# PARAQUANTUM BRST TRANSFORMATIONS*

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Applying the paraquantization of order two to a non abelian gauge theory, we show that the action is invariant under some non trivial BRST transformations. The corresponding modified BRST charge and Slavnov–Taylor identities are derived.

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## 1. Introduction

Parafield theories differ from field theories in that the dynamical variables satisfy not bilinear but trilinear relations [1–6]. In the Hilbert space $A$, associated with the parafield theory, the observables are determined by the requirement of the locality condition [2, 3, 5]. Although it is, in principle, possible to study various features of these theories within the Hilbert space $A$, it is often convenient to put this space in correspondence with a larger Hilbert space $B$, in which the operators satisfy bilinear relations [2, 3, 5]. Traditionally, for Fock-type irreducible representation of paraquantum theories with an unique vacuum state

$$a_k|0\rangle = 0 \tag{1.1}$$

this is done by means of the Green ansatz [3–5]:

$$a_k = \sum_{\alpha=1}^{Q} a_k^{(\alpha)}, \tag{1.2}$$

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where $Q$ is the order of the parafield, $\alpha$ is the Green index and $a_k^{(\alpha)}$ are Green components satisfying bilinear but anomalous (anti) commutation relations:

$$\left[a_k^{(\alpha)}, a_l^{(\alpha)}\right]_{\pm} = \delta_{kl}\,,$$
$$\left[a_k^{(\alpha)}, a_l^{(\beta)}\right]_{\pm} = 0\,, \quad \alpha \neq \beta\,. \tag{1.3}$$

("+" (resp. "−") sign is for parabosons (resp. parafermions)). One can remove this anomaly by means of a Klein transformation [2, 5]. Thus, for real fields, the Green components $a_k^{(\alpha)}$ will transform as a representation of the group SO($Q$). Once a parafield theory is formulated in the Hilbert space $A$, one can put the states and observables of this space into correspondence with a subset of the states and observables of the Hilbert space $B$, which has a larger number of states and class of observables. Now, the structure of observables in parafield theories is restricted by the requirement of the locality. To be more specific, and as an example, for a parafermion theory of odd order, the observables are limited to the functionals of the commutators $:[\widehat{\psi}(x_1), \widehat{\psi}(x_2)]_{\pm}:$ ( " : : " means the normal ordering), and of even order, they may at most be functionals of this commutators and the symmetric product $\vdots[\widehat{\psi}(x_1)\ldots\widehat{\psi}(x_Q)]_{\pm}\vdots$ defined as [2, 3, 5, 7]:

$$\vdots[\widehat{\psi}(x_1)\ldots\widehat{\psi}(x_Q)]_{\pm}\vdots = Q! \sum_{\substack{\alpha_1,\ldots,\alpha_Q \\ \alpha_1\neq\alpha_2\cdots\neq\alpha_Q}} \widehat{\psi}^{(\alpha_1)}(x_1)\ldots\widehat{\psi}^{(\alpha_Q)}(x_Q)\,, \tag{1.4}$$

where $Q$ is the order of the quantization and $\widehat{\psi}^{(\alpha_i)}(x_i)$ is the Green's component of $\widehat{\psi}(x_i)$ ( the symbol " $\widehat{}$ " means $\psi^+$ or $\psi$).

It is worth to mention that the above observables can be further restricted by additional symmetries which the theory might carry. For example, in most cases of interest, chiral or conformal invariance rule out the symmetric product as a possible term in action.

Throughout this paper and to simplify matter, we restrict ourselves to parafermion fields of order two and ordinary vector field. In Section 2, we discuss our model and show that it is invariant under some nontrivial transformations called paraquantum BRST transformations (PBRST). The corresponding PBRST charge is constructed. Moreover, the violation of the Lagrangian symmetry under the anti PBRST transformations is shown explicitly. Finally, in Section 3, we derive the paraquantum Slavnov–Taylor identities and draw our conclusions.

## 2. The model

In our model, we take as a paraquantum Lagrangian $\mathcal{L}_{\text{tot}}$, the one describing a massive Dirac parafermion $\psi$ and a massless ordinary vector boson $A^a_\mu$ and verifying the strong locality condition [2, 3, 5, 7]:

$$\mathcal{L}_{\text{tot}} = \sum_{i=1}^{4} \mathcal{L}_i \, ,$$

where

$$\begin{aligned}
\mathcal{L}_1 =& \tfrac{1}{2}\left[\overline{\psi}, \not{D}\psi\right]_- + \tfrac{1}{2}\lambda_a A^a_\mu (-1)^{N_\psi} \vdots \left[\overline{\psi}, \gamma^\mu \psi\right]_- \vdots + \tfrac{m}{2}\left[\overline{\psi}, \psi\right]_- \\
& + \tfrac{\lambda}{2}(-1)^{N_\psi} \vdots \left[\overline{\psi}, \psi\right]_- \vdots \, , \\
\mathcal{L}_2 =& -\tfrac{\alpha}{2}\left[\partial_\mu A^\mu, \partial_\nu A^\nu\right]_+ \, , \\
\mathcal{L}_3 =& \tfrac{1}{2}\left[\partial_\mu \bar{\omega}_a, \partial^\mu \omega^a\right]_- - \tfrac{1}{2} g f_{abc} A^{c\mu}\left[\partial_\mu \bar{\omega}_b, \omega^b\right]_- \\
& + \tfrac{\lambda_{abc}}{2} A^{c\mu} (-1)^{N_\omega} \vdots \left[\partial_\mu \bar{\omega}^a, \omega^b\right]_- \vdots \, , \\
\mathcal{L}_4 =& -\tfrac{1}{4} F^{\mu\nu}_a F^a_{\mu\nu} \, .
\end{aligned} \tag{2.1}$$

and

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc}\left[A^b_\mu, A^a_\nu\right]_+ \, .$$

Now, $\not{D}$ is the usual covariant derivative, $m$ the parafermionic mass, $\alpha$ the gauge parameter, $g$ the gauge coupling, $\omega$ the parafield ghost and $N_\psi$ (resp. $N_\omega$) the parafermion (resp. paraghost) number operator defined as [2, 3, 5, 7]:

$$N_\psi = \int d^3\vec{x}\left[\overline{\psi}(x), \psi(x)\right]_-$$

and

$$N_\omega = \int d^3\vec{x}\left[\bar{\omega}_a(x), \omega^a(x)\right]_- \, . \tag{2.2}$$

The couplings $\lambda_a$ and $\lambda_{abc}$ are given by:

$$\lambda^a = \lambda_1 T^a$$

and

$$\lambda_{abc} = \lambda_2 f_{abc} \, , \tag{2.3}$$

where the $T^a$'s and $f_{abc}$'s are the generators and the structure constants of the gauge group respectively. The symmetric forms $\vdots[\psi,\gamma^\mu\psi]_-\vdots$ and $\vdots[\psi,\psi]_-\vdots$ are defined as [2, 3, 5, 7]:

$$\vdots[\overline{\psi},\gamma^\mu\psi]_-\vdots = 2!\,[\overline{\psi}^{(1)}\gamma^\mu\psi^{(2)} + \overline{\psi}^{(2)}\gamma^\mu\psi^{(1)}]$$

and

$$\vdots[\overline{\psi},\psi]_-\vdots = 2!\,[\overline{\psi}^{(1)}\psi^{(2)} + \overline{\psi}^{(2)}\psi^{(1)}]\,. \tag{2.4}$$

One has to notice the presence of the symmetric form $\vdots[\overline{\psi},\widehat{\Gamma}\psi]\vdots$ ($\widehat{\Gamma}$ is the Dirac or identity matrix) which is justified by the fact that in our model the order of the quantization is even (two). Thus, according to Kamefuchi and Ohnuki theorem concerning fields verifying the strong locality condition [2], the above mentioned symmetric form is allowed. This means that extra terms will appear and therefore, the ordinary BRST transformations will be altered. It is worth to mention that the non local factors $(-1)^{N_\psi}$ and $(-1)^{N_\omega}$ do not cause any essential difficulty because the even and odd sectors of the state vector space, which are defined with respect to the parity of the eigenvalues of the number operators $N_\psi$ an $N_\omega$, are completely separated from each other [5].

### *2.1. Paraquantum BRST transformations*

Before one gets to the BRST transformations, it is more convenient to work with ordinary fermions and bosons. This is essentially due to the complicated parafermionic and parabosonic commutation relations. To get rid of this, and as it is mentioned in Section 1, one has to transform corresponding parafields to ordinary fields. This is done by means of the Klein transformations [1–6]. Moreover, and in order to deal with classical rather than quantum fields, we use the paraquantum path integral formalism developed in our Ref. [5]. Thus a straightforward calculation gives (Appendix A):

$$\mathcal{L}_1 = \sum_{\alpha=1}^{2}\Big[\overline{\varphi}_j^{(\alpha)}\partial\!\!\!/\varphi_j^{(\alpha)} - (g+\eta_\alpha\lambda_1)T^a_{jk}A_{\mu a}\overline{\varphi}_j^{(\alpha)}\gamma^\mu\varphi_k^{(\alpha)} + (m+\eta_\alpha\lambda)\overline{\varphi}_j^{(\alpha)}\varphi_j^{(\alpha)}\Big]\,, \tag{2.1.1}$$

$$\mathcal{L}_2 = -\,\alpha\big(\partial_\mu A^{a\mu}\big)^2\,, \tag{2.1.2}$$

$$\mathcal{L}_3 = \sum_{\alpha=1}^{2} \Big[\partial^\mu \bar{\theta}_a^{(\alpha)} \partial_\mu \theta^{a(\alpha)} - (g + \eta_\alpha \lambda_3) f_{abc} A_\mu^c \partial^\mu \bar{\theta}^{a(\alpha)} \theta^{b(\alpha)}\Big] \tag{2.1.3}$$

and

$$\mathcal{L}_4 = -\tfrac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}\,, \tag{2.1.4}$$

where

$$F_{\mu\nu}^a = \partial_\mu A_\mu^a - \partial_\nu A_\mu^a + 2g f_{abc} A_\mu^b A_\nu^c \tag{2.1.5}$$

and

$$\eta_\alpha = \begin{cases} 1 & \text{if} \quad \alpha = 1\,, \\ -1 & \text{if} \quad \alpha = 2\,. \end{cases} \tag{2.1.6}$$

Now, the $\varphi$'s (resp. $\theta$'s) are ordinary fermions (resp. ghost bosons) defined as:

$$\varphi^{(\alpha)} = \frac{1}{\sqrt{2}}\Big[\phi^{(1)} + i\eta_\alpha \phi^{(2)}\Big] \tag{2.1.7a}$$

and

$$\theta_a^{(\alpha)} = \frac{1}{\sqrt{2}}\Big[\Omega_a^{(1)} + i\eta_\alpha \Omega_a^{(2)}\Big]\,. \tag{2.1.7b}$$

The $\phi$'s (resp. $\Omega$'s) are the Green components of the parafermionic (resp. paraghost) field $\psi$ (resp. $\omega$) given by [2, 5]:

$$\psi = \phi^{(1)} - ik_2 \phi^{(2)} \tag{2.1.8}$$

and

$$\omega_a = \Omega_a^{(1)} - ik_2' \Omega_a^{(2)} \tag{2.1.9}$$

where $k_2$ and $k_2'$ are the parafermion and paraghost Klein operators respectively whose expressions are [2, 5]:

$$k_2 = (-1)^{N_\psi} \tag{2.1.10}$$

and

$$k_2' = (-1)^{N_\omega}\,, \tag{2.1.11}$$

where $N_\psi$ and $N_\omega$ are given in Eq. (2.2).

Now it is very important to notice that, if one sets $\lambda_1 = g$ and $\lambda = m$, the variation of $\mathcal{L}_1$ under the following transformations:

$$\delta A^a_\mu = -\varepsilon\left[\delta^{ab}\partial_\mu - 2gf^{abc}A_{c\mu}\right]f_b\,, \tag{2.1.12}$$

$$\delta\varphi^{(1)}_j = -2ig\varepsilon T^a_{jk}f_a\varphi^{k(1)}\,, \tag{2.1.13}$$

$$\delta\overline{\varphi}^{(1)}_j = 2ig\varepsilon T^a_{kj}f_a\overline{\varphi}^{k(1)}\,, \tag{2.1.14}$$

$$\delta\varphi^{(2)}_j = 0 \tag{2.1.15}$$

and

$$\delta\overline{\varphi}^{(2)}_j = G\overline{\varphi}^{(2)}_j\,, \tag{2.1.16}$$

gives (see Appendix B)

$$\delta\mathcal{L}_1 = G\overline{\varphi}^{(2)}_j\not{\partial}\varphi^{j(2)}\,. \tag{2.1.17}$$

The $f$'s and $G$ are functions of $\theta$'s, $\varphi$'s and the gauge field $A^a_\mu$ whose expressions will be determined later. The only condition one can impose on the $f$'s is:

$$f^a\varphi^{(1)}_k = \varphi^{(1)}_k f^a \tag{2.1.18}$$

(a similar condition holds for $\varphi^{+(1)}_k$). The parameter $\varepsilon$ is a Grassmann number (independent of the space-time coordinates) which anticommutes with the $\theta$'s, $\varphi$'s and $f$'s. Now, using the transformations (2.1.12), (2.1.14) we obtain[1]:

$$\delta\mathcal{L}_2 = 2\alpha\varepsilon\left(\partial_\nu A^{a\nu}\right)\left[\Box f_a - 2gf_{abc}\partial^\mu\left(A^c_\mu f^b\right)\right], \tag{2.1.19a}$$

$$\begin{aligned}\delta\mathcal{L}_3 = \sum_{\alpha=1}^{2}\Big\{&\left[-\Box\theta^{(\alpha)}_a + (g+\eta_\alpha\lambda_3)f_{abc}\partial^\mu\left(A^c_\mu\theta^{b(\alpha)}\right)\right]\delta\bar{\theta}^{a(\alpha)}\\ &+\left[-\Box\bar{\theta}^{(\alpha)}_a + (g+\eta_\alpha\lambda_3)f_{abc}A^c_\mu\partial^\mu\bar{\theta}^{b(\alpha)}\right]\delta\theta^{a(\alpha)}\\ &+\varepsilon(g+\eta_\alpha\lambda_3)f_{abc}\left[\partial_\mu f^c - 2gf^{cde}A_{\mu e}f_d\right]\partial^\mu\bar{\theta}^{a(\alpha)}\theta^{b(\alpha)}\Big\}\\ &+\text{a total derivative}\end{aligned} \tag{2.1.19b}$$

and

$$\delta\mathcal{L}_4 = 0. \tag{2.1.19c}$$

[1] Using the antisymmetric property of the strength field tensor $F^a_{\mu\nu}$ together with Eq. (2.1.12) and the gauge field equation of motion one gets $\delta\mathcal{L}_4 = 0$.

Setting:

$$f^a = h \sum_{\alpha=1}^{2} \theta^{a(\alpha)}, \tag{2.1.20}$$

where

$$h = \overline{\varphi}_j^{(2)} \not{\partial} \varphi^{j(2)} \tag{2.1.21}$$

and choosing:

$$\delta \bar{\theta}^{a(\alpha)} = 0, \tag{2.1.22}$$

$$\delta \theta^{a(\alpha)} = -\varepsilon (g + \eta_\alpha \lambda_3) f^{abc} h \theta_b^{(\alpha)} \theta_c^{(\alpha)} \tag{2.1.23}$$

and

$$\begin{aligned} G = -\,\varepsilon\Big\{\alpha\Big[\Box\,(\partial_\nu A_b^\nu) + 2g f_{abc}\partial_\mu\partial_\nu A^{a\nu}A^{c\mu}\Big]\sum_{\alpha=1}^{2}\theta^{b(\alpha)} \\ + (g+\lambda_3) f^{abc}\Big[f_{dae}A^{e\mu}\partial_\mu\bar{\theta}^{d(1)}\theta_b^{(1)}\left(\lambda_3\theta_c^{(1)} - g\theta_c^{(2)}\right) \\ - \partial_\mu\bar{\theta}_a^{(1)}\partial^\mu\theta_b^{(1)}\sum_{\alpha=1}^{2}\theta_c^{(\alpha)}\Big] + \text{similar terms with} \\ \left(\lambda_3 \longleftrightarrow -\lambda_3,\quad \theta^{(1)} \longleftrightarrow \theta^{(2)}\right)\Big\} \end{aligned} \tag{2.1.24}$$

we obtain:

$$\sum_{i=1}^{4} \delta \mathcal{L}_i = \delta \mathcal{L}_{\text{tot}} = 0 \tag{2.1.25}$$

and therefore the Lagrangian $\mathcal{L}_{\text{tot}}$ is invariant under the paraquantum BRST transformations (PBRST) (2.1.12)–(2.1.16) and (2.1.22)–(2.1.23). It is to be noted that this PBRST invariance is a natural consequence of the local gauge invariance of the original Lagrangian (before introducing the gauge fixing and Faddev–Popov ghost terms) and the paraquantum symmetry [2–7].

Now the crucial point is that in ordinary non Abelian quantum field theories and in spite of the drastic changes of the theory due to the introduction of the Fadeev–Popov ghosts (FP) and gauge fixing terms (GF), the difference between the original Lagrangian density (before the introduction of FP and GF) and the total one is just a BRST coboundary term of the form $\delta(\ldots)$ [8]. This fact is just a consequence of the nilpotency property of the BRST anti derivation $\delta$ *i.e.* $\delta^2 = 0$ [9, 10]. However, and as it will be clear in what follows, the nilpotency property is violated in our model. This is one of the fundamental differences between ordinary and paraquantum theories. In fact, using Eqs (2.1.12) and (2.1.20) one gets:

$$\delta^2 A^a_\mu = -\varepsilon\Big[D^{ab}_\mu(Gf_b) + D^{ab}_\mu\Big(h\sum_{\alpha=1}^{2}\delta\theta^{(\alpha)}_b\Big) + 2\varepsilon g f^{abc} f_b\big(D_{\mu cd} f^d\big)\Big] \neq 0\,, \tag{2.1.26}$$

where $\delta\theta^{(\alpha)}_b$ is given by Eq. (2.1.23) and the covariant derivative $D_{\mu cd}$ is defined as:

$$D_{\mu cd} = \delta_{cd}\partial_\mu - 2g f_{cda} A^a_\mu\,. \tag{2.1.27}$$

### *2.2. Paraquantum BRST charge $Q_{\mathrm{PB}}$*

As in ordinary quantum gauge theories, one can construct a PBRST charge $Q_{\mathrm{PB}}$ as follows:

$$Q_{\mathrm{PB}} = \int d^3\vec{x}\, j^0_{\mathrm{PB}}\,, \tag{2.2.1}$$

where the PBRST current $j^\mu_{\mathrm{PB}}$ is given by

$$j_{\mathrm{PB}} = \delta\chi^{(n)}\frac{\partial\mathcal{L}_{\mathrm{tot}}}{\partial(\partial_\mu\chi^{(n)})} \tag{2.2.2}$$

and $\chi^{(n)}$ stands for $A^a_\mu$, $\varphi^{(\alpha)}_k$, $\overline{\varphi}^{(\alpha)}_k$, $\theta^{(\alpha)}_a$ and $\bar{\theta}^{(\alpha)}_a$ $(\alpha = \overline{1,2})$. With the help of the PBRST Eqs (2.1.12)–(2.1.16) and (2.1.22)–(2.1.23), a straightforward calculation gives:

$$\begin{aligned} j^\mu_{\mathrm{PB}} = & -\varepsilon\Big[2\big(D^{ab}_\nu f_b\big)\Big(\alpha\delta^{\mu\nu}\partial_\rho A^\rho_a + F^{\mu\nu}_a\Big) + 2igT^a_{kl} f_a \overline{\varphi}^{k(1)}\gamma^\mu\phi^{l(1)} \\ & + h\sum_{\alpha=1}^{2}(g+\eta_\alpha\lambda_3)\Big(D^{\mu(\alpha)ab}\bar{\theta}^{(\alpha)}_b\Big) f_{ade}\theta^{d(\alpha)}\theta^{e(\alpha)}\Big]\,, \end{aligned} \tag{2.2.3}$$

where $D^{ab}_\nu$ is given by Eq. (2.1.27) and $D^{(\alpha)\mu ab}$ has the following expression:

$$D^{(\alpha)\mu ab} = \Big[\delta^{ab}\partial^\mu - (g+\eta_\alpha\lambda_3) f^{abc} A^\mu_c\Big]\,. \tag{2.2.4}$$

Now, and contrary to the usual ordinary quantum non abelian theories where the BRST charge is nilpotent [8, 9], the PBRST charge $Q_{\mathrm{PB}}$ does not have this property. In fact, if one takes the expression (2.2.2) ($\chi^{(n)}$ stands for quantum fields) as well as the following equal time canonical commutation relations:

$$\left[\chi^{(n)}(x),\, \Pi_{\chi^{(m)}}(y)\right]_{\mp} = -i\delta_{n,m}\delta^3(\vec{x}-\vec{y})$$

and

$$\left[\chi^{(n)}(x),\chi^{(m)}(y)\right]_{\mp} = \left[\Pi_{\chi^{(n)}}(x),\Pi_{\chi^{(m)}}(y)\right]_{\mp} = 0\,, \qquad (2.2.5)$$

where $\Pi_{\chi^{(n)}}$ is the momentum variable conjugate to $\chi^{(n)}$ and the anticommutators (with + sign) are taken only if both fields are fermions, one can show easily that:

$$\left[Q_{\rm PB},\chi^{(n)}\right] = -i\delta\chi^{(n)}\,. \qquad (2.2.6)$$

Now, thanks to the relation (2.2.6), it is straightforward to deduce that

$$\left[Q^2_{\rm PB},\, \theta^{(\alpha)}_a\right]_- = \left[Q_{\rm PB},\delta\theta^{(\alpha)}_a\right]_+\,. \qquad (2.2.7)$$

Moreover, from the expressions of $Q_{\rm PB}$ and $h$, one gets

$$\left[Q_{\rm PB},\, h\right]_- = 0\,. \qquad (2.2.8)$$

Thus, Eqs (2.2.7), (2.2.8), (2.1.23) and (2.1.24) imply that

$$\left[Q_{\rm PB},\, \delta\theta^{(\alpha)}_a\right]_+ = -\,2\varepsilon(g+\eta_\alpha\lambda_3)f_{abc}h\big[-i\delta\theta^{(\alpha)b}\theta^{(\alpha)c} + \theta^{(\alpha)b}\theta^{(\alpha)c}Q_{\rm PB}\big] \qquad (2.2.9)$$

and since

$$i\delta\theta^{(\alpha)b}\theta^{(\alpha)c} \neq \theta^{(\alpha)b}\theta^{(\alpha)c}Q_{\rm PB}\,,$$
$$[Q^2_{\rm PB},\, \theta^{(\alpha)}_a]_- \neq 0$$

which yields

$$Q^2_{\rm PB} \neq 0\,. \qquad (2.2.10)$$

Another important symmetry one can have from our Lagrangian (2.1), is its invariance under a ghost phase transformation:

$$\theta^{(\alpha)}_a \to e^{i\beta}\theta_a{}^{(\alpha)}$$

and

$$\bar\theta_a{}^{(\alpha)} \to e^{-i\beta}\bar\theta_a{}^{(\alpha)}\,, \qquad (2.2.11)$$

where $\beta$ is a real parameter independent of the space time coordinates. The corresponding Noether conserved current and charge are given respectively by

$$J^{\mu}_{\theta(\alpha)} = -i\partial^{\mu}\bar{\theta}^{(\alpha)}_{a}\theta^{(\alpha)a} - i\bar{\theta}^{(\alpha)}_{a}D^{(\alpha)ab}_{\mu}\theta_{b}{}^{(\alpha)} \tag{2.2.12}$$

$$Q_{\theta(\alpha)} = \int d^{3}\vec{x}J^{0}_{\theta(\alpha)}\,. \tag{2.2.13}$$

($D^{(\alpha)ab}_{\mu}$ is given by Eq. (2.2.4).) Since $Q_{\theta(\alpha)}$ generates the transformations (2.2.11) on Fadeev–Popov (FP) ghost fields $\theta^{(\alpha)}$ and leaves the other fields invariant:

$$\left[Q_{\theta(\alpha)}, \theta_{a}{}^{(\alpha)}\right] = -i\theta_{a}{}^{(\alpha)}$$

and

$$\left[Q_{\theta(\alpha)}, \bar{\theta}_{a}{}^{(\alpha)}\right] = -i\bar{\theta}_{a}{}^{(\alpha)}\,, \tag{2.2.14}$$

we call it the paraquantum FP (PFP) ghost charge. The latter, and contrary to the ordinary non abelian quantum theories [9, 10], does not constitute with the PBRST charge $Q_{\rm PB}$ a closed algebra. In fact, it is obvious that

$$\left[Q_{\rm PB}, Q_{\rm PB}\right]_{+} = 2Q^{2}_{\rm PB}\,,$$
$$\left[Q_{\theta(\alpha)}, Q_{\theta(\alpha)}\right]_{-} = 0\,,$$

but

$$\begin{aligned}\left[Q_{\theta(\alpha)}, Q_{\rm PB}\right]_{-} &= i\delta\theta_{a}{}^{(\alpha)}\Pi_{\theta_{a}(\alpha)} + i\Big\{\theta^{(\alpha)}\left[\Pi_{\theta(\alpha)}, \delta\chi^{(m)}\right] \\ &\quad - \bar{\theta}^{(\alpha)}\left[\Pi_{\bar{\theta}(\alpha)}, \delta\chi^{(m)}\right]\Big\} \\ &\neq \Lambda_{1}Q_{\theta(\beta)} \quad (\beta = 1, 2)\end{aligned}$$

or

$$\neq \Lambda_{2}Q_{\rm PB}\,, \tag{2.2.15}$$

where $\Lambda_1$ and $\Lambda_2$ are complex numbers. Thus, from Eqs (2.2.15) $Q_{\rm PB}$ and $Q_{\theta(\alpha)}$ cannot constitute a closed algebra.

### *2.3. Anti-BRST transformations*

As in ordinary non abelian quantum theories, we can define the paraquantum anti-BRST transformations (PABRST) as follows:

$$\bar{\delta}A^{a}_{\mu} = -\varepsilon\left[\delta^{ab}\partial_{\mu} - 2gf^{abc}A_{c\mu}\right]\bar{f}_{b}\,,$$

$$
\begin{aligned}
\bar{\delta}\varphi_j^{(1)} &= -2ig\varepsilon T^a_{jk}\bar{f}_a\varphi_k^{(1)}\,,\\
\bar{\delta}\overline{\varphi}_j^{(1)} &= 2ig\varepsilon T^a_{kj}\bar{f}_a\overline{\varphi}_k^{(1)}\,,\\
\bar{\delta}\varphi_j^{(2)} &= 0\,,\\
\bar{\delta}\overline{\varphi}_j^{(2)} &= -\overline{G}\overline{\varphi}_j^{(2)}\,,\\
\bar{\delta}\bar{\theta}_a^{(\alpha)} &= -\varepsilon(g+\eta_\alpha\lambda_3)f_{abc}h\bar{\theta}^{(\alpha)b}\bar{\theta}^{c(\alpha)}\,,\\
\bar{\delta}\theta_a{}^{(\alpha)} &= 0\,,
\end{aligned}
\tag{2.3.1}
$$

where

$$
\bar{f}_a = h\sum_{\alpha=1}^{2}\bar{\theta}_a{}^{(\alpha)}\,. \tag{2.3.2}
$$

The PABRST $\bar{\delta}$ operation is defined as

$$
\bar{\delta} \equiv C_{\mathrm{PFP}}\delta C^{-1}_{\mathrm{PFP}}\,, \tag{2.3.3}
$$

where $\delta$ is the previous PBRST anti-derivation and $C_{\mathrm{PFP}}$ is the PFP charge conjugation given by

$$
\begin{aligned}
C_{\mathrm{PFP}}A^a_\mu &= A^a_\mu\,,\\
C_{\mathrm{PFP}}\widehat{\varphi}^{(\alpha)} &= \widehat{\varphi}^{(\alpha)}\,, \quad (\widehat{\varphi} = \varphi \text{ or } \overline{\varphi})\\
C_{\mathrm{PFP}}\theta_a{}^{(\alpha)} &= \bar{\theta}_a{}^{(\alpha)}
\end{aligned}
$$

and

$$
C_{\mathrm{PFP}}\bar{\theta}_a{}^{(\alpha)} = -\theta_a{}^{(\alpha)}\,. \tag{2.3.4}
$$

For ordinary non abelian gauge theories, the invariance under the anti-BRST transformations follows immediately from the invariance under the BRST transformations and the FP charge conjugation [8, 11]. However, it is not the case here. In fact, working in the Landau gauge ($\alpha = 0$) (to keep our proof transparent) it is easy to show that:

$$
\bar{\delta}\mathcal{L}_{\mathrm{tot}} = C_{\mathrm{FP}}[Gh - \delta\bar{\mathcal{L}}_3]\,, \tag{2.3.5}
$$

where

$$
\bar{\mathcal{L}}_3 = \mathcal{L}_3\big(\theta_a{}^{(\alpha)} \longleftrightarrow \bar{\theta}_a{}^{(\alpha)}\big)\,. \tag{2.3.6}
$$

Moreover, and from Subsection 2.2., we have found that

$$
\delta\mathcal{L}_3 = -Gh\,.
$$

Thus Eq. (2.3.5) becomes

$$\bar{\delta}\mathcal{L}_3 = -C_{\mathrm{FP}}\,\delta\,[\mathcal{L}_3 + \bar{\mathcal{L}}_3]\,. \tag{2.3.7}$$

Now, using Eqs (2.1.3) and (2.3.6) we obtain

$$\delta[\mathcal{L}_3 + \bar{\mathcal{L}}_3] = -\sum_{\alpha=1}^{2}\Big[2\,\Box\,\bar{\theta}_a{}^{(\alpha)} + (g + \eta_\alpha\lambda_3)f_{abc}\bar{\theta}^{(\alpha)b}\partial^\mu A^c_\mu\Big]\delta\theta^{(\alpha)a}\,. \tag{2.3.8}$$

Hence, it is clear from Eq. (2.3.8) that $\delta[\mathcal{L}_3 + \bar{\mathcal{L}}_3] \neq 0$ and therefore

$$\bar{\delta}\mathcal{L}_{\mathrm{tot}} \neq 0\,. \tag{2.3.9}$$

This is a striking result! It implies that the PBRST invariance does not imply the PABRST one and vice-versa. As a consequence, the fundamental relation

$$\delta\bar{\delta} + \bar{\delta}\delta = 0\,, \tag{2.3.10}$$

which holds in ordinary non abelian theories [8, 11] is violated in our model. In fact, using the definition of $\delta$ as well as Eqs (2.1.16) and (2.1.25) we obtain (in the Landau gauge):

$$(\bar{\delta}\delta + \delta\bar{\delta})\overline{\varphi}_j^{(2)} = \Big\{C_{\mathrm{PFP}}\delta\overline{G} - \delta\overline{G} + [\overline{G},\, G]_-\Big\}\overline{\varphi}_j^{(2)} \neq 0\,, \tag{2.3.11}$$

where

$$\overline{G} = G\big(\theta_a{}^{(\alpha)} \longleftrightarrow \bar{\theta}_a{}^{(\alpha)}\big)\,.$$

## 3. Paraquantum Slavnov–Taylor identities

As we know, any symmetry of a given quantum field theory is manifested through some relations between various Green's functions called Slavnov–Taylor identities [12]. It is the purpose of this section to derive such relations for a paraquantum field theory of order two (our present model).

Now, from the invariance of the vacuum under the PBRST transformations *i.e.* from

$$Q_{\mathrm{PB}}|0\rangle = 0$$

we obtain

$$\langle 0|\delta\mathcal{L}_{\mathrm{tot}}|0\rangle = 0\,. \tag{3.1}$$

It is convenient to formulate the identities (3.1) as equations for the generating functional $\Gamma$ (effective action) of one particle irreducible (or proper)

Green's function as follows: to define $\Gamma$, we need a source functional $S$ defined by

$$S[s,x,y,I,J] \equiv \int d^4x \Big[ s_\mu^a A_a^\mu + \sum_{\alpha=1}^{2} \big( \theta_a{}^{(\alpha)} x_\alpha^a + \bar{\theta}^{(\alpha)} y_\alpha^a + \varphi_k{}^{(\alpha)} J_\alpha^k + \overline{\varphi}_k{}^{(\alpha)} I_\alpha^k \big) \Big], \quad (3.2)$$

where $s$'s, $x$'s, $y$'s $j$'s and $I$'s are the Schwinger sources ($x$'s, $y$'s, $j$'s and $I$'s are Grassmann $C$-numbers). As it is developed in our Refs [5] and [7], one can write the partition function $Z$ as

$$Z = \int DA_\mu D\theta^{(1)} \ldots \exp\left[ i \int d^4x \mathcal{L}_{\text{tot}} + iS \right] \quad (3.3)$$

since

$$\delta \mathcal{L}_{\text{tot}} = 0 .$$

Then, the invariance of the partition function under PBRST implies that

$$i \int d^4x \langle 0|T \Big[ s_\mu^a \delta A_a^\mu + \sum_{\alpha=1}^{2} \Big( \delta\theta_a{}^{(\alpha)} x_\alpha^a + \delta\bar{\theta}_a{}^{(\alpha)} y_\alpha^a + \delta\varphi_k{}^{(\alpha)} j_\alpha^k + \delta\overline{\varphi}_k{}^{(\alpha)} I_\alpha^k \Big) \Big] \exp iS|0\rangle = 0 . \quad (3.4)$$

(T is the time ordering product). Defining the functional $\Gamma$ as the Legendre transform of the partition function $W$ of the connected Green's functions as follows [13, 14]

$$W[s,x,y,I,J] = \Gamma(A,\theta,\bar{\theta}) + \langle 0|TS|0\rangle , \quad (3.5)$$

one can show that

$$s_\mu^a = -\frac{\delta'\Gamma}{\delta' A_\mu^a}, \quad x_\alpha^a = -\frac{\delta'\Gamma}{\delta'\theta_a{}^{(\alpha)}}, \quad y_\alpha^a = -\frac{\delta'\Gamma}{\delta'\bar{\theta}_a{}^{(\alpha)}}$$

and

$$j_\alpha^k = \frac{\delta' W}{\delta'\varphi_k{}^{(\alpha)}}, \quad I_\alpha^k = \frac{\delta' W}{\delta'\overline{\varphi}_k{}^{(\alpha)}} . \quad (3.6)$$

Moreover, from Eq. (3.5) we obtain

$$\langle 0|A_\mu^a|0\rangle = \frac{\delta' W}{\delta' S_\mu^a}, \quad \langle 0|\theta_\alpha^a|0\rangle = \frac{\delta' W}{\delta' x_\alpha^a},$$

$$\langle 0|\bar{\theta}_\alpha^a|0\rangle = \frac{\delta' W}{\delta' y_\alpha^a}, \quad \langle 0|\varphi_k{}^{(\alpha)}|0\rangle = \frac{\delta' W}{\delta' j_\alpha^k}$$

and

$$\langle 0|\overline{\varphi}_k{}^{(\alpha)}|0\rangle = \frac{\delta' W}{\delta' I^k_\alpha}\,. \tag{3.7}$$

"$\delta'$" means the functional variation. A straightforward calculation using Eqs (3.2), (3.6) and (3.7) as well as Eqs (2.1.12)–(2.1.16) and (2.1.22)–(2.1.24), gives

$$\int d^4x \left[\frac{\delta'\Gamma}{\delta' A^a_\mu}\Omega^1_{\mu,a} + \sum_{\alpha=1}^{2}\Big(\frac{\delta'\Gamma}{\delta'\theta_a{}^{(\alpha)}}\Omega^{2,\alpha}_{\mu,a} + \frac{\delta' W}{\delta'\varphi_k{}^{(\alpha)}}\Omega^{3,\alpha}_k + \frac{\delta' W}{\delta'\overline{\varphi}_k{}^{(\alpha)}}\Omega^{4,\alpha}_k\Big)\right] = 0\,, \tag{3.8}$$

where

$$\Omega^1_{\mu,a} = \sum_{\alpha=1}^{2}\Big[\partial^\mu\Big(\frac{\delta' W}{\delta' x^a_\alpha}\frac{\delta' W}{\delta' I^k_2}\not{\partial}\frac{\delta' W}{\delta' j^k_2}\Big) - 2g f_{abc}\frac{\delta' W}{\delta' s^a_\mu}\frac{\delta' W}{\delta' x^a_\alpha}\frac{\delta' W}{\delta' I^k_2}\not{\partial}\frac{\delta' W}{\delta' j^k_2}\Big]\,, \tag{3.9a}$$

$$\Omega^{2,\alpha}_{\mu,a} = (g+\eta_\alpha\lambda_3) f_{abc}\frac{\delta' W}{\delta' x^b_\alpha}\frac{\delta' W}{\delta' x^c_\alpha}\frac{\delta' W}{\delta' I^k_2}\not{\partial}\frac{\delta' W}{\delta' J^k_2}\,, \tag{3.9b}$$

$$\Omega^{3,1}_{\mu,a} = 2igT^a_{jk}\frac{\delta' W}{\delta' j^k_1}\sum_{\alpha=1}^{2}\frac{\delta' W}{\delta' x^a_\alpha}\frac{\delta' W}{\delta' I^i_2}\not{\partial}\frac{\delta' W}{\delta' j^i_2}\,, \tag{3.9c}$$

$$\Omega^{3,2}_{\mu,a} = 0\,, \tag{3.9d}$$

$$\Omega^{4,1}_k = -\,2igT^a_{kl}\frac{\delta' W}{\delta' I^l_1}\sum_{\alpha=1}^{2}\frac{\delta' W}{\delta' x^a_\alpha}\frac{\delta' W}{\delta' I^i_2}\not{\partial}\frac{\delta' W}{\delta' j^i_2}\,, \tag{3.9e}$$

$$\Omega^{4,2}_k = F\frac{\delta' W}{\delta' I^2_k} \tag{3.9f}$$

and

$$F = -\Bigg\{\alpha\Big[\Box\Big(\partial_\nu\frac{\delta' W}{\delta' s_{\mu b}}\Big) + 2g f_{abc}\Big(\partial_\mu\partial_\nu\frac{\delta' W}{\delta' s_{\mu a}}\Big)\frac{\delta' W}{\delta' s_{\mu c}}\Big]\sum_{\alpha=1}^{2}\frac{\delta' W}{\delta' x^b_\alpha} + (g+\lambda_3) f_{abc}\Big[f_{ead}\frac{\delta' W}{\delta' s_{\mu d}}\Big(\partial_\mu\frac{\delta' W}{\delta' y^e_1}\Big)\frac{\delta' W}{\delta' x^b_1}\Big(\lambda_3\frac{\delta' W}{\delta' x^c_1} - g\frac{\delta' W}{\delta' x^c_2}\Big) - \Big(\partial_\mu\frac{\delta' W}{\delta' y^a_1}\Big)\Big(\partial_\mu\frac{\delta' W}{\delta' x^b_1}\Big)\sum_{\alpha=1}^{2}\frac{\delta' W}{\delta' x^c_\alpha}\Big] + \text{similar terms with}$$

$$\left.(\lambda_3 \longleftrightarrow -\lambda_3, \quad x_1 \longleftrightarrow x_2, \quad y_1 \longleftrightarrow y_2)\right\} . \tag{3.10}$$

Thus, Eq. (3.8) is our paraquantum Slavnov–Taylor identity.

## 4. Conclusions

We conclude from the previous study of our simple model (a simple extension of ordinary quantum field theory) that the transition from ordinary to paraquantum field theory (of order two or more) yields:

*(i)* The modification of the PBRST transformations.
*(ii)* The violation of the nilpotency property $\delta^2 \neq 0$ and $Q^2_{\text{PB}} \neq 0$.
*(iii)* The violation of the closure of the PBRST algebra.
*(iv)* The violation of the anti-PBRST invariance and therefore the fundamental relation $\bar{\delta}\delta + \delta\bar{\delta} = 0$.
*(v)* The modification of the Slavnov–Taylor identity.

Hence, a simple extension of a non abelian gauge theory through the paraquantization changes drastically all its fundamental properties. More details are under investigation and study [16].

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## Appendix A

The Klein operator $k_{\alpha-\rho(\alpha)}$ (or $K'_{\alpha-\rho(\alpha)}$) (where $\alpha = \overline{1,Q}$ and $Q$ is the order of paraquantization) is defined as [5, 6]

$$k_{\alpha-\rho(\alpha)} = \exp\left(i\pi \sum_{\beta=1}^{\alpha-\rho(\alpha)} N^{(\beta)}\right) \quad \text{for} \quad 2 \geq \alpha - \rho(\alpha) \geq Q \tag{A.1}$$

and

$$k_0 = 1\,,$$

where $N^{(\beta)}$ is the number operator for the corresponding paraquantum field Green's component $\hat{\chi}^{(\beta)}(x)$ ($\hat{\chi}^{(\beta)}$ stands for $\hat{\Psi}^{(\beta)}$'s and $\hat{\omega}^{(\beta)}$'s and "$\hat{\chi}$" means $\overline{\chi}$ or $\chi$) given by

$$N^{(\beta)} = \int d^3\vec{x}\, \overline{\chi}^{(\beta)}(x)\chi^{(\beta)}(x)\,, \tag{A.2}$$

where

$$\widehat{\chi}(x) = \sum_{\beta=1}^{Q} \widehat{\chi}^{(\beta)} .$$

This Klein operator has the following properties:

$$k_{\alpha-\rho(\alpha)} = k^{+}_{\alpha-\rho(\alpha)} = k^{-1}_{\alpha-\rho(\alpha)} , \tag{A.3}$$

$$\left[k_{\alpha-\rho(\alpha)}, \widehat{\chi}^{(\beta)}(x)\right]_{-} = 0 \quad \text{for} \quad \alpha-\rho(\alpha) < \beta \tag{A.4}$$

and

$$\left[k_{\alpha-\rho(\alpha)}, \widehat{\chi}^{(\beta)}(x)\right]_{+} = 0 \quad \text{for} \quad \alpha-\rho(\alpha) \leq \beta , \tag{A.5}$$

with

$$\rho(\alpha) = \begin{cases} 0 & \text{if} \quad \alpha \quad \text{is even} \\ 1 & \text{if} \quad \alpha \quad \text{is odd} \end{cases} \tag{A.6}$$

Now, the Klein transformed fields are given by [5, 6]:

$$\widehat{\chi}(x) = \sum_{\alpha=1}^{Q} (-1)^{1-\rho(\alpha)} k_{\alpha-\rho(\alpha)} \hat{\xi}^{(\alpha)}(x) , \tag{A.7}$$

where in this case the $\xi^{(\alpha)}$'s obey the ordinary canonical commutation relations namely

$$\left[\bar{\xi}^{(\alpha)}, \xi^{(\beta)}\right] = 0 \quad \text{if} \quad \alpha \neq \beta . \tag{A.8}$$

In our present case we have $Q = 2$ which implies that $\rho = 0$. Moreover, the resulting Klein transformed fields (see Eq. (2.1.7) are given by:

$$\begin{aligned} \widehat{\psi}^{(1)} &= \widehat{\phi}^{(1)} , \\ \widehat{\omega}^{(1)}_a &= \widehat{\Omega}^{(1)} , \\ \widehat{\psi}^{(2)} &= \pm i k_2 \widehat{\chi}^{(2)} \quad (+ \text{ for } \widehat{\psi} = \overline{\psi}) , \\ \widehat{\omega}^{(2)}_a &= \pm i k'_2 \widehat{\Omega}^{(2)}_a \quad (+ \text{ for } \widehat{\omega}_a = \overline{\omega}_a) \end{aligned} \tag{A.9}$$

with

$$k_2 = (-1)^{N_\psi}$$

and

$$k'_2 = (-1)^{N_\omega} . \tag{A.10}$$

Now, Eq. (A.3) implies that:

$$k_2 \partial_\mu k_2 = k'_2 \partial_\mu k'_2 = 0 . \tag{A.11}$$

By using Eqs (A.8)–(A.11) as well as Eq. (1.4) we obtain:

$$\mathcal{L}_1 = \tfrac{1}{2}\sum_{\alpha=1}^{2}\left[\bar{\phi}^{(\alpha)}, \not{D}\phi^{(\alpha)}\right] + i\lambda_a A_\mu^a\left[-\bar{\phi}^{(1)}\not{D}\phi^{(2)} + \bar{\phi}^{(2)}D\phi^{(1)}\right]$$
$$+ \tfrac{m}{2}\sum_{\alpha=1}^{2}\left[\bar{\phi}^{(\alpha)}, \phi^{(\alpha)}\right] + i\lambda\left[-\bar{\phi}^{(1)}\phi^{(2)} + \bar{\phi}^{(2)}\phi^{(1)}\right], \tag{A.12}$$

setting

$$\widehat{\phi}^{(1)} = \frac{\varphi^{(1)} + \varphi^{(2)}}{\sqrt{2}}, \quad \widehat{\Omega}_a^{(1)} = \frac{\theta_a^{(1)} + \theta_a^{(2)}}{\sqrt{2}},$$

and

$$\widehat{\phi}^{(2)} = \frac{\varphi^{(1)} - \varphi^{(2)}}{\sqrt{2}}, \quad \widehat{\Omega}_a^{(2)} = \frac{\theta_a^{(1)} - \theta_a^{(2)}}{\sqrt{2}}.$$

One gets thus

$$\mathcal{L}_1 = \tfrac{1}{2}\sum_{\alpha=1}^{2}\left[\overline{\varphi}^{(\alpha)}, \not{D}\varphi^{(\alpha)}\right] + \lambda_a A_\mu^a\left[-\overline{\varphi}^{(1)}\varphi^{(2)} + \overline{\varphi}^{(2)}\varphi^{(1)}\right]$$
$$+ \tfrac{m}{2}\sum_{\alpha=1}^{2}\left[\overline{\varphi}^{(\alpha)}, \varphi^{(\alpha)}\right] + \lambda\left[-\overline{\varphi}^{(1)}\varphi^{(1)} + \overline{\varphi}^{(2)}\varphi^{(2)}\right]. \tag{A.13}$$

Using the path integral formalism of Ref. [5] we get the classical Lagrangian $\mathcal{L}_1$ of Eq. (2.1.1). Similarly, following the same procedure one can get the expression (2.1.3) of $\mathcal{L}_3$.

## Appendix B

Starting from the expression (2.1.1) of $\mathcal{L}_1$ with $\lambda_1 = g$ and $\lambda = m$ (our model) one gets:

$$\delta\mathcal{L}_1 = \sum_{\alpha=1}^{2}\Big\{\delta\overline{\varphi}_j^{(\alpha)}\not{\partial}\varphi_j^{(\alpha)} + \overline{\varphi}_j^{(\alpha)}\not{\partial}\varphi_j^{(\alpha)} - (1+\eta_\alpha)gT_{jk}^a\left[A_{\mu a}\overline{\varphi}_j^{(\alpha)}\partial^\mu\delta\varphi_k^{(\alpha)}\right.$$
$$\left. + \delta A_{\mu a}\overline{\varphi}_j^{(\alpha)}\partial^\mu{\varphi_k}^{(\alpha)} + A_{\mu a}\delta\overline{\varphi}_j^{(\alpha)}\partial^\mu\varphi_k^{(\alpha)}\right]$$
$$+ (1+\eta_\alpha)m\left[\delta\overline{\varphi}_j^{(\alpha)}\varphi_j^{(\alpha)} + \overline{\varphi}_j^{(\alpha)}\delta\varphi_j^{(\alpha)}\right]\Big\}, \tag{B.1}$$

where $\eta_\alpha$ is defined in Eq. (2.1.6).

Now, using the transformations (2.1.12)–(2.1.16) together with the relation

$$\varepsilon\varphi_j^{(\alpha)} = -\varphi_j^{(\alpha)}\varepsilon$$

and

$$f_a\varphi_j^{(\alpha)} = -\varphi_j^{(\alpha)} f_a\,, \tag{B.2}$$

we obtain:

$$\delta\mathcal{L}_1 = \sum_{i=1}^{6} T_i \tag{B.3}$$

with

$$\begin{aligned}
T_1 &= 2ig\varepsilon T^a_{jk}\overline{\varphi}_a^{j(1)}\not\partial\left[f_a\varphi^{k(1)}\right],\\
T_2 &= 2ig\varepsilon T^a_{jk} f\overline{\varphi}^{j(1)}\not\partial\varphi^{k(1)},\\
T_3 &= 2ig\varepsilon T^a_{jk}\partial_\mu f_a\overline{\varphi}^{j(1)}\gamma^\mu\varphi^{k(1)},\\
T_4 &= -4ig^2\varepsilon T^a_{jk} f_{abc}A^c_\mu f^b\overline{\varphi}^{j(1)}\gamma^\mu\varphi^{k(1)},\\
T_5 &= 4g^2\varepsilon T^a_{jk}A_{\mu a}T^d_{lj} f_d\overline{\varphi}^{l(1)}\gamma^\mu\varphi^{k(1)},\\
T_6 &= 4g^2\varepsilon T^a_{jk}A_{\mu a}T^d_{kl}\overline{\varphi}^{j(1)}\gamma^\mu f_d\varphi^{l(1)}.
\end{aligned} \tag{B.4}$$

Notice that with the help of Eqs (B.2) we get:

$$T_1 + T_2 = -2ig\varepsilon T^a_{jk}\partial_\mu f_a\overline{\varphi}^{j(1)}\gamma^\mu\varphi^{k(1)} + \text{a total derivative},$$

thus

$$\sum_{i=1}^{3} T_i = \text{a total derivative (negligible)}. \tag{B.5}$$

Thanks to the Lie Algebra

$$\left[T^a, T^b\right] = f^{abc}T_c\,. \tag{B.6}$$

We deduce that:

$$\sum_{i=4}^{6} T_i = 0\,. \tag{B.7}$$

Finally,

$$\delta\mathcal{L}_1 = \delta\left[\overline{\varphi}_j^{(2)}\not\partial\varphi^{j(2)}\right] = G\overline{\varphi}_j^{(2)}\not\partial\varphi^{j(2)}, \tag{B.8}$$

which is Eq. (2.1.17).

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