6\varepsilon_P^p$).

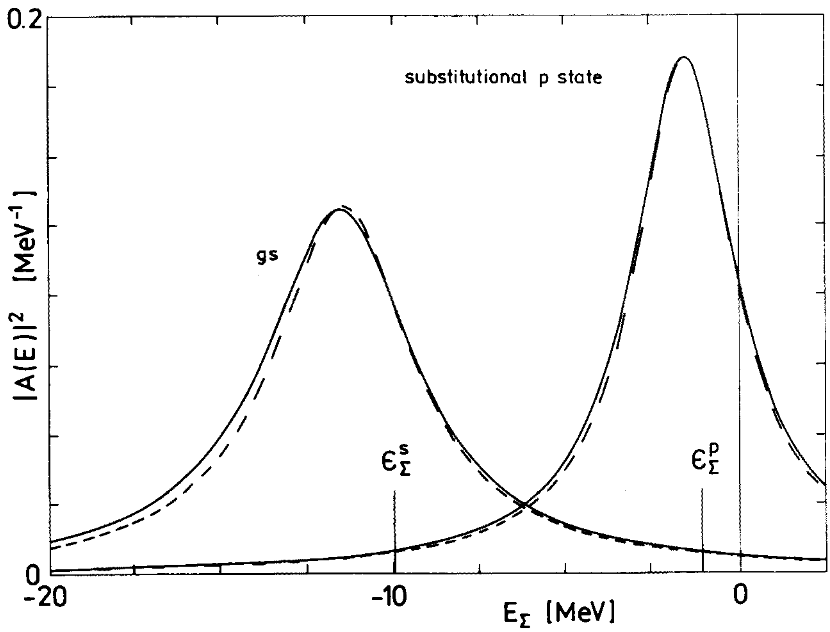

Fig. 2. Distributions $|A(E)|^2$ for the hypernuclear g.s. and the substitutional p state, obtained with energy dependent F's and Γ's (solid lines) and with constant F's and Γ's (broken lines).

All the F's and Γ's are functions of E_Σ. However, this energy dependence is very weak, particularly for the Γ's. This is seen in Fig. 2, where in the solid curves this dependence is taken into account, whereas the broken curves have been obtained by using for all the F's and Γ's the constant values given in (3.3) (thus they are Breit–Wigner distributions). The small difference between the "exact" distributions (solid lines) and the Breit–Wigner distributions (broken lines) is irrelevant for our discussion in which we shall identify the F's and Γ's in (3.3) with the energy shifts and widths. (The curves in Fig. 3 have been obtained with constant values of all the F's and Γ's.)

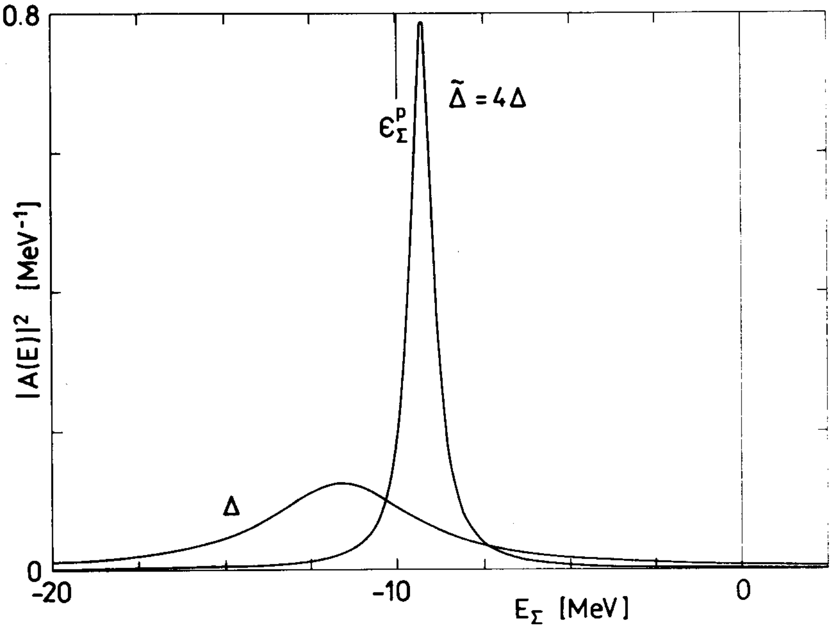

Fig. 3. Distributions $|A(E)|^2$ for substitutional s state obtained with $\Delta = \tilde{\Delta}$ and $\Delta = 4\tilde{\Delta}$.

Our results for the widths, $\Gamma \sim (3-5)$ MeV, appear reasonable compared with the widths observed in experiment. In the case of the substitutional s state, the state of the nuclear core (s state hole in ^{16}O) is by itself unstable with the width Γ_c of about 14 MeV [26], and thus the observed width should be $\Gamma^s + \Gamma_c \cong 19$ MeV, *i.e.*, much bigger than Γ^s.

Our results for the energy shifts F are of the order of 1 MeV and are negative. This means that in the presence of $\Sigma\Lambda$ coupling the binding of Σ increases. On the other hand, the rigid nuclear core model [15] with $\Delta \cong 80$ MeV leads to positive energy shifts. To simulate the excitation of the nuclear core in the Λ channel, a smaller value of Δ (denoted by $\tilde{\Delta}$) was also consider in [15], and for $\tilde{\Delta} = \Delta/4$ the resulting energy shifts were found to be negative (and the resulting widths bigger than for $\Delta \cong 80$ MeV). In our present model, the non-rigidity of the nuclear core is fully taken into account. Consequently, we expect to obtain results comparable with those of the rigid nuclear model (with $\Delta \cong 80$ MeV), if in our present model we replace Δ by a bigger value $\tilde{\Delta}$. Let us consider the value of $\tilde{\Delta} = 4\Delta$, obtained by replacing M_Λ by $\widetilde{M}_\Lambda = 0.88 M_\Lambda$ and $\widetilde{M}_{\rm N} = 0.88 M_{\rm N}$. In this case we get (in MeV):

$$\begin{aligned} \tilde{F}^s\left(\varepsilon_\Sigma^s\right) &= 0.70\,, & \tilde{F}^{gs}\left(\varepsilon_\Sigma^s\right) &= 0.65\,, \\ \tilde{\Gamma}^s\left(\varepsilon_\Sigma^s\right) &= 0.81\,, & \tilde{\Gamma}^{gs}\left(\varepsilon_\Sigma^s\right) &= 0.75\,, \end{aligned} \tag{3.4}$$

The resulting distribution $|\tilde{A}^s|^2$ is shown in Fig. 3. As expected, the present results obtained with $\tilde{\Delta} = 4\Delta$ are in a qualitative agreement with the rigid nuclear core results of [15] (obtained with $\Delta \cong 80$ MeV). The energy shifts $\tilde{F}$ are positive, and the widths $\tilde{\Gamma} < \Gamma$.

The dependence of the energy shifts F and Δ (increasing of F with increasing Δ) is easy to explain. Let us consider, *e.g.*, the contribution of $\mathcal{F}^{s(s)}$ to F^s, *i.e.*, $\mathrm{Re}\mathcal{F}^{s(s)}$ (the same reasoning applies to other contributions) which may be written in the form (see Eq. (2.34)):

$$\begin{aligned} \mathrm{Re}\mathcal{F}^{s(s)}\left(\varepsilon_\Sigma^s\right) &= \tfrac{1}{2}\,(2\pi)^{-6}\int d\vec{k}_\Lambda d\vec{k}_{\rm N} \\ &\times \frac{\mathcal{P}}{\varepsilon_\Sigma^s + \varepsilon_{\rm P}^s + \Delta - \varepsilon_\Lambda\left(k_\Lambda\right) - \varepsilon_{\rm N}\left(k_{\rm N}\right)}\left|\left(\varphi_{\vec{k}_\Lambda}\varphi_{\vec{k}_{\rm N}}\right|v\left|\psi_s^\Sigma\psi_s^{\rm P}\right)\right|^2, \end{aligned} \tag{3.5}$$

where ψ_s stands for ψ_{lm} with $l = m = 0$.

Let us divide the six-dimensional momentum space of $\vec{k}_\Lambda$ and $\vec{k}_{\rm N}$ into regions I and II defined by:

$$\varepsilon_\Lambda^s + \varepsilon_{\rm N}^s + \Delta - \varepsilon_\Lambda\left(k_\Lambda\right) - \varepsilon_{\rm N}^s\left(k_{\rm N}\right)\begin{cases} > 0 & \text{in region I}\,, \\ < 0 & \text{in region II}\,. \end{cases} \tag{3.6}$$

Since the contribution of region I to the integral in (3.5) is positive and that of region II is negative, the value of $\mathrm{Re}\mathcal{F}^{s(s)}$ increases when we increase the size of region I (and thus decrease the size of region II). This is exactly what happens when we increase Δ.

Another factor affecting the magnitude of $\mathrm{Re}\mathcal{F}^{s(s)}$ is the square of the Fourier transform of $v|\psi_s^\Sigma\psi_s^{\rm P})$, $\left|\left(\varphi_{\vec{k}_\Lambda}\varphi_{\vec{k}_{\rm N}}\right|v\left|\psi_s^\Sigma\psi_s^{\rm P}\right)\right|^2$, which for our v, Eq. (3.1), decreases with increasing Λ and N momenta, enhances the contribution of region I compared to that of region II, and thus makes the energy shift larger (algebraically).

This effect is stronger when the Fourier transform decreases faster with increasing momenta, which may be realized by increasing the range of $v(r)$. Our $v(r)$, Eq. (3.1), has the OPE range which appears to be the longest range physically acceptable for the $\Sigma\Lambda$ conversion interaction. Thus the negative energy shifts F listed in (3.3) would become more negative for more realistic v containing components of shorter range.

The discussion of the widths Γ is similar. *E.g.*, the contribution of $\mathcal{F}^{s(s)}$ to $\Gamma^s/2$, *i.e.*, $-\operatorname{Im}\mathcal{F}^{s(s)}$, may be written as

$$-\operatorname{Im}\mathcal{F}^{s(s)} = \tfrac{1}{2}\pi\,(2\pi)^{-6}\int d\vec{k}_\Lambda d\vec{k}_{\mathrm{N}}\delta\left(\varepsilon^s_\Sigma + \varepsilon^s_{\mathrm{P}} + \Delta - \varepsilon_\Lambda\left(k_\Lambda\right) - \varepsilon_{\mathrm{N}}\left(k_{\mathrm{N}}\right)\right)$$
$$\times\left|\left(\varphi_{\vec{k}_\Lambda}\varphi_{\vec{k}_{\mathrm{N}}}\middle|v\middle|\psi^\Sigma_s\psi^{\mathrm{P}}_s\right)\right|^2. \tag{3.7}$$

Here only the Λ and N momenta contribute, which satisfy the condition $\varepsilon^s_\Sigma + \varepsilon^s_{\mathrm{P}} + \Delta - \varepsilon_\Lambda\left(k_\Lambda\right) - \varepsilon_{\mathrm{N}}\left(k_{\mathrm{N}}\right) = 0$. With increasing Δ, these momenta increase, and the square of the Fourier transform decreases. Thus with increasing Δ, the width becomes smaller. Similarly as in the case of the energy shifts, this effect of the Fourier transform is stronger for $v(r)$ with a longer range. The widths Γ listed in (3.3) would decrease if we used $v(r)$ with a longer range (with χ readjusted to preserve the fit to the $\Sigma^-\mathrm{P} \to \Lambda\mathrm{N}$ cross section).

In Fig. 4, we show the double differential cross sections $d^2\sigma/d\hat{k}_\pi dE_\pi$ at $\Theta = 0°$ and $p_{\mathrm{K}} = \hbar k_{\mathrm{K}} = 450$ MeV/c for the (K^-,π^+) reactions on our "^{16}O" target, leading to the Σ hypernuclear g.s., and to the substitutional s and p states. The cross sections have been calculated in the PWIA with zero range for the elementary process $\mathrm{K}^-\mathrm{P} \to \Sigma^-\pi^+$. The respective formulae are derived in Appendix B. The cross sections are shown as functions of $\Delta\mathrm{M} = (\mathrm{M_{HY}} - \mathrm{M_T})c^2$ (see Eq. (B.10)). For comparison, the corresponding values of Σ binding energy B_Σ, Eq. (B.19), are also shown. In calculating the $|A|^2$ factors in Eqs (B.7), (B.15), the energy dependence of the F's and Γ's was taken into account. The units on the ordinate axis are arbitrary (only counting rates are measured in experiment). In case of no $\Sigma\Lambda$ coupling, the Σ hypernuclear states become discrete, and instead of the double differential cross section, we have the single differential cross section $d\sigma/d\hat{k}_\pi$ for transitions to these discrete states, whose magnitude and position is indicated by the vertical lines in Fig. 4.

The cross section for the transition to the hypernuclear g.s., Eqs (B.15), (B.17), contains the factor $\left|\left(R^\Sigma_0\left|j_1(qr)\right|R^{\mathrm{P}}_1\right)\right|^2$ which makes it much smaller than the cross section for the transition to the substitutional p state, which contains the much bigger factor $\left|\left(R^\Sigma_1\left|j_0(qr)\right|R^{\mathrm{P}}_1\right)\right|^2$. For kaon momenta p_{K} closer to the "magic momentum" of about 270 MeV/c, the relative probability of producing the substitutional p state (compared to that for the g.s.)

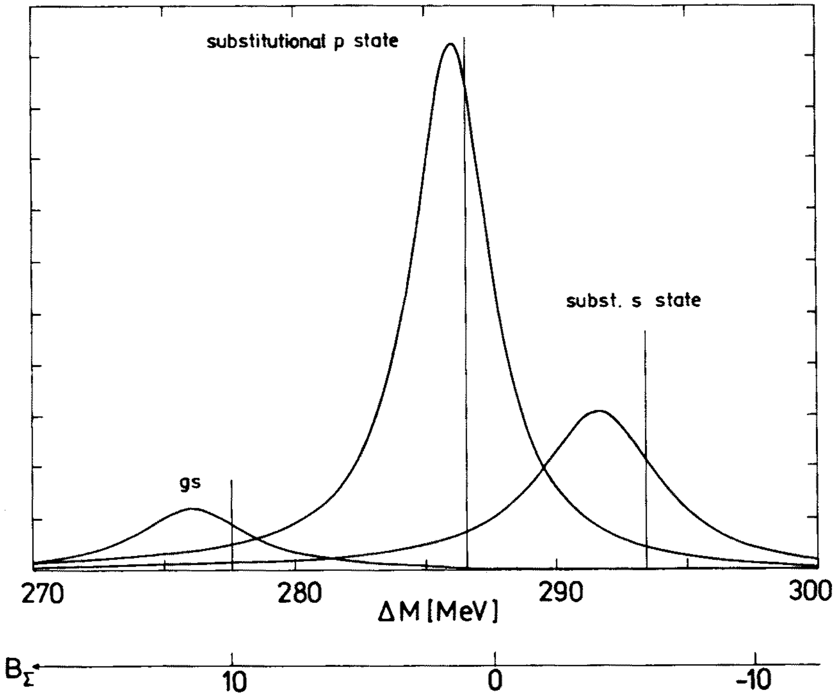

Fig. 4. The double differential cross sections $d^2\sigma/d\hat{k}_\pi dE_\pi$ at $\theta = 0°$ and $p_K = 450$ MeV/c for the excitation of the g.s., and the s and p substitutional states in the (K^-, π^+) reaction on our model of ^{16}O target. Vertical lines are the respective differential cross sections $d\sigma/d\hat{k}_\pi$ in case of no ΣΛ coupling.

would be even more pronounced. For the substitutional s state, we also have a large factor $\left|\left(R_0^\Sigma\,|j_0(qr)|\,R_0^p\right)\right|^2$, however $Z_i = 2(2l_i+1)$ in expressions (B.7), (B.12) makes the cross section for the s state about three times smaller than that for the p state.

In the more realistic case of the presence of the ls coupling our scheme would have to be slightly modified. In this case the peak in Fig. 4 corresponding to the substitutional p state would split into $p_{1/2}$ and $p_{3/2}$ peaks, and the g.s. peak would be smaller — its height would be proportional to the number of protons in the $p_{1/2}$ shell, $Z_{p_{1/2}} = 2$ (instead of $Z_1 = 6$, see Eq. (B.15)).

In the effective Λ channel approximation, the present scheme was applied in [15]. The conclusion was that because of the ΣΛ coupling the Σ hypernuclear bound states not only acquire a width, but may be shifted to positive energies, which would explain the pion spectra observed in (K, π) reactions. This conclusion, however, should be questioned for two reasons. First, the results of [15] have been sensitive to the value of $\widetilde{\Delta}$ which could not be determined precisely. Second, for those values of $\tilde{\Delta}$ for which the Σ bound states are shifted into continuum, the following difficulty arises. Whereas the starting assumption in [15] was that one may restrict oneself to Σ bound states (Ansatz (4) in [15]), the end result was a Σ state in the continuum.

The two difficulties of [15] do not exist in our present approach. First, it does not contain any adjustable parameter $\tilde{\Delta}$. Second, the energy shifts obtained in the present approach are small and negative. Thus the restriction to bound states in the Σ channel (our Ansatz (2.10)) appears justified.

The present paper suggests that the $\Sigma\Lambda$ coupling shifts the Σ bound states to even more bound states and thus can not explain the peaks observed in (K, π) reactions, which correspond to Σ states with positive energy. In reaching this conclusion, we have assumed that we know the Σ s.p. potential V_Σ, although in principle it should be derived from the full hyperon-nucleon interaction

$$\begin{pmatrix} V(\Sigma\mathcal{N},\Sigma\mathcal{N}) & , & V \\ V & , & V(\Lambda\mathcal{N},\Lambda\mathcal{N}) \end{pmatrix} .$$

Thus in the present approach, as in the approach of other authors, V_Σ is treated more or less phenomenologically. In this situation, the precise position of the Σ bound states in the presence of the $\Sigma\Lambda$ coupling is uncertain because of the uncertainty in V_Σ.

Appendix A

Expressions for $\mathcal{F}_D^{i(k)}$ and $\mathcal{F}_{Ex}^{i(i)}$

With the transformation

$$\begin{aligned} \vec{k}_\Lambda &= \left(2M_\Lambda/\hbar^2\right)^{\frac{1}{2}} \vec{k}_1 , & \vec{r}_1 &= \left(\hbar^2/2M_\Lambda\right)^{\frac{1}{2}} \vec{r}_1 , \\ \vec{k}_\mathrm{N} &= \left(2M_\mathrm{N}/\hbar^2\right)^{\frac{1}{2}} \vec{k}_2 , & \vec{r}_2 &= \left(\hbar^2/2M_\mathrm{N}\right)^{\frac{1}{2}} \vec{r}_2 , \end{aligned} \tag{A.1}$$

we may write expression (2.31) as:

$$\mathcal{G}_\mathcal{E}\left(\vec{r}_1\vec{r}_2;\vec{r}_1{}'\vec{r}_2{}'\right) = -\left(\left(2M_\Lambda/\hbar^2\right)\left(2M_\mathrm{N}/\hbar^2\right)\right)^{3/2} G_\kappa\left(\vec{r}_1\vec{r}_2;\vec{r}_1{}'\vec{r}_2{}'\right), \tag{A.2}$$

where

$$G_\kappa(\vec{r}_1\vec{r}_2;\vec{r}_1{}'\vec{r}_2{}') = \left(\frac{1}{2\pi}\right)^6 \int d\vec{k}_1 \int d\vec{k}_2 \frac{\exp\left(i\vec{k}_1(\vec{r}_1-\vec{r}_1{}') + i\vec{k}_2(\vec{r}_2-\vec{r}_2{}')\right)}{\left(k_1^2+k_2^2-\kappa^2-i\eta\right)} \tag{A.3}$$

where $\kappa = \mathcal{E}^{1/2}$.

It appears that an analytical expression for two-particle Green's function (A.3) was first derived by Sommerfeld [18]. Later it was derived again by Chew [19]. Here, we shall apply the results for G_κ and its partial-wave expansion as presented by Morse and Feshbach [21].

We denote the polar coordinates of $\vec{r}_i (i = 1,2)$ by r_i, ϑ_i, φ_i. Notice that the polar coordinates of $\vec{r}_i$ are r_i, ϑ_i, φ_i, with the same angles ϑ_i, φ_i, and with

$$r_1 = \left(\hbar^2/2M_\Lambda\right)^{\frac{1}{2}} r_1 , \qquad r_2 = \left(\hbar^2/2M_{\mathrm{N}}\right)^{\frac{1}{2}} r_2 . \tag{A.4}$$

We denote by r the length of the six-dimensional vector $\vec{r} = (\vec{r}_1\vec{r}_2)$, *i.e.* $r = (r_1^2 + r_2^2)^{1/2}$, and introduce the angle α by the relations:

$$r_1 = r\cos\alpha , \qquad r_2 = r\sin\alpha , \qquad 0 \le \alpha \le \pi/2 . \tag{A.5}$$

We may describe the two vectors $\vec{r}_1$, $\vec{r}_2$ by the two-particle hyperspherical coordinates r, α, ϑ_1, φ_1, ϑ_2, φ_2. We have:

$$\int d\vec{r}_1 \int d\vec{r}_2 = \left[\left(\hbar^2/2M_\Lambda\right)\left(\hbar^2/2M_{\mathrm{N}}\right)\right]^{3/2} \int d\vec{r}_1 \int d\vec{r}_2$$

$$= \left[\left(\hbar^2/2M_\Lambda\right)\left(\hbar^2/2M_{\mathrm{N}}\right)\right]^{3/2} \int_0^\infty dr\, r^5 \int_0^{\pi/2} d\alpha \cos^2\alpha \sin^2\alpha \int d\hat{r}_1 \int d\hat{r}_2 , \tag{A.6}$$

where $\int d\hat{r}_i \equiv \int d\hat{r}_i = \int_0^\pi d\vartheta_i \sin\vartheta_i \int_0^{2\pi} d\varphi_i$.

The partial-wave expansion of G_κ in the hyperspherical coordinates is:

$$G_\kappa\left(\vec{r}_1\vec{r}_2;\vec{r}_1'\vec{r}_2'\right) = \sum\{l_1l_2m_1m_2\} Y_{l_1m_1}(\hat{r}_1)\, Y^*_{l_1m_1}(\hat{r}_1')\, Y_{l_2m_2}(\hat{r}_2)\, Y^*_{l_2m_2}(\hat{r}_2')$$
$$\times \left[i\pi/\left(rr'\right)^2\right] g_{\kappa\; l_1l_2}\left(r_1r_2; r_1'r_2'\right) , \tag{A.7}$$

$$g_{\kappa\; l_1l_2}\left(r_1r_2;r_1'r_2'\right) = \sum_{n=0}^{\infty} C_{n\; l_1l_2} \cos^{l_1}\alpha \sin^{l_2}\alpha \cos^{l_1}\alpha' \sin^{l_2}\alpha'$$
$$\times F\left(-n, l_1 + l_2 + n + 2 \left| l_2 + \frac{3}{2} \right| \sin^2\alpha\right)$$
$$\times J_{l_1+l_2+2n+2}\left(\kappa r_<\right) H^{(1)}_{l_1+l_2+2n+2}\left(\kappa r_>\right) , \tag{A.8}$$

where $r_> = \mathrm{Max}(r, r')$, $r_< = \mathrm{Min}(r, r')$, J_l denotes the Bessel function and $H_l^{(1)}$ the Hankel function of first kind, F is the hypergeometric function, and

$$C_{n\; l_1l_2} = \frac{(l_1 + l_2 + 2n + 2)(l_1 + l_2 + n + 1)!\,\Gamma\left(l_2 + n + \frac{3}{2}\right)}{n!\,\left[\Gamma\left(l_2 + \frac{3}{2}\right)\right]^2 \Gamma\left(l_1 + n + \frac{3}{2}\right)} . \tag{A.9}$$

To calculate $\mathcal{F}_D^{i(k)}$ defined in Eq. (2.34), we insert into this equation expression (A.2) with G_κ expanded into partial waves, Eq. (A.7). For the coupling potential $v(r_{12}) = \chi\tilde{v}(r_{12})$ (χ = strength parameter, $\tilde{v}$ = shape function), we use the expansion:

$$v(r_{12}) = -4\pi\chi \sum \{LM\} v_L(r_1 r_2) Y_{LM}^*(\hat{r}_1) Y_{LM}(\hat{r}_2) . \tag{A.10}$$

After taking into account (A.6), and introducing the notation:

$$\kappa_k = \left(E_\Sigma + \Delta + \varepsilon_{\mathrm{P}}^k\right)^{\frac{1}{2}} , \tag{A.11}$$

we get:

$$\mathcal{F}_D^{i(k)}(E_\Sigma) = -i\chi^2 \mathcal{C} \sum (l_1 l_2 L L') \mathcal{R}^{i(k)}(l_1 l_2 L L'; \kappa_k) \mathcal{X}_D^{i(k)}(l_1 l_2 L L') , \tag{A.12}$$

where $\mathcal{C} = (\pi/4)\left(\hbar^2/\sqrt{M_\Lambda M_{\mathrm{N}}}\right)^3$,

$$\begin{aligned} \mathcal{R}^{i(k)}&(l_1 l_2 L L'; \kappa) \\ &= \tfrac{1}{2}\int_0^\infty dr\, r^3 \int_0^\infty dr'\, r'^3 \int_0^{\pi/2} d\alpha \cos^2\alpha \sin^2\alpha \int_0^{\pi/2} d\alpha' \cos^2\alpha' \sin^2\alpha' \\ &\times R_{l_i}^\Sigma(r_1) R_{l_k}^{\mathrm{P}}(r_2) v_L(r_1 r_2) g_{\kappa\, l_1 l_2}(r_1 r_2; r_1' r_2') v_{L'}(r_1' r_2') \\ &\times R_{l_i}^\Sigma(r_1') R_{l_k}^{\mathrm{P}}(r_2') \end{aligned} \tag{A.13}$$

results from the integration over r_1, r_2, r_1', r_2', and

$$\begin{aligned} \mathcal{X}_D^{i(k)}(l_1 l_2 L L') &= (4\pi)^2 \left((2l_i+1)(2l_k+1)\right)^{-1} \\ &\times \sum \left(Y_{l_i m_a} Y_{l_k m_b} \left| Y_{LM}^* Y_{LM} \right| Y_{l_1 m_1} Y_{l_2 m_2}\right) \\ &\quad \left(Y_{l_1 m_1} Y_{l_2 m_2} \left| Y_{L'M'}^* Y_{L'M'} \right| Y_{l_i m_a} Y_{l_k m_b}\right) , \end{aligned} \tag{A.14}$$

(the summation is over all magnetic numbers) results from the integration over $\hat{r}_1, \hat{r}_2, \hat{r}_1', \hat{r}_2'$.

Similarly, we get for $\mathcal{F}_{Ex}^{i(i)}(E_\Sigma)$, Eq. (2.35), an expression identical with expression (A.12), but with $\mathcal{X}_D^{i(i)}$ replaced by $\mathcal{X}_{Ex}^{i(i)}$. In turn, the expression for $\mathcal{X}_{Ex}^{i(i)}$ is identical with expression (A.14) for $\mathcal{X}_D^{i(i)}$, but with $|Y_{l_i m_a} Y_{l_i m_b})$ replaced by $|Y_{l_i m_b} Y_{l_i m_a})$.

Using expression (A.8) for $g_{\kappa\, l_1 l_2}$, we may write Eq. (A.13) in the form:

$$\mathcal{R}^{i(k)}(l_1 l_2 L L'; \kappa) = \sum_n C_{n l_1 l_2} f^{i(k)}(n l_1 l_2 L L'; \kappa), \tag{A.15}$$

$$f^{i(k)}(n l_1 l_2 L L'; \kappa) = \int_0^\infty dr\, r^3 H^{(1)}_{l_1+l_2+2n+2}(\kappa r) \int_0^r dr'\, r'^3 J_{l_1+l_2+2n+2}(\kappa r')$$
$$\times \tfrac{1}{2}\left[A^{i(k)}_{n l_1 l_2 L}(r) A^{i(k)}_{n l_1 l_2 L'}(r') + A^{i(k)}_{n l_1 l_2 L'}(r) A^{i(k)}_{n l_1 l_2 L}(r')\right], \tag{A.16}$$

where

$$A^{i(k)}_{n l_1 l_2 L}(r) = \int_0^{\pi/2} d\alpha\, \cos^{l_1+2}\alpha\, \sin^{l_2+2}\alpha\, F(-n, l_1+l_2+n+2 | l_2+\tfrac{3}{2} | \sin^2\alpha)$$
$$\times R^{\Sigma}_{l_i}(r_1) R^{\mathrm{P}}_{l_k}(r_2) v_L(r_1 r_2). \tag{A.17}$$

Notice that $r_1 = (\hbar^2/2M_\Lambda)^{1/2} r\cos\alpha$ and $r_2 = (\hbar^2/2M_{\mathrm{N}})^{1/2} r\sin\alpha$. For $L = L'$, Eq. (A.16) is simplified because the two terms in the square brackets in Eq. (A.16) are identical.

A straightforward calculation leads to the following expressions for $\mathcal{X}_D$ and $\mathcal{X}_{Ex}$ in terms of Wigner 3-j and 6-j symbols:

$$\mathcal{X}^{i(k)}_D(l_1 l_2 L L') = (2L+1)(2l_1+1)(2l_2+1)$$
$$\begin{pmatrix} l_i & l_1 & L \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{pmatrix} l_k & l_2 & L \\ 0 & 0 & 0 \end{pmatrix}^2 \delta_{LL'}, \tag{A.18}$$

$$\mathcal{X}^{i(i)}_{Ex}(l_1 l_2 L L') = (2L+1)(2L'+1)(2l_1+1)(2l_2+1) \begin{pmatrix} l_i & l_1 & L \\ 0 & 0 & 0 \end{pmatrix}$$
$$\times \begin{pmatrix} l_i & l_2 & L \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_i & l_1 & L' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_i & l_2 & L' \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} l_i & l_2 & L \\ l_i & l_1 & L' \end{Bmatrix}. \tag{A.19}$$

Appendix B

PWIA for (K^-, π^+) *in-flight reactions*

Let us consider in PWIA the production of the l_i substitutional state. The initial and final states are:

$$|I\rangle = \exp(i\vec{k}_{\mathrm{K}}\vec{r})\begin{pmatrix}|0\rangle\\ 0\end{pmatrix}, \quad |F^i\rangle = \exp(i\vec{k}_{\pi}\vec{r})\begin{pmatrix}A^i(E)|\Psi^i_{0\Sigma}\rangle\\ |\Psi^i_{E\Lambda}\rangle\end{pmatrix}. \tag{B.1}$$

We assume a zero-range spin-independent interaction for the elementary process $\mathrm{K}^-\mathrm{P} \to \pi^+\Sigma^-$ (with a constant T matrix denoted by t), and obtain for the transition amplitude

$$\langle F^i|T|I\rangle = tA^i(E)^*\left\langle \Psi^i_{0\Sigma}\right|\int d\vec{r}\int d\xi\, \exp(-i\vec{q}\vec{r})\Phi^\dagger_\Sigma(\vec{r}\xi)\Phi_{\mathrm{P}}(\vec{r}\xi)\left|0\right\rangle, \tag{B.2}$$

where the momentum transfer

$$\vec{q} = \vec{k}_\pi - \vec{k}_K, \tag{B.3}$$

and $\Phi^\dagger_\Sigma(\vec{r}\xi)$, $\Phi_{\mathrm{P}}(\vec{r}\xi)$ are the operators of Σ creation and P annihilation at $\vec{r},\xi$:

$$\Phi^\dagger_\Sigma(\vec{r}\xi) = \sum\{\lambda_\Sigma\}a^\dagger_{\Sigma\lambda_\Sigma}\psi^\Sigma_{\lambda_\Sigma}(\vec{r}\xi)^*, \quad \Phi_{\mathrm{P}}(\vec{r}\xi) = \sum\{\lambda_{\mathrm{P}}\}a_{\mathrm{P}\lambda_\Sigma}\psi^{\mathrm{P}}_{\lambda_\Sigma}(\vec{r}\xi). \tag{B.4}$$

The continuum states are not included in expansions (B.4), because they are irrelevant in the present considerations.

When we insert expansions (B.4) into (B.2) (notice that with our form of $\psi^\Sigma_{\lambda_\Sigma}$ and $\psi^{\mathrm{P}}_{\lambda_\Sigma}$, Eqs (2.1-2), the ξ integration (summation) results in $\delta_{\nu_\Sigma\nu_{\mathrm{P}}}$), and use for $|\Psi^i_{0\Sigma}\rangle$ expression (2.3), we get:

$$\begin{aligned}\langle F^i|T|I\rangle &= tA^i(E)^*\sum\{l_{\mathrm{P}}\ m_{\mathrm{P}}\ l_\Sigma\ m_\Sigma\ \tilde{\nu}\}\int d\vec{r}\,\exp(-i\vec{q}\vec{r})\psi^{\Sigma^*}_{l_\Sigma m_\Sigma}(\vec{r})\psi^{\mathrm{P}}_{l_{\mathrm{P}}m_{\mathrm{P}}}(\vec{r})\\ &\times (Z_i)^{-1/2}\sum\{m\nu\}\langle 0|a^\dagger_{\mathrm{P}l_i m\nu}a_{\Sigma l_i m\nu}a^\dagger_{\Sigma l_\Sigma m_\Sigma\tilde{\nu}}a_{\mathrm{P}l_{\mathrm{P}}m_{\mathrm{P}}\tilde{\nu}}|0\rangle\\ &= tA^i(E)^*(Z_i)^{1/2}(R^\Sigma_{l_i}|j_0(qr)|R^{\mathrm{P}}_{l_i}),\end{aligned} \tag{B.5}$$

where

$$\left(R^\Sigma_{l''}\middle|j_l(qr)\middle|R^{\mathrm{P}}_{l'}\right) = \int dr\, r^2 R^\Sigma_{l''}j_l(qr)R^{\mathrm{P}}_{l'}. \tag{B.6}$$

With transition amplitude (B.5), we get for cross section:

$$\left(\frac{d\sigma^2}{d\hat{k}_\pi dE_\pi}\right)^i = C|A^i(E)|^2 Z_i\left|(R^\Sigma_{l_i}|j_0(qr)|R^{\mathrm{P}}_{l_i})\right|^2, \tag{B.7}$$

where

$$C = |t|^2 \left[E_{\mathrm{K}} E_\pi / \left(2\pi\hbar^2 c^2\right)^2\right] \frac{k_\pi}{k_{\mathrm{K}}}, \tag{B.8}$$

$$E_{\mathrm{K}(\pi)} = \left[\left(M_{\mathrm{K}(\pi)} c^2\right)^2 + \left(c\hbar k_{\mathrm{K}(\pi)}\right)^2\right]^{1/2}. \tag{B.9}$$

The pion energy E_π is connected with E_Σ by energy conservation (the recoil of hypernucleus is ignored in our model):

$$E_\pi = E_{\mathrm{K}} - \varDelta M, \tag{B.10}$$

where $\varDelta M = (M_{\mathrm{HY}} - M_{\mathrm{T}})c^2$ is the difference between the mass of the hypernucleus M_{HY} (in the l_i state) and the mass of the target nucleus M_{T} (in its g.s.). In our model

$$\varDelta M = E_\Sigma + M_\Sigma c^2 - \left(\varepsilon^i_{\mathrm{P}} + M_{\mathrm{P}} c^2\right). \tag{B.11}$$

Consequently, for a given kaon momentum k_{K} and a fixed $\cos\theta = \hat{k}_\pi \hat{k}_{\mathrm{K}}$, the pion energy E_π (and momentum k_π), the momentum transfer q, and E_Σ are functions of $\varDelta M$.

If we switch off the ΣΛ coupling ($v \to 0$), we have $F^i \to 0$, $\Gamma^i \to 0$, $|A^i|^2 \to \delta\left(E_\Sigma - \varepsilon^i_\Sigma\right)$, and the substitutional l_i states become discrete. The differential cross section for the transition to these discrete states is

$$\left(\frac{d\sigma}{d\hat{k}_\pi}\right)^i = C Z_i \left| \left(R^\Sigma_{l_i} \middle| j_0(qr) \middle| R^{\mathrm{P}}_{l_i}\right) \right|^2. \tag{B.12}$$

The discrete value of $\varDelta M$ is then:

$$(\varDelta M)_0 = \varepsilon^i_\Sigma + M_\Sigma c^2 - \left(\varepsilon^i_{\mathrm{P}} + M_{\mathrm{P}} c^2\right). \tag{B.13}$$

A similar procedure in the case of the transition to the g.s.(m_1) leads to:

$$\langle F^{\mathrm{gs}(m_1)} | T | I \rangle = t A^{\mathrm{gs}}(E)^* \sqrt{4\pi}\, i^{-l_1} Y_{l_1\, m_1}(\hat{q}) \sqrt{2} \left(R^\Sigma_0 \middle| j_{l_1}(qr) \middle| R^{\mathrm{P}}_{l_1}\right), \tag{B.14}$$

$$\begin{aligned} \left(\frac{d^2\sigma}{d\hat{k}_\pi dE_\pi}\right)^{\mathrm{gs}} &= \sum\{m_1\} \left(\frac{d^2\sigma}{d\hat{k}_\pi dE_\pi}\right)^{\mathrm{gs}(m_1)} \\ &= C \left| A^{\mathrm{gs}}(E) \right|^2 Z_1 \left| \left(R^\Sigma_0 \middle| j_{l_1}(qr) \middle| R^{\mathrm{P}}_{l_1}\right) \right|^2, \end{aligned} \tag{B.15}$$

$$\Delta M = E_\Sigma + M_\Sigma c^2 - \left(\varepsilon_{\mathrm{P}}^1 + M_{\mathrm{P}} c^2\right). \tag{B.16}$$

If the $\Sigma\Lambda$ coupling is switched off, then

$$\left(\frac{d\sigma}{d\hat{k}_\pi}\right)^{\mathrm{gs}} = C Z_1 \left| \left(R_0^\Sigma \middle| j_{l_1}(qr) \middle| R_{l_1}^{\mathrm{P}} \right) \right|^2, \tag{B.17}$$

$$(\Delta M)_0 = \varepsilon_\Sigma^0 + M_\Sigma c^2 - \left(\varepsilon_{\mathrm{P}}^1 + M_{\mathrm{P}} c^2\right). \tag{B.18}$$

If we denote by B_Σ the separation energy of Σ from the hypernucleus (in its ground or excited state) with the nuclear core left in its ground state, then $\Delta M = B_{\mathrm{P}} - B_\Sigma + M_\Sigma c^2 - M_{\mathrm{P}} c^2$, where B_{P} is the proton binding (separation) energy in the target nucleus. In our model, we identify B_{P} with $-\varepsilon_{\mathrm{P}}^1$, and have:

$$\Delta M = -B_\Sigma + M_\Sigma c^2 - \left(M_{\mathrm{P}} c^2 + \varepsilon_{\mathrm{P}}^1\right). \tag{B.19}$$

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