

ON THE COBE DISCOVERY — FOR PEDESTRIANS*

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A recent discovery by the COBE satellite is presented, and its importance discussed in the context of the physics of the early Universe. The various implications for our understanding of the structure of the Universe are outlined.

PACS numbers: 98.70. Vc, 98.80. Es

1. Introduction

It has by now become virtually a dogma that the Big Bang did indeed happen some 10 to 20 billion years ago. The Standard Big Bang Model attempts to describe the evolution of the Universe from the earliest moments up to its present stage. The model consists of several components, some of which are based on solid astronomical observations, some are at best supported by them, while others still remain attractive theoretical hypotheses. In spite of those experimental loopholes, it is indeed encouraging that the Standard Big Bang Model which spans some 30 orders of magnitude in energy (some 60 orders in time, 32 orders in temperature) provides a relatively self-consistent framework for describing the whole evolution of the Universe.

The recent discovery¹ by the COBE satellite [1] of the non-zero angular variations in the sky temperature provides another important experimental piece of evidence supporting the Standard Big Bang Model. It also provides a solid basis on which various cosmological models and hypotheses (*eg.*, large-scale structure formation, dark matter, cosmic inflation) can be tested.

* Presented at the XXXII Cracow School of Theoretical Physics, Zakopane, Poland, June 2–12, 1992.

¹ The Appendix provides a self-contained summary of the COBE satellite and its recent discovery.

This lecture is intended to explain the importance of the COBE discovery to those unfamiliar with the physics of the early Universe. I will keep it at a rather basic level, partly because I have been asked by the organizers to deliver it to a general audience of physicists, and partly because I myself am not an expert on the subject. I will make an effort to explain, in hopefully simple terms, what the COBE result tells us about the Universe in its infancy. I will also attempt to briefly outline various recent attempts to reconcile the implications of the COBE results with other astronomical observations. There exist many extensive reviews of the physics of the early Universe, like, *eg.*, Refs. [2–4]. This lecture may hopefully serve as an introduction for outsiders to these and other professional reviews. Unfortunately, I am not aware of any semi-popular reviews of the COBE discovery, other than those published in various science magazines (Ref. [5]). (To those unfamiliar with the whole subject I recommend one of the many academic textbooks, *eg.*, Ref. [6], before reading an excellent but much more advanced monograph of Kolb and Turner [7] which I have extensively used in preparing this talk.)

2. Evidence for the Big Bang

Our belief that the Big Bang actually took place is based on two crucial observations:

- The expansion of the (observed part) of the Universe, as first discovered by Hubble in 1920.
- The cosmic microwave background radiation (CMBR), discovered by Penzias and Wilson in 1965, whose thermal spectrum is that of a black-body of temperature 2.7 K.

While the observed expansion could still be explained by the steady-state theory, both observations taken together rule it out, as they imply that at some time in the past the Universe was hot and dense, and that it has cooled down due to an expansion from a primordial fireball of a tiny size to the present size of 3Gpc. ($1\text{pc} = 3.26 \text{ light-year} = 3.1 \times 10^{16}\text{m}$)

Another important, although perhaps not as direct, argument for the Big Bang is provided by the Big Bang Nucleosynthesis theory which correctly predicts the primordial abundance of light elements in the Universe. This theory is also based on a basic assumption that light elements were formed when the Universe was very hot and dense.

It is worth stressing that the COBE discovery does *not* provide evidence for the Big Bang itself, contrary to what has been claimed in some popular literature. It, however, provides a strong support for the Standard Big Bang Model which I will now briefly review.

3. The standard Big Bang model

I will first describe the major events in the history of the Universe (see the table). Presently, the earliest moments of the Universe one can attempt to describe are those starting with the Planck era ($t > t_{\text{Planck}} = 10^{-43}\text{s}$) and energies smaller than the Planck mass $m_{\text{Planck}} = 1.2 \times 10^{19}\text{GeV}$. (Times earlier than t_{Planck} could presumably be described by the theory of superstrings but the theory has not been developed to that extent yet.) After the Planck era, the Universe is believed to have gone through a period of cosmic inflation at some time *very* roughly around the grand unification era ($t \simeq 10^{-34}\text{s}$). The next important phases of the Universe were the end of the electroweak unification ($t \simeq 10^{-12}\text{s}$), and the confinement of quarks into hadrons ($t \simeq 10^{-6}\text{s}$). Nuclei of light elements (D , ^3He , ^4He , and ^7Li) began to form at much later times, at around 1s (plus or minus some two order of magnitudes) after the Big Bang, as described by the Big Bang Nucleosynthesis (BBN) theory.

TABLE I
Major events in the history of the Universe

| Time | Energy | Temp. | Event |
|------------------------------|---------------------------|-------------------------|-------------------------------------|
| $\sim 10^{-43}\text{ s}$ | $\sim 10^{19}\text{ GeV}$ | $\sim 10^{32}\text{ K}$ | Quantum Gravity (superstrings?) |
| $\sim 10^{-34}\text{ s}$ | $\sim 10^{16}\text{ GeV}$ | $\sim 10^{28}\text{ K}$ | Cosmic Inflation; Grand Unification |
| $\sim 10^{-12}\text{ s}$ | $\sim 10^2\text{ GeV}$ | $\sim 10^{15}\text{ K}$ | End of Electroweak Unification |
| $\sim 10^{-6}\text{ s}$ | $\sim 1\text{ GeV}$ | $\sim 10^{13}\text{ K}$ | Quark Hadron Phase Transition |
| $\sim 1\text{ s}$ | $\sim 1\text{ MeV}$ | $\sim 10^{10}\text{ K}$ | Big Bang Nucleosynthesis |
| $\sim 10^4\text{ yrs}$ | $\sim 10\text{ eV}$ | $\sim 10^4\text{ K}$ | Matter Domination Begins |
| $\sim 10^5\text{ yrs}$ | $\sim 1\text{ eV}$ | $\sim 10^3\text{ K}$ | Decoupling and Recombination |
| $\gtrsim 10^{10}\text{ yrs}$ | $\sim 10^{-2}\text{ eV}$ | 2.73 K | today |

The Universe kept expanding and cooling, but remained in a hot, radiation-dominated (RD) state until very roughly tens to hundreds of thousands of years after the Big Bang. At that time it had cooled down enough to become matter-dominated (MD) (which means that at that point the total mass-energy density started being dominated by matter, and not anymore by photons). This was soon followed by the phase of recombination of electrons and protons into mostly hydrogen, after which the electromagnetic radiation effectively decoupled from matter. At that time large baryonic

structures in the Universe could start growing, as the radiation effects had become too weak to truncate them.

The COBE result tells us “directly” about this last phase of decoupling, after which the Universe became virtually transparent to electromagnetic radiation, as will be shown in more detail shortly.

Mathematically, the Standard Big Bang Model is based on general relativity with Friedmann–Robertson–Walker (FRW) metric

$$ds^2 = -dt^2 + R^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right) \quad (1)$$

which assumes isotropy and homogeneity of the space-time. The coordinates (t, r, θ, ϕ) are all co-moving coordinates (meaning co-movers have $(r, \theta, \phi) = \text{const}$ in time), $R(t)$ is the scale factor and $k = 0, 1, -1$ refers to the curvature (flat, closed, and open, respectively) of the Universe. The scale factor $R(t)$ relates co-moving quantities to their physical counterparts via $x_{\text{phys}} = R(t)x$.

Einstein’s equations with the FRW metric are known as the Friedmann equation which relates the expansion rate given by $R(t)$ to the mass-energy density ρ and curvature of the Universe

$$H^2 \equiv \left(\frac{\dot{R}(t)}{R(t)} \right)^2 = \frac{8\pi G_N}{3} \rho - \frac{k}{R^2(t)} + \frac{\Lambda}{3}, \quad (2)$$

where H is the Hubble parameter, $G_N = 1/m_{\text{Planck}}^2$ is Newton’s constant, and Λ is the cosmological constant which I will set equal to zero. A detailed study of the solutions of the Friedmann equation is beyond the scope of this lecture [7]. I will merely mention here that from Eq. (2) it follows that during the radiation-dominated epoch $\rho \simeq \rho_{\text{rad}} \propto R^{-4}$ and $R(t) \propto t^{1/2}$, while during the matter-dominated epoch $\rho \simeq \rho_{\text{matter}} \propto R^{-3}$ and $R(t) \propto t^{2/3}$. (The extra power in the case of ρ_{rad} comes simply from the fact that the photons’ wavelength also “stretches” during the expansion). Heuristically, one may say that the hot Universe was expanding “more slowly” than during its later matter-dominated phase.

Present limits on the Hubble parameter H are [4, 7, 8]

$$0.4 \lesssim h_0 \lesssim 1, \quad (3)$$

where

$$h_0 = \frac{H_0}{(100 \text{ km/Mpc/s})} \quad (4)$$

and H_0 is the present value of the Hubble parameter H . The Hubble law relates the velocity v at which galaxies recede from us to their distance r

from us: $v = H_0 r$. Thus light from more distant objects is more redshifted. Astrophysicists often find it convenient to use the redshift parameter $z \equiv \Delta\lambda/\lambda_0$ not only as a measure of the spatial separations of distant objects, but also as a way of referring to past events. Indeed, the deeper we look into space, the larger is the redshift z , and the deeper we look into the past.

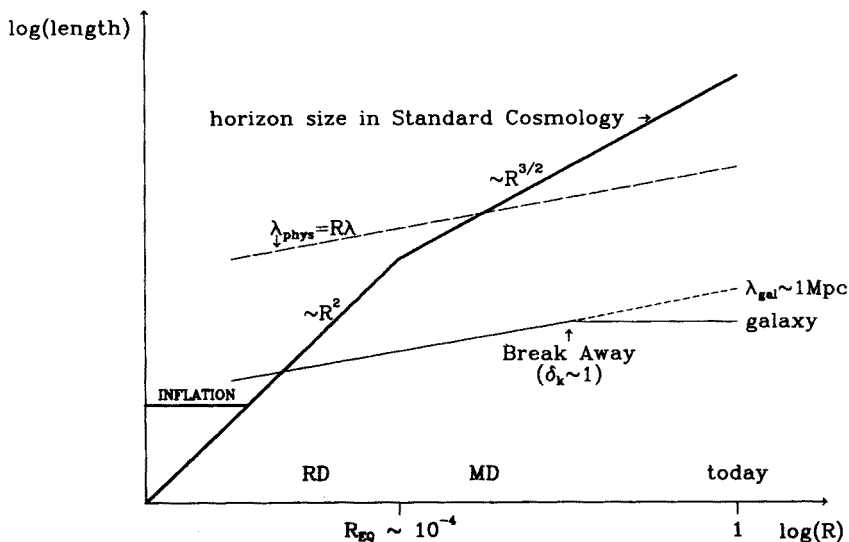


Fig. 1. Evolution of various physical scales with the scale factor R in the Standard Big Bang Model. The scale factor is normalized so that $R(t = \text{today}) = 1$. The thick solid line shows the evolution of the Hubble distance H^{-1} (more or less the horizon size, i.e., the radius of the observable Universe) in standard FRW cosmology during the radiation-dominated (RD) epoch ($H^{-1} \sim R^2$) and in the matter-dominated (MD) Universe ($H^{-1} \sim R^{3/2}$). During the brief period of inflation the Hubble constant, and thus the Hubble distance remained constant (thick dotted line), while the scale factor and therefore physical sizes and the horizon size grew extremely rapidly (not shown). After the end of inflation, physical scales characterized by $\lambda_{\text{phys}} = R\lambda$, started reentering the horizon, first smaller scales and later larger ones. A thin solid line represents a scale corresponding to typical galactic sizes. Such scales would at present be of the order of about 1 Mpc, had matter density perturbations over those scales not grown into gravitationally self-bound systems like galaxies. The dashed line shows a typical scale which was super-horizon-size until long after matter-radiation equity (EQ). (The figure is discussed in Secs. 3 and 6.)

The critical density $\rho_{\text{crit}} \equiv 3H^2/(8\pi G_N) = 1.88 \times 10^{-29} h^2 \text{ g/cm}^3$ corresponds to the flat Universe $k = 0$. One also often uses the concept of the Hubble distance, or time (I assume $c = 1$), H^{-1} , as an intuitive measure

of the radius of the observable part of the Universe. At present it is roughly given by $H_0^{-1} \simeq 3h_0^{-1} \text{ Gpc} = 9.78 \times 10^9 h^{-1} \text{ yrs}$. More precisely, one defines the horizon size $d_H(t)$ at a given time t as the (proper) distance that light travelled from $t = 0$ to the time t . The present horizon thus separates the Universe that is at least in principle visible to us from the part from which light has not reached us. (One can show that today the horizon size is $d_H = 2H_0^{-1}$.) The thick solid line in the figure (adopted from Ref. [7]) shows the Hubble distance in the standard FRW cosmological model during the radiation-dominated and matter-dominated epochs as a function of the scale factor R which is here normalized so that $R(t = \text{today}) = 1$. In the standard FRW cosmology the Hubble distance and the horizon size are essentially equal. In the inflationary scenario, typically shortly after the Planck time the Universe underwent a process of extremely rapid expansion to become billions of times larger than before inflation started. (The relative distances between particles grew rapidly not because the particles moved in space but because the space itself "stretched" rapidly.) Inflation thus moved the nearby space regions far beyond the observable Universe. During that brief period the Hubble distance remained essentially constant (thick dotted line in the figure) while the horizon size (not shown in the figure) inflated. In the inflationary scenario the observable Universe is thus now, as it has been ever since the end of inflation, expanding into the regions that were in close contact with us before inflation. In other words, those regions have been reentering the horizon, first the smaller scales (like the one indicated by the thin solid line) and later the larger scales. (I will come back to the figure later.)

Another convenient parameter is Ω defined as

$$\Omega = \frac{\rho}{\rho_{\text{crit}}}, \quad (5)$$

where ρ is the (present) total mass-energy density. Notice that $\Omega <, =, > 1$ corresponds to $k = -1, 0, +1$ and thus to the open, flat, and closed Universe, respectively. This can be seen by rewriting Eq. (2) in terms of Ω . As before, Ω_0 denotes the present value of Ω .

The case $\Omega = 1$ (or very close to one) is strongly preferred by theorists because it follows from cosmic inflation and because of other theoretically motivated reasons. At present observations only limit Ω to the range between about 0.1 and a few. A significant progress is, however, expected to be made within the next few years [4, 8].

Similarly, one can introduce partial ratios, *eg.*, Ω_B will be the fraction of the total mass-energy density due to baryons, and Ω_{DM} will be the respective fraction due to dark matter (DM) for which there is now convincing evidence. It is commonly assumed that most of DM is likely to be

non-baryonic and neutral. (A generic name of weakly interacting massive particles (WIMPs) accommodates several candidates, like a massive neutrino, an axion or the lightest supersymmetric particle, predicted by many high energy physics models.) As we will see shortly, the relative ratio of the baryonic and non-baryonic contributions to the total mass-energy density, will be important in describing the epochs of matter domination, recombination and decoupling. We will see that matter domination (MD) begins earlier if there is more matter in the Universe. Especially, if most of the matter were non-baryonic, then MD began significantly earlier than decoupling of radiation and (baryonic) matter and the recombination of protons and electrons into hydrogen atoms.

4. The cosmic microwave background radiation

I will now briefly review properties of the cosmic microwave background radiation. The spectrum of the uniform radiation coming to us uniformly from all directions is to a very high precision that of a blackbody of temperature $T = 2.735 \pm 0.06$ K. The existence of the CMBR was originally guessed by Gamow in the thirties. In the mid-sixties Dicke and Peebles of Princeton extended Gamow's idea and were preparing to look for the faint remnant of the glow of the primordial fireball. At the same time, and just a few miles away, at Bell Labs, Penzias and Wilson tested a new low-noise microwave antenna. Despite numerous attempts, they could not remove a persistent noise coming uniformly from all directions.² Finally, they learned from a colleague that they actually discovered the most ancient remnant of the early hot period of the Universe.

Since 1965 many groups have confirmed this discovery and, with increasing precision, established that the spectrum of the radiation is that of a perfect blackbody. The experiments measure the radiation intensity over a wide range of wavelengths ranging from about a millimeter to almost one meter (from almost 10^3 GHz down to less than one GHz) at several different points in the sky. Then they fit the blackbody spectrum and find its temperature.

The second crucial feature of the CMBR is its isotropy: the intensity of the radiation from all directions has been found to a high precision to be equal ($\delta T/T \lesssim 10^{-4}$ (prior to the COBE discovery)).

Several important consequences were derived from these properties. First, the steady-state theory was ruled out as it did not predict the hot, dense period of the Universe whose remnant the CMBR is.

² According to one source, even removing bird droppings from the antenna surface did not help.

Second, the CMBR tells us about the very early phase of the Universe's evolution when radiation and matter ceased to interact with each other. It tells us that already some tens to hundreds of thousands of years after the Big Bang the Universe was very smooth and isotropic. This of course raises questions as to what caused the Universe to be so "featureless" at that early stage, especially since at present matter in the Universe is not smoothly distributed at all.

Third, the background radiation is incredibly uniform across the whole sky. Take two antipodal points in the sky. The microwave radiation coming from opposite direction has travelled freely for nearly as long as the Universe existed (since decoupling) while the total distance between the antipodal points is two times larger. Thus, in standard FRW cosmology, those points never interacted with each other. Why then is the CMBR radiation so incredibly isotropic? The hypothesis of cosmic inflation has been invented to solve those questions. It says that those far regions were in causal contact before space inflated and made them seem, in standard cosmology, causally disconnected.

Finally, the smoothness of the CMBR has important implications for galaxy formation. Baryonic matter could not form large clumps until it decoupled from radiation. This sets the time scale for the period of galaxy formation. Moreover, large-scale structures seem to have had to originate from very tiny perturbations in matter density. If they had been larger at the time of decoupling it would have been reflected in the larger anisotropies of the CMBR as I will discuss later.

I will now review in some more detail the periods of matter domination, and decoupling and recombination.

4.1. Radiation- vs matter-dominated Universe

At present the Universe is very cold. In other words, its total mass-energy density is completely dominated by matter. Even the mass-energy density due only to luminous matter ($\sim 5 \times 10^{-31} \text{ g/cm}^3$) exceeds that of radiation ($\sim 7 \times 10^{-34} \text{ g/cm}^3$; mostly from CMBR photons!) by a few order of magnitudes. This was not always the case. The early Universe was very hot and thus radiation-dominated, i.e., the mass-energy density due to radiation exceeded that due to matter ($\rho_{\text{rad}} > \rho_{\text{matter}}$). As I have already said, due to the expansion both densities decreased with time (or in other words with the scale factor $R(t)$), but not at the same rate: $\rho_{\text{rad}} \propto R^{-4}$ while $\rho_{\text{matter}} \propto R^{-3}$. Eventually, matter started dominating the evolution of the Universe at the time $t_{\text{EQ}} \simeq 1.4(1/(\Omega_0 h^2)^2) \times 10^3 \text{ yrs}$ (at temperature $T_{\text{EQ}} \simeq 5.5(\Omega_0 h^2) \text{ eV}$, and redshift $1 + z_{\text{EQ}} \simeq 2.32(\Omega_0 h^2) \times 10^4$). This was an important moment in the Universe's history: it was only after t_{EQ} that perturbations

in matter density could start growing. Furthermore, baryonic matter still did not decouple from electromagnetic radiation, and thus baryonic ripples (local variations of baryonic mass density) were still erased by radiation. But non-baryonic matter could start clumping, *i.e.*, non-baryonic instabilities could start growing, as soon as matter domination occurred. Notice that the beginning of matter domination occurs earlier if the total mass-energy density Ω_0 is larger. Thus a large and non-baryonic matter component allows for the large cosmic structures to start growing earlier. *E.g.*, if $\Omega_0=1$ and $0.5 \lesssim h \lesssim 0.7$ then $5.6 \times 10^3 \text{ yrs} < t_{\text{EQ}} < 2.25 \times 10^4 \text{ yrs}$. For further reading see Ref. [7].

4.2. Recombination and decoupling

Recombination and decoupling happened nearly simultaneously and are seldom distinguished even though strictly speaking they refer to physically distinct processes [7].

Recombination refers to the process of combining electrons and protons into hydrogen atoms. At higher temperatures both *e*'s and *p*'s interacted rapidly with photons. As the temperature was decreasing, at some point photons failed (on average) to keep them apart. The process was gradual although relatively rapid. As discussed in Ref. [7], at time $t_{\text{rec}} \simeq 1.5(1/\sqrt{\Omega_0 h^2}) \times 10^5 \text{ yrs}$ (corresponding to temperature $T_{\text{rec}} \simeq 0.31 \text{ eV}$ and redshift $1200 \lesssim 1 + z_{\text{rec}} \lesssim 1400$, depending on $\Omega_B h^2$) more than 90 % of protons (re)combined into atoms.

Decoupling of radiation from matter was a direct consequence of recombination. Electrons and protons, before they formed hydrogen atoms, co-existed with photons in the form of hot plasma. The mean free path of photons was thus short relative to the Hubble distance (measure of the radius of the Universe), $\lambda_\gamma \ll H^{-1}$, mainly due to interactions with electrons. The Universe was opaque. After recombination the Universe suddenly became transparent to photons. Since they do not interact with hydrogen, their mean free path suddenly became very large, in fact much larger than the size of the Universe, $\lambda_\gamma \gg H^{-1}$. The time of decoupling is given by $t_{\text{dec}} \simeq 1.9(1/\sqrt{\Omega_0 h^2}) \times 10^5 \text{ yrs}$ ($T_{\text{dec}} \simeq 0.26 \text{ eV}$, $1100 \lesssim 1 + z_{\text{dec}} \lesssim 1200$).

An important point for us here is that photons from the time of decoupling are the most ancient photons that we can still see today. The present shape of the CMBR spectrum thus provides us with (almost) direct information about the inhomogeneities in the baryonic matter on the last scattering surface, *i.e.*, at the last time radiation and baryonic matter interacted. Any angular distortions of the CMBR spectrum can be related to those early (baryonic) matter density perturbations, as I will discuss in more detail later.

One more concept needs to be touched upon here which deals with relating physical scales at a given time in the past to angular separations in the sky that we see today. Obviously, two antipodal points in the sky have never been causally connected as light has not had enough time to travel twice the radius of the visible Universe in the time less than its age. The size of the observable Universe (more strictly, of the horizon d_H) at a given time in the past thus corresponds to only a finite angular separation between two (arbitrary) points in the sky. In the case of particular interest to us here the size of the horizon at the time of decoupling was roughly $d_H(t_{\text{dec}}) \simeq 200h^{-1}$ Mpc. One can show that the corresponding angular separation is given by $\theta_{\text{dec}} = 0.87^\circ \Omega_0^{1/2} (z_{\text{dec}}/1100)^{-1/2}$. (For comparison, the angular size of the Moon is about 1° .) Thus angular separations larger than θ_{dec} correspond to length scales larger than the horizon size at the time of decoupling, or in other words to super-horizon-size scales. Similarly, sub-horizon-size scales at decoupling were those which related to distances less than $d_H(t_{\text{dec}})$, and now to angles less than θ_{dec} .

At this point it is perhaps worthwhile to read the Appendix which summarizes the COBE results.

5. Angular distribution of CMBR fluctuations

There are several sources that can cause angular temperature fluctuations δT of the CMBR. These are [7]:

- our motion relative to the cosmic rest frame (the dipole anisotropy);
- fluctuations δT intrinsic to the radiation field itself on the last scattering surface;
- peculiar velocity (*i.e.*, not due to the expansion of the Universe) of the last scattering surface;
- damping of initial temperature anisotropies if the Universe re-ionizes after decoupling;
- fluctuations in temperature caused by the irregularities of matter distribution (*i.e.*, of the gravitational potential) on the last scattering surface.

The dipole anisotropy has long been known, and COBE has reconfirmed it at $\delta T = 3.36 \pm 1$ mK. It is interpreted as the Doppler effect caused by our motion (*i.e.*, of our Local Group of galaxies) relative to the cosmic rest frame.

The next three sources are of microphysical nature and are dominant for small angular scales $\theta \ll 1^\circ$. In other words, they affected (in a calculable way) the spectrum of the CMBR over the scales that were sub-horizon-size at the time of decoupling.

The last source is dominant at large angular scales $\theta \gg 1^\circ$ (super-horizon-size scales). $\delta T/T$ on such large scales were not affected after t_{EQ}

by microphysical processes 2 to 4 listed above. They are thus likely to reflect truly primordial (from the time of inflation?) mass density fluctuations. Typically one finds that the size of the matter density fluctuations $\delta\rho/\rho$ at the time of decoupling is related to the angular fluctuations $\delta T/T$ in the CMBR by

$$\frac{\delta\rho}{\rho} = \text{const} \times \left(\frac{\delta T}{T} \right), \quad (6)$$

where the constant is very roughly of the order of ten. This dependence reflects the effect of the mass density fluctuations $\delta\rho/\rho$, and thus associated gravitational potential fluctuations over large scales. Specific predictions are, however, very model-dependent, and unfortunately cannot be easily explained. (See Chapter 9 of Ref. [7] for a detailed discussion.) In one commonly assumed spectrum of density fluctuations, given by a power law (see next section), one can show that over large angular scales ($\theta \gg 1^\circ$)

$$\frac{\delta T}{T} \sim \theta^{(1-n)/2} \quad (7)$$

between two points on the sky separated by the angle θ (the so-called Sachs–Wolfe effect). When fitting the data to the power law spectrum, the DMR group finds [1] $n = 1.15^{+0.45}_{-0.65}$.

It is important to remember that COBE has measured $\delta T/T$ over (angular) scales $\theta > 7^\circ$ (see the Appendix) and thus over scales that were super-horizon-size until long after decoupling (remember that $\theta_{\text{dec}} \simeq 1^\circ$). Thus it is only in this regime that the COBE satellite tells us “directly” about the (model-dependent) scale of density perturbations.

On the other hand, one is tempted to use the COBE result to figure out the size of density perturbations over much smaller scales, in particular to the scales corresponding to present galaxies.

Is an extrapolation to smaller angular scales ($\theta < 1^\circ$, or physical scales $\lambda \lesssim 100$ Mpc) justified? Frankly speaking not really. Even by a relative scale independence of $\delta T/T$ over the angular range between 10° and 180° found by COBE. (Again, it is easier to say this than to explain.) One may argue that microphysical processes mentioned above are not likely to significantly alter the size of $\delta\rho/\rho$. Still, it cannot be overemphasized that it is only in this context that the COBE measurement tells us about the size of matter density perturbations at the scales that were already within the horizon during decoupling, in particular of those being seeds of large structures like galaxies.

In the next section I describe a currently popular scheme of large-scale structure formation, and discuss how the data from COBE supports it.

6. Large-scale structure formation — an unsettled issue

The Universe today is very “lumpy” on smaller scales (less than hundreds of megaparsecs). Indeed, most matter is confined in the form of galaxies (sizes of \sim several kiloparsecs), or clusters of galaxies (\sim few megaparsecs), and some in even larger structures, like super-clusters. On the other hand, when viewed over very large or global scales, the Universe looks relatively homogeneous and isotropic. As I have already said, structures could start significantly growing only after matter domination occurred ($t > t_{\text{EQ}}$) in the case of non-baryonic matter, and only after decoupling ($t > t_{\text{dec}} > t_{\text{EQ}}$) in the case baryons. COBE’s measurement of $\delta T/T$ sets the scale of $(\delta\rho/\rho) \sim 10^{-5}$. How then did the matter fragment from a very smooth distribution at the times of decoupling to the presently observed large-scale structures?

Various models of structure formation have been proposed. They all require some form of seeds and a mechanism describing the growth of these seeds into present structures, like galaxies and their clusters. The mechanism that relies on the growth of gravitational instabilities seems particularly attractive. I will briefly describe it here. The basic point of this picture is that small initial mass density inhomogeneities will grow if their size is larger than a certain critical scale.

The problem is in fact very old. It was initially posed by Newton: take a uniform static distribution of massive particles of mass m with density ρ and temperature T ; and next introduce tiny perturbations in the density $\delta\rho$. Then gravity will work to enhance the perturbations. On the other hand the gas pressure in denser regions will also increase thus resisting growth and compression. Will then the perturbations $\delta\rho$ grow or be damped? Jeans solved the problem. He showed that gravity can amplify even tiny matter density inhomogeneities *provided* that their overall size λ satisfies

$$\lambda > l_J \equiv \sqrt{\frac{\pi k T}{m G_N \rho}}, \quad (8)$$

where l_J is called the Jeans length. The growth has been shown to be exponential in time. On the other hand, density perturbations smaller than l_J are damped.

When applied to the expanding Universe, this mechanism still works, however classical (exponential) growth of $\delta\rho/\rho$ is *moderated*. In the matter-dominated epoch $\delta\rho/\rho$ grows only as a power law. In the radiation-dominated epoch the growth of $\delta\rho/\rho$ can be shown to be further moderated to the extent of being effectively quenched.

Thus the problem of large-scale structure growth is determined by the initial conditions at the time of t_{EQ} : i) the total amount of (non-relativistic)

matter density in the Universe Ω ; *ii*) the relative ratios of the various species contributing to Ω (baryons, WIMPs, cosmological constant Λ , *etc.*); and *iii*) the spectrum and type (adiabatic or “isocurvature”) of primordial density perturbations. A discussion of these important and difficult issues is beyond the scope of this lecture. (I refer those interested to, *eg.*, Refs. [2–4, 7].) I will just mention here that two general categories of seeds have been considered. One is based on random density fluctuations, while the other is provided by topological defects (like cosmic strings, textures, *etc.*).

It is often assumed that massive non-baryonic WIMPs dominate the mass density of the Universe. In this case their initial inhomogeneities could start growing already after matter domination occurred and thus start forming gravitational potential wells, into which later baryonic matter fell following their decoupling from photons after t_{dec} .

In describing the growth of (spatial) density perturbations $\delta\rho(\vec{x})$ it is convenient to decompose them into their Fourier spectral modes. Define

$$\delta(\vec{x}) \equiv \frac{\delta\rho(\vec{x})}{\rho} = \frac{\rho(\vec{x}) - \rho}{\rho}, \quad (9)$$

where $\rho(\vec{x})$ is the mass-energy density, ρ is the average density, and for definiteness I use co-moving coordinates \vec{x} . Then the spectral modes are given by

$$\delta_k = \frac{1}{V} \int_V d^3x \delta(\vec{x}) e^{i\vec{k}\vec{x}}, \quad (10)$$

and usually $\delta_k V$ is used to eliminate the volume. Another often used quantity is the so-called power spectrum $P(k)$, where

$$P(k) \sim |\delta_k|^2 \quad (11)$$

which shows which spectral modes (or length scales $\lambda = 2\pi/k$) dominate a given spatial density perturbation. For simplicity, astrophysicists usually assume a simple, “featureless” power law spectrum $|\delta_k|^2 \sim k^n$ mentioned in the last section. In particular, the case $n = 1$ corresponds to the Harrison–Zel’dovich, scale-invariant spectrum which is not only simple but also follows from many models of cosmic inflation.

As I said in Sec. 3, physical and co-moving quantities are related by

$$dx_{\text{phys}} = R(t)dx, \quad k_{\text{phys}} = \frac{k}{R(t)}, \quad \lambda_{\text{phys}} = R(t)\lambda. \quad (12)$$

Let us come back to the figure. It shows the evolution of various physical length scales as a function of the scale factor $R(t)$ which has been normalized

here so that $R(t = \text{today}) = 1$. As I have already discussed in Sec. 3, the Hubble distance (roughly the horizon size) evolved as R^2 during the RD era and as $R^{3/2}$ during the MD era, corresponding to the fact that the rate of the Universe's expansion was different in the two phases. During inflation the scale factor stretched exponentially by many orders of magnitude (the space stretched), while the Hubble distance did not.

Now take two scales, both of which had been moved from the sub-horizon to the super-horizon regions by inflation. (Imagine for simplicity two density perturbations of the shape of sine waves of definite λ .) The smaller scale (denoted by a thin solid line), which corresponds to smaller λ , reenters the horizon earlier; in this case before R_{EQ} and thus becomes subject to causal physical processes before matter domination and decoupling. The other scale (denoted by long dashes) reenters the horizon long after R_{EQ} , and therefore after decoupling of matter and radiation. This scale has not been affected by the microphysical processes of the early Universe, and thus reflects truly primordial mass-energy density fluctuations from the earliest moments after the Big Bang. As I have already stressed, these are the scales that the DMR detector of the COBE satellite is actually probing.

An important input in trying to trace the large-scale structure formation is the size of the initial relative amplitude $\delta\rho/\rho$ which later evolved into present bound systems like galaxies and their clusters. Thus in particular, the galactic scales are of the order $\lambda_{\text{gal}} \sim 1$ Mpc, and thus at the time of matter-radiation equity were already back within the horizon. (The scales that reentered the horizon at t_{EQ} were of the size of $\sim 13(\Omega_0 h^2)^{-1}$ Mpc.) By saying $\lambda_{\text{gal}} \sim 1$ Mpc I mean the size those perturbations would have grown to by today had they not later undergone non-linear growth which made them transform into gravitationally self-bounded systems, like galaxies, of typical sizes of a few tens of kiloparsecs [7]. Extrapolating the COBE measurement of $\delta T/T$ to smaller angles and using some model dependence gives $\delta\rho/\rho \sim 10^{-5}$. Those working on explaining the evolution of large-scale structures have now gained an important constraint on the size of the initial conditions. I am in no position to discuss these complicated and, to my mind far from being fully clarified, issues. But I think that there is now a consensus that the growth of large-scale structures could only (or most plausibly) be achieved when one assumes that most matter (the exact ratio being subject of hot debates) is of non-baryonic nature and massive (non-relativistic). This stringent constraint on $\delta\rho$ rules out the so-called explosive models. It also seems to put in a rather uncomfortable position several other models of structure formation, like textures-seeded ones, which typically require somewhat larger initial density perturbations. But one should also bear in mind that the whole subject of galaxy formation is a very complex one and it may be too early to make any definite statements here.

7. Implications for dark matter

As I have already mentioned several times, one seems to need a lot of non-baryonic matter in the Universe [4, 7, 8]. It seems necessary, for example, to reconcile the strongly theoretically motivated value $\Omega = 1$ with the upper limits on the baryonic fraction $\Omega_B \lesssim 0.15$ coming from BB Nucleosynthesis; it is desired to allow for (non-baryonic) structures to start growing already after t_{EQ} ; and in order for matter domination to begin earlier in the first place. These and other arguments have led to a hypothesis that the bulk of matter ($\sim 90\%$) in the (flat) Universe is non-baryonic and non-shining (dark), although baryonic matter could also contribute some fraction (a few percent or so).

Observations also provide us with some convincing, although indirect, evidence for dark matter (DM) at different astronomical scales, most notably in galactic halos, but also in clusters of galaxies and at very large cosmic scales [4, 7, 8].

Not much is known about the specific nature of DM. For a long time cold dark matter (CDM) in a generic form of particles with mass in the several GeV range, was considered as the most attractive candidate. The argument was based on the fact that with CDM it was much easier to generate large structures from tiny initial density perturbations, unlike with so-called hot DM (HDM), like neutrinos, which, being by nature relativistic, do not cluster on galactic scales.

On the other hand, however, it has also been known that the CDM model is not fully consistent with the measured angular correlation function of galaxies, and also with their pair-wise velocity dispersions.

The implications of the COBE discovery are twofold. On one hand, the smallness of the initial density perturbations implied by COBE favors the more attractive CDM scenario which predicts very small $\delta\rho/\rho$ (typically $\sim 10^{-5}$). At the same time, however, in the purely CDM scenario it is now even more difficult to correlate the observed pair-wise velocity dispersions at small scales (\sim few Mpc) with the COBE measurement at very large scales (~ 1 Gpc).

Several possibilities have already been considered. One is to invoke a non-zero cosmological constant, which is probably theoretically not very attractive. Other attempts involve various variations of the simplest CDM scenario by either modifying the primordial power spectrum; or by allowing for new particle physics like new interactions; or by introducing an admixture of hot dark matter into the picture, to mention just a few possibilities. (The literature on the subject is in fact rapidly growing.) But no matter what final picture emerges, it seems likely that the simple (simplistic?) scenario with just one type of (cold) DM, although surprisingly successful, most likely will have to be altered.

Astrophysicists would presumably prefer to have just one kind of new species. Otherwise, they fear, they will be forced to open Pandora's box. Being after all a high energy physicist, I am tempted to make a few comments here. I would like to stress that from the particle physics point of view it is completely natural to expect several new weakly interacting particles.

First, if DM indeed consists of some particles, as it seems to be generally accepted, then they should be predicted by high-energy physics models. The Standard Model does not provide any DM candidate. Of course, it can be trivially extended to accommodate massive neutrinos and give one of them the right mass of about 10 eV to close the Universe. But at the same time it would be most natural to introduce neutrino mixings, hence decays and oscillations, on which there are stringent bounds from the solar neutrino problem, double-beta decay, supernova, and from elsewhere. Besides, massive neutrinos, being HDM, are not favored by models of large-scale structure formation. I want to stress that attempts to accommodate WIMPs most likely point us to significant extensions of the Standard Model (with a notable exception of the axion). Two basic frameworks have emerged from over a decade of theoretical efforts to go beyond the Standard Model (which in itself has theoretical problems): composite models and supersymmetry. I think it is fair to say that there is a prevailing opinion now that only supersymmetry seems to remain an attractive possibility.³ (While many consider it a success, others find it very regrettable.) Supersymmetry, even in its minimal version, offers a very attractive CDM candidate, namely the lightest supersymmetric particle (LSP) [8]. It also provides a framework for neutrino masses. Thus in this particular and attractive example of supersymmetry, one can easily find candidates for both cold (*eg.*, LSPs) and hot dark matter (massive neutrinos).

8. Final comments

This lecture has been meant as an introduction to further reading. It is by no means complete, nor is the discussion of covered topics exhaustive. I have made an attempt to keep it at a relatively basic level, sometimes (I confess) at the cost of significant oversimplifications. My intention has been, however, to sketch the general framework in which the recent COBE discovery should be viewed, in other words, its importance for our understanding of the early Universe. I did not want to bury myself in the "fine details" of gauge-dependencies, uncertainties in the nature of primordial fluctuations, and many other in principle important issues. Sometimes I have alluded to them and have referred to more advanced literature. I can only hope that

³ Which of course does not make it a proof for its existence!

more professional and more technically precise reviews will soon be available to non-experts.

Let me briefly summarize the most important implications of the recent COBE discovery:

- COBE confirms the thermal origin of the CMBR spectrum ($\delta T/T$ is independent of frequency), thus strengthening even further our confidence in the Standard Big Bang Model.
- Observed fluctuations $\delta T/T = 1.1 \times 10^{-5}$ reflect associated fluctuations in the gravitational potential on the last scattering surface (i.e., at the time of decoupling at roughly 300,000 years after the Big Bang), and thus in the mass density $\delta \rho/\rho \sim 10^{-5}$. (A caveat to remember is that this conclusion is based on an extrapolation from larger angles (over 7°) down to less than one degree.) This is strong confirmation of the basic hypothesis that the present large-scale structures have grown by the (Jeans) gravitational instability mechanism from very tiny mass density fluctuations at the early Universe. I should note, however, that models based on cosmic strings are not in conflict with the present bounds. (Other approaches, like the explosive models and the ones where the seeds are provided by textures, seem to be out as they typically predict $\delta T/T \gtrsim 10^{-4}$.)
- COBE data is consistent with the Harrison-Zel'dovich scale-invariant power spectrum ($n = 1$), also predicted by many models of inflation. Thus cosmic inflation is possible in the light of the COBE result.
- COBE does not confirm or rule out the flat Universe ($\Omega = 1$).
- COBE also does not discriminate between different candidates for dark matter (hot or cold DM, cosmological constant) but it provides new useful constraints on the models of large-scale structure formation. In particular, it rules out a purely baryonic Universe which requires $\delta T/T \sim 10^{-4}$ or more.

With much more data coming from COBE, and anticipated results from other experiments, one should expect significant progress in this field during the next few years. The physics of the early Universe is gaining a solid experimental framework.

I would like to thank the organizers of the XXXII Cracow School on Theoretical Physics for their kind invitation. I am also greatly indebted to Gus Evrard and Alexandre Dolgov for many clarifying discussions, and to Subir Sarkar for his sharp but often useful comments. I am also very grateful to my many colleagues, especially Gordy Kane, Enrico Nardi, and Rick Watkins for their detailed remarks about the manuscript.

Appendix A

The COBE satellite

The COsmic Background Explorer (COBE) satellite was launched by NASA in November 1989 to an orbit at 900 km above the Earth's surface. It carried three major detectors.

The job of the Far InfraRed Absolute Spectrophotometer (FIRAS) was to scan a wide range of CMBR frequencies. The instrument, which carried its own temperature-controlled blackbody calibration source, provided beautiful results confirming with unprecedented accuracy that the CMBR spectrum is that of a blackbody of $T = 2.735 \pm 0.06$ K.

The Diffuse InfraRed Background Experiment (DIRBE) searched for radiation due to galactic evolution, and has provided us with very valuable data about the early formation of luminous matter, as well as interstellar dust.

The Differential Microwave Radiometers (DMR) has been designed to perform a complete angular mapping of the CMBR. It consists of three pairs of radiometers, operating at three frequencies: 31.5 GHz, 53 GHz, and 90 GHz (corresponding to wavelengths of 9.5 mm, 5.7 mm, and 3.3 mm, respectively) where the CMBR signal has been known to be significantly larger than the galactic background. Within each pair, the antennas are separated by 60° . They measure the temperature difference between points in the sky separated by that angle from each other, each pair of antennas in a different frequency. As the satellite rotates, it covers all the points in the sky, and thus eventually the temperature difference between each point and all the points 60° apart from it are measured many thousand times. Thus the detector effectively measures the temperature of each point relative to all the other points. By "points" I mean here bins of angular size of 7° which is the beam size resolution of the detectors. The DMR detector has by now almost completed three years of running (it makes a complete map of the sky in half a year), and is expected to operate for another two or so years.

The COBE measurement

In April 1992 the DMR team [1] announced a discovery of residual non-zero temperature differences $\delta T = 30 \pm 5 \mu\text{K}$ between different points in the sky at least 7° apart that were attributed to the CMBR. The corresponding relative temperature variations are very small

$$\frac{\delta T}{T} = 1.1 \times 10^{-5}, \quad (13)$$

showing again an extreme smoothness of the CMBR spectrum over angles larger than 7° .

Other announcements confirmed the dipole anisotropy at $T = 3.36 \pm 1$ mK, and the quadrupole anisotropy at $T = 13 \pm 4 \mu\text{K}$.

When using as a fit the power spectrum of the form $P(k) \propto k^n$, where k is the wavenumber, and assuming n and the rms magnitude of the quadrupole anisotropy as free parameters, the group has found $n = 1.15^{+0.45}_{-0.65}$ and $\langle Q_{\text{rms}}^2 \rangle^{1/2} = 16 \pm 4 \mu\text{K}$. In the case of the theoretically motivated case $n = 1$, corresponding to the Harrison-Zel'dovich spectrum (see Secs. 5 and 6), the result is $\langle Q_{\text{rms}}^2 \rangle^{1/2} = 17 \pm 5 \mu\text{K}$.

The data and background analyses that has led to these results were very complex, and I will merely sketch them here. (For more detailed descriptions see the original papers [1].) The largest contribution to the raw signal comes of course from the dipole anisotropy caused by our motion relative to the cosmic rest frame. The next largest "pollutant" is the microwave radiation coming from our own Milky Way. In order to minimize it the detectors were designed to operate at carefully chosen frequencies (see above) where the contribution from the Galaxy is known to be very small and the CMBR signal is close to its maximum. By comparing the data at the three frequencies, the frequency dependent contributions due to dust at high frequency and synchrotron emission were subtracted out. Finally, further analysis was limited to altitudes larger than 20° above the Galactic plane, and it was checked that the residual signal was independent of this angle.

The remaining step was to remove the intrinsic noise of the antennas themselves. It was assumed that, at each frequency, the observed point to point variance of the maps is the quadrature sum of the instrumental noise and the intrinsic fluctuations on the sky

$$\sigma_{\text{obs}}^2 = \sigma_{\text{instr}}^2 + \sigma_{\text{sky}}^2, \quad (14)$$

where $\sigma_{\text{obs}}^2 \sim (A + B)/2$ and $\sigma_{\text{instr}}^2 \sim (A - B)/2$, and A and B are the signals at the two channels at each frequency. Based on this analysis, the group found

$$\sigma_{\text{sky}} = 30 \pm 5 \mu\text{K}, \quad (15)$$

over angles of 10° (which was the result of the 7° beam resolution angle being smeared by smoothing the data). The above analysis was based on the data collected during only the first year. The new data collected since then, and new data from few more years of running will provide us with significantly sharper signals. Other experiments which look for angular anisotropies at smaller angles are within a factor of two from seeing the effect and should reach the level implied by COBE within the next year or so.

REFERENCES

- [1] G.F. Smoot, *et al.*, draft printed April 21, 1992, submitted to the *Astrophys. J. Lett.*; E.L. Wright, *et al.*, draft printed April 14, 1992, submitted to the *Astrophys. J. Lett.*.
- [2] A.D. Dolgov, in *Surveys in High Energy Physics*, Vol. 6, 57 (1991).
- [3] D. Schramm, in the *Proceedings of the PASCOS-91 Symposium*, Boston, March 1991, Fermilab-Conf-91/266-A.
- [4] M. Turner, in the *Proceedings of the Trends in Astroparticle Physics*, Santa Monica, November 1990, Fermilab-Conf-91-78-A, Mar 1991.
- [5] See, *eg.*, the June 1992 issue of *Physics Today*.
- [6] W. Kaufmann, *Universe* W.H. Freeman and Company, New York, 1991.
- [7] E. Kolb, M. Turner, *The Early Universe* Addison Wesley, Reading, MA, 1990 and references therein.
- [8] For more details, see my lecture *Dark Matter and Supersymmetry* given at the School.