N=1, D=4 NONSYMMETRIC SUPERGRAVITY*

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Using the complex hyperbolic formalism, a model of nonsymmetric supergravity is constructed. It is shown that in order to have in four dimensions and with one supersymmetric charge, the same fermionic and bosonic degrees of freedom, one has to introduce a real scalar field. The local supersymmetric transformations are derived and the equations of motion are shown explicitly.

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1. Introduction

The gravitational force is the oldest one known to man and the least understood. It is the dimensionful character of the gravitational constant which destroys the predictivity of the theory. That is, it is impossible to have a renormalizable theory! Since all the present day experimental data confirm General Relativity (GR), any future quantum or other kind of a gravitation theory should be an extension rather than a replacement of GR.

While things do not work as it should be in GR, the nongravitational forces seem to describe nature remarkably. They are described by renormalizable quantum field theories and are based essentially on the electroweak theory of Weinberg, Salam and Glashow, and the theory of strong interactions (QCD) within the framework of the so-called Standard Model $SU(3)\otimes SU(2)\otimes U(1)$. In spite of the remarkable unified scheme of these interactions, gravity is absent!

It is one of the main objectives of science to bring order in chaos and explain the many diverse physical phenomena by one underlying theory.

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Thus, it becomes clear that a unification of the gravitational and non gravitational forces, based on the same general principles, is needed. This is the most outstanding problem of our century!

Supergravity is proposed as such a theory. It is based on a new symmetry principle between fermions and bosons. The remarkable thing is that the local version of this symmetry can be achieved only if space is curved and thus gravity is present. However, this dream was stopped before it grew up! Supergravity was shown to be a nonrenormalizable theory! [1].

To solve this delicate problem, many other attitudes were considered. One alternative is to treat GR as a special case of an extended (or general) theory, namely nonsymmetric gravitation theory (NGT) [2]. It was Einstein's idea first to consider the metric $g_{\mu\nu}$ not as a symmetric but rather as a general (consisting of symmetric and antisymmetric parts) tensor. The Einstein's goal was to unify gravity with electromagnetism. However, this dream has also remained unfulfilled.

Despite this failure, Einstein's successors were convinced that there is a part of truth in the idea. It was Moffat [3–8] who revived Einstein's dream, but in the context of the gravitation itself, the so-called NGT. The most appealing feature of this theory, the corrected formulation of which, using the tetrad formalism, is given in Ref. [9], is that all its theoretical predictions are compatible with the existing classical experimental tests which have confirmed the validity of GR. Moreover, the recently proposed fifth force [10], can be contained naturally in the NGT formalism [6]. On the other hand, and from the gauge theory point of view, the gravitational field $g_{\mu\nu}$ of GR represents a spin 2 massless particle (graviton) in which the bosonic degree of freedom is 2. In NGT, however, the field $g_{\mu\nu}$ represents two particles: the graviton (spin 2) and a massless spin 0 (scalar) particle with one degree of freedom called the skewon and related to the skew part of the tensor $g_{\mu\nu}$ [8].

These differences between GR and NGT are definite proofs that the supersymmetric versions of these theories (supergravity and nonsymmetric supergravity) will have different structures and phenomenology.

In Section 2, we give a brief review of the Moffat's pure NGT and the revised formulation of the theory. In Section 3, we describe our supersymmetric model of NGT and derive the corresponding equations of motion. In Section 4, we deduce the supersymmetric transformations for the graviton and the skewon. Finally, in Section 5, we draw our conclusions.

2. Revised pure NGT formalism

In the Moffat's pure NGT, formulated successfully in a hyperbolic complex space [11], the fundamental tensor $g_{\mu\nu}$ is Hermitian ($\tilde{g}_{\mu\nu} = g_{\nu\mu}$, where

"∼" means hyperbolic complex conjugate) and consist of symmetric and antisymmetric parts $g_{(\mu\nu)}$ and $g_{[\mu\nu]}$ respectively:

$$g_{\mu\nu} = g_{(\mu\nu)} + jg_{[\mu\nu]} \qquad (j^2 = 1). \tag{2.1}$$

A Hermitian contravariant tensor $g^{\mu\nu}$ is also defined, and verifies the following relations:

$$g^{\mu\nu} g_{\mu\rho} = g^{\nu\mu} g_{\rho\mu} = \delta^\nu_\rho .$$

The affine connection $W^\lambda_{\mu\nu}$ is defined by [3–8]:

$$W^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \tfrac{2}{3}\delta^\lambda_\mu W_\nu \tag{2.2}$$

with

$$W_\nu = \tfrac{1}{2}(W^\alpha_{\nu\alpha} - W^\alpha_{\alpha\nu}), \tag{2.3}$$

where the displacement Hermitian field $\Gamma^\lambda_{\mu\nu}$ defines the variation of a vector A^λ in an infinitesimal parallel transport:

$$\delta A^\lambda = -\Gamma^\lambda_{\mu\nu} A^\nu dx^\mu . \tag{2.4}$$

It is worth to mention that, according to reference [9], it is not possible to have the hermiticity of $W^\lambda_{\mu\nu}$ and $\Gamma^\lambda_{\mu\nu}$ simultaneously, but rather one can apply the hermiticity condition to one of them (*e.g.* $\tilde{\Gamma}^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu}$) together with another choice like $\tilde{W}_\nu = -W_\nu$.

A hyperbolic complex vierbein e^a_μ was also introduced and thus, the sesquilinear form of the tensor $g_{\mu\nu}$ is:

$$g_{\mu\nu} = e^a_\mu \tilde{e}^b_\nu \eta_{ab} \tag{2.5}$$

(η_{ab} is the Minkowski flat space metric). The proposed pure NGT action is:

$$S = \frac{1}{8\pi G}\int \mathcal{L}_g d^4x , \tag{2.6}$$

where G is the Newton's gravitation constant[1] and

$$\mathcal{L}_g = -\tfrac{1}{2}(e\tilde{e})^{1/2} e^{\mu a}\tilde{e}^{\nu b}(R_{\mu\nu})_{ab}(\omega), \tag{2.7a}$$

where

$$(R_{\mu\nu})_{ab}(\omega) = (\omega_\nu)_{ab,\mu} - (\omega_\mu)_{ab,\nu} + [\omega_\mu, \omega_\nu]_{ab} . \tag{2.7b}$$

[1] From Eq. (2.7a) and on, and in order to simplify our formulas, we work in the system where $8\pi G = 1$.

Here $(\omega_\mu)_{ab}$ is the spin connection, assumed to verify the property [7]

$$(\omega_\mu)_{ab} = -(\tilde{\omega}_\mu)_{ba} \tag{2.8}$$

and e (respectively $\tilde{e}$) is the determinant of e^a_μ (respectively $\tilde{e}^a_\mu$). The correct compatibility condition (see Ref. [9]) is given by:

$$e^a_{\mu,\sigma} + (\omega_\sigma)^a_c e^c_\mu - W^\rho_{\mu\sigma} e^a_\rho = 0\,. \tag{2.9}$$

Now, the most appealing characteristic of NGT, and contrary to general relativity, is that the tangent space is the GL(4,ℝ) group manifold [7–11], instead of the SO(3,1) Lorentz group manifold (Minkowski space). Of course, the latter is a subspace of the former.

3. The NGT supersymmetric model

To get a realistic model we have taken as a tangent space (locally), the Lorentz group manifold (Minkowski space). In other words, our starting point is the special relativity. It is important to note that this choice does not contradict the well known general properties and features of NGT, namely the compatibility condition (2.9) and reality (in the sense of hyperbolic complex numbers) of the Lagrangian (2.7a). Moreover, and as it will be seen later, it simplifies matter in constructing a supersymmetric model of NGT and deriving the supersymmetric transformations. In fact, one immediate consequence of this choice is the reality of the spin connection:

$$(\omega_\mu)^a_b = (\tilde{\omega}_\mu)^a_b\,. \tag{3.1}$$

Now, one part of our total Lagrangian is the one given by (2.7a) together with the relation (3.1). It describes the spin 2 and spin 0 particles related to the symmetric and antisymmetric parts of the metric and called graviton and skewon, respectively [7–8].

In the context of $N = 1$ supersymmetry (SUSY), one has to associate to each bosonic particle a fermionic partner. This means that one has to have a SUSY doublet of a graviton (respectively skewon) and a fermionic spin 3/2 (respectively 1/2) fields. However, since the vierbein e^a_μ does not represent directly the graviton and skewon, our strategy consists of first constructing in the same manner as for the bosonic part of the Lagrangian (hyperbolic complex formalism) the fermionic part, then deriving the corresponding local SUSY transformations and finally, identifying the fields combinations representing the real particles in the model.

The fermionic part of the Lagrangian is:

$$\mathcal{L}_\psi = \mathcal{L}^{(1)}_\psi + \tilde{\mathcal{L}}^{(1)}_\psi\,, \tag{3.2}$$

where

$$\mathcal{L}_{\psi}^{(1)} = -\tfrac{i}{2}\Omega^{\mu\nu\alpha\tilde{\rho}}\bar{\psi}_{\nu}\gamma^{5}\gamma_{\mu}D_{\alpha}\psi_{\rho} \tag{3.3}$$

and the massless fermionic field ψ_ρ is of Majorana type and hyperbolic complex, *i.e.*

$$\psi_{\rho} = \psi_{1\rho} + j\psi_{2\rho} \qquad (j^2 = 1) \tag{3.4}$$

($\psi_{1\rho}$ and $\psi_{2\rho} \in \mathbb{C}$, $\mathbb{C}$ being the ring of complex numbers). The tensor $\Omega^{\mu\nu\alpha\tilde{\rho}}$ is given by:

$$\Omega^{\mu\nu\alpha\tilde{\rho}} = (e\tilde{e})^{1/2}\eta^{icjk}e_{i}^{\mu}e_{c}^{\nu}e_{j}^{\alpha}\tilde{e}_{k}^{\rho}\,. \tag{3.5}$$

Here η^{icjk} is the totally antisymmetric tensor (the Latin (respectively Greek) letters are used as a flat (respectively curved) space-time indices) and D_α is the covariant derivative defined as:

$$D_{\alpha}\psi_{\rho} = \partial_{\alpha}\psi_{\rho} + \tfrac{1}{2}\omega_{\alpha}{}^{ab}\sigma_{ab}\psi_{\rho} \tag{3.6}$$

with the Lorentz group (SO(3.1)) generators σ_{ab} given by:

$$\sigma_{ab} = \tfrac{1}{4}[\gamma_a, \gamma_b] \tag{3.7}$$

(γ's are the Dirac matrices).

The Majorana condition as well as the hyperbolic complex structure allow the fermionic field ψ_ρ to have just four degrees of freedom. This is similar to the Schwinger term in the ordinary supergravity theories (SUGRA), where the fermionic field (the gravitino in this case) is a spin 3/2 particle and has two degrees of freedom. Here, our massless fermionic field is hyperbolic complex and thus has four degrees of freedom. Now, if one takes as a supersymmetric Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\rm g} + \mathcal{L}_{\psi}\,, \tag{3.8}$$

it turns out that in $d = 4$ dimensions the bosonic and fermionic degrees of freedom mismatch. In fact, we have three bosonic and four fermionic degrees of freedom corresponding to the graviton, skewon and the hyperbolic complex Majorana fermionic field ψ_μ.

To overcome this unwanted difficulty, one has to introduce another spin 0 particle (one bosonic degree of freedom). This is done by adding to the Lagrangian (3.8) another part:

$$\mathcal{L}_{\varphi} = (e\tilde{e})^{1/2}g_{(\mu\nu)}\partial^{\mu}\varphi\partial^{\nu}\varphi\,, \tag{3.9}$$

where

$$g_{(\mu\nu)} = \tfrac{1}{2}(e_{\mu a}\tilde{e}_{\nu}^{a} + \tilde{e}_{\mu a}e_{\nu}^{a}\,. \tag{3.10}$$

The Lagrangian (3.9) contains the kinetic and interacting terms of the scalar field φ with the graviton. It should be noted that φ is a dynamical and not an auxiliary field. This is similar to the case of $N = 1,\ d = 11$ supergravity of Cremmer, Julia and Scherk [12], where in order to match the bosonic and fermionic degrees of freedom, an antisymmetric tensor of order three was introduced.

One may wonder whether one can formulate the theory in higher dimensions (à la Kaluza–Klein). It turns out that it is possible in $d = 5$ dimensions [13]. This is a very interesting result! In fact, in "d" dimensions, one has the graviton (symmetric part of $g_{\mu\nu}$) and the skewon (antisymmetric part of $g_{\mu\nu}$) with $\frac{1}{2}d(d-3)$ and $\frac{1}{2}(d-3)(d-2)$ bosonic degrees of freedom, respectively. This gives a total of $(d-1)(d-3)$ bosonic degrees of freedom. For the Majorana spinorial field ψ_μ and by taking into account its hyperbolic complex structure, one has $2 \times 2^{\alpha-1}$ fermionic degrees of freedom, where:

$$\alpha = \begin{cases} \frac{d}{2} & \text{if } d \text{ even}, \\ \frac{d-1}{2} & \text{if } d \text{ odd}. \end{cases}$$

Thus, in order to have the same bosonic and fermionic degrees of freedom (as it is required by SUSY), one has to work in $d = 5$ dimensions.

Now, upon variation of the action with respect to e^a_μ, $\tilde{e}^a_\mu$, ψ_μ, $\tilde{\psi}_\mu$, $(\omega_\mu)_{ab}$ and φ (application of the least action principle), we obtain the following equations of motion:

$$\partial^\mu\left[(e\tilde{e})^{1/2} g_{(\mu\nu)}\partial^\nu\varphi\right] = 0\,, \tag{3.11}$$

$$\Omega^{\mu\nu\alpha\tilde{\rho}}\gamma^5\gamma_\mu D_\alpha\psi_\rho = 0\,, \tag{3.12}$$

$$(\Lambda_\mu + \tilde{\Lambda}_\mu)\Sigma^{\mu\nu}_{ab} + (\vartheta^\nu_{ab} - \tilde{\vartheta}^\nu_{ba}) = \zeta^\nu_{ab} + \tilde{\zeta}^\nu_{ab} \tag{3.13}$$

and

$$\begin{aligned} R_{\mu a} = {}& \tfrac{1}{2}\Omega^{\sigma\nu\alpha\tilde{\rho}}\chi_{\sigma\nu\alpha\rho}e_{\mu a} + \tilde{\Omega}^{\rho\nu\alpha\tilde{\sigma}}\tilde{\chi}_{\nu\rho\alpha\mu}e_{\sigma a} + \Omega^{\nu\sigma\alpha\tilde{\rho}}[\chi_{\mu\nu\alpha\rho} + \chi_{\nu\alpha\mu\rho}]e_{\sigma a} \\ & + 2[g_{(\alpha\beta)}\partial^\alpha\varphi\partial^\beta\varphi e_{\mu a} - g_{\mu\nu}e_{\rho a}\partial^\rho\varphi\partial^\nu\varphi]\,, \end{aligned} \tag{3.14}$$

where

$$\begin{aligned} \Lambda_\mu &= e^\rho_a e^a_{\rho,\mu}\,, \\ \Sigma^{\mu\nu}_{ab} &= \tfrac{1}{2}(e\tilde{e})^{1/2}[e^\mu_a\tilde{e}^\nu_b - e^\nu_a\tilde{e}^\mu_b]\,, \\ \vartheta^\nu_{ab} &= (e\tilde{e})^{1/2}(e^\mu_a D_\mu\tilde{e}^\nu_b - e^\nu_a D_\mu\tilde{e}^\mu_b)\,, \\ \zeta^\nu_{ab} &= \tfrac{i}{2}\Omega^{\mu\alpha\nu\tilde{\rho}}\bar{\psi}_\alpha\gamma^5\gamma_\mu\sigma_{ab}\psi_\rho\,, \\ \chi_{\mu\nu\alpha\rho} &= -i(e\tilde{e})^{1/2}\bar{\psi}_\nu\gamma^5\gamma_\mu D_\alpha\psi_\rho\,. \end{aligned}$$

It is straightforward to show that the action and equations of motion are invariant under the following symmetries:

a) $d = 4$ general covariance with parameter ξ:

$$\begin{aligned}
\delta\varphi &= -\xi^{\alpha}{}_{,}\varphi_{\alpha} \\
\delta e^{\mu a} &= -\xi^{\alpha} e^{\mu a}{}_{,\alpha} + e^{\alpha a}\xi^{\mu}{}_{,\alpha} \\
\delta \bar{e}^{\nu b} &= -\xi^{\alpha} \bar{e}^{\nu b}{}_{,\alpha} + \bar{e}^{\alpha b}\xi^{\nu}{}_{,\alpha} \\
\delta\psi_{\mu} &= -\xi^{\alpha}\psi_{\mu,\alpha} - \psi_{\alpha}\xi^{\alpha}{}_{,\mu} \\
\delta\tilde{\psi}_{\nu} &= -\xi^{\alpha}\tilde{\psi}_{\nu,\alpha} - \tilde{\psi}_{\alpha}\xi^{\alpha}{}_{,\nu} \\
\delta\omega_{\mu}{}^{ab} &= -\xi^{\alpha}\omega_{\mu}{}^{ab}{}_{,\alpha} - \omega_{\alpha}{}^{ab}\xi^{\alpha}{}_{,\mu} \\
\delta(R_{\mu\nu})_{ab} &= -\xi^{\alpha}(R_{\mu\nu})_{ab,\alpha} - (R_{\alpha\nu})_{ab}\xi^{\alpha}{}_{,\mu} - (R_{\mu\alpha})_{ab}\xi^{\alpha}{}_{,\nu}
\end{aligned} \tag{3.15}$$

b) local SO(3,1) Lorentz transformations with parameter $\alpha_{ab} = -\alpha_{ba}$:

$$\begin{aligned}
\delta\varphi &= 0 \\
\delta e^{\mu a} &= \alpha^{a}{}_{b} e^{\mu b} \\
\delta \bar{e}^{\mu a} &= \alpha^{a}{}_{b} \bar{e}^{\mu b} \\
\delta\psi_{\mu} &= \alpha^{ab}\sigma_{ab}\psi_{\mu} \\
\delta\tilde{\psi}_{\mu} &= \alpha^{ab}\sigma_{ab}\tilde{\psi}_{\mu} \\
\delta\omega_{\mu}{}^{ab} &= -\partial_{\mu}\alpha^{ab} + \tfrac{1}{2} f_{cdef}{}^{ab}\alpha^{cd}\omega_{\mu}{}^{ef} ,
\end{aligned} \tag{3.16}$$

where

$$f_{abcd}{}^{ef}\sigma_{ef} = \eta_{bc}\sigma_{ad} + \eta_{ad}\sigma_{bc} - \eta_{bd}\sigma_{ac} - \eta_{ac}\sigma_{bd} \tag{3.17}$$

(σ_{ab} are the Lorentz group generators). Of course, in addition to this symmetry, one has invariance under local SUSY transformations.

4. Local SUSY transformations

To derive the SUSY transformations, we will use throughout this section, the 1.5 order formalism [14]. In other words, one only needs to vary the explicit tetrad fields e^{a}_{μ} and $\bar{e}^{a}_{\mu}$, the scalar field φ and the fermionic fields ψ_{μ} and $\tilde{\psi}_{\mu}$ and although the spin connection $\omega_{\mu}(e, \bar{e}, \psi_{\mu}, \tilde{\psi}_{\mu}, \varphi)$ is not invariant, one may put $\delta\omega = 0$. As explained in reference [15], this is due essentially to the fact that the spin connection $\omega_{\mu}(e, \bar{e}, \psi_{\mu}, \tilde{\psi}_{\mu}, \varphi)$ satisfies its own field equation.

Starting from the Lagrangian (2.7a) and applying the 1.5 order formalism, one can show easily that:

$$\delta\mathcal{L}_{g} = \delta\mathcal{L}_{g}^{(1)} + \delta\tilde{\mathcal{L}}_{g}^{(1)} , \tag{4.1}$$

where

$$\delta\mathcal{L}_{\mathrm{g}}^{(1)} = -\tfrac{1}{4}(e\tilde{e})^{1/2}[e_i^{\rho}R - 2\tilde{e}^{\nu b}e^{\rho a}e_i^{\mu}(R_{\mu\nu})_{ab}]\delta e_{\rho}^{i} \tag{4.2}$$

and R is the scalar curvature defined as:

$$R = e^{\mu a}\tilde{e}^{\nu b}(R_{\mu\nu})_{ab}\,. \tag{4.3}$$

Now, if one sets:

$$\delta e_{\rho}^{i} = i\bar{\varepsilon}\gamma^{i}\psi_{\rho}\,, \tag{4.4}$$

where ε is an ordinary Majorana complex spinor parameter, the expression (4.2) becomes:

$$\delta\mathcal{L}_{\mathrm{g}}^{(1)} = -\tfrac{i}{4}(e\tilde{e})^{1/2}[e_i^{\rho}R - 2\tilde{e}^{\nu b}e^{\rho a}e_i^{\mu}(R_{\mu\nu})_{ab}](\bar{\varepsilon}\gamma^{i}\psi_{\rho})\,. \tag{4.5}$$

To cancel the contribution (4.5), one has to consider $\delta\mathcal{L}_{\psi}^{(1)}$ and $\delta\mathcal{L}_{\varphi}$. In fact, taking the variation $\delta\mathcal{L}_{\psi}^{(1)}$ and setting:

$$\Omega^{\mu\nu\alpha\bar{\rho}}\delta\bar{\psi}_{\nu} = [\Omega^{\mu\rho\nu\bar{\alpha}} - \Omega^{\mu\rho\alpha\bar{\nu}}]D_{\nu}\bar{\varepsilon} + f^{\mu\alpha\rho}\bar{\varepsilon} \tag{4.6}$$

and

$$\begin{aligned}\Omega^{\mu\nu\alpha\bar{\rho}}\delta(D_{\alpha}\psi_{\rho}) =& \Omega^{\mu\nu\alpha\bar{\rho}}\{[D_{\alpha}, D_{\rho}]\varepsilon - \tfrac{i}{2}G^{\sigma}{}_{\sigma\alpha\rho} + iG^{\sigma}{}_{\alpha\sigma\rho}\}\\ &+ i\Omega^{\sigma\nu\alpha\bar{\rho}}G^{\mu}{}_{\sigma\alpha\rho} + i\Omega^{\mu\sigma\alpha\bar{\rho}}G^{\nu}{}_{\sigma\alpha\rho} + 2if\gamma^{5}e_j^{\mu}e^{\nu j}\varepsilon\,,\end{aligned} \tag{4.7}$$

where

$$G^{\mu}{}_{\sigma\alpha\rho} = (\bar{\varepsilon}\gamma^{\mu}\psi_{\sigma})(D_{\alpha}\psi_{\rho}) \tag{4.8}$$

(f and $f^{\mu\alpha\rho}$ will be determined later) together with the definition (3.5) of $\Omega^{\mu\nu\alpha\bar{\rho}}$ and the orthonormality relations:

$$e_a^{\mu}e_{\mu}^{b} = \delta_a^{b}\,, \quad e_a^{\mu}e_{\nu}^{a} = \delta_{\nu}^{\mu}\,,$$

(and similarly for $\tilde{e}_{\mu}^{a}$) one gets:

$$\begin{aligned}\delta\Omega^{\mu\nu\alpha\bar{\rho}} =& \tfrac{1}{2}\Omega^{\mu\nu\alpha\bar{\rho}}[e_k^{\sigma}\delta e_{\sigma}^{k} + \tilde{e}_k^{\sigma}\delta\tilde{e}_{\sigma}^{k}] + \Omega^{\mu\nu\alpha\bar{\sigma}}\tilde{e}_{\sigma}^{k}\delta\tilde{e}_k^{\rho}\\ &+ \Omega^{\sigma\nu\alpha\bar{\rho}}e_{\sigma}^{k}\delta e_k^{\mu} + \Omega^{\mu\sigma\alpha\bar{\rho}}e_{\sigma}^{k}\delta e_k^{\nu} + \Omega^{\mu\nu\sigma\bar{\rho}}e_{\sigma}^{k}\delta e_k^{\alpha}\,.\end{aligned} \tag{4.9}$$

Thus, $\delta\mathcal{L}_{\psi}^{(1)}$ can be written as:

$$\delta\mathcal{L}_{\psi}^{(1)} = \sum_{i=1}^{5} T_i \tag{4.10}$$

with

$$T_1 = -\tfrac{i}{2}\Omega^{\mu\nu\alpha\bar{\rho}}[(D_\alpha\bar{\varepsilon})\gamma^5\gamma_\mu D_\rho\psi_\nu - (D_\rho\bar{\varepsilon})\gamma^5\gamma_\mu D_\alpha\psi_\nu],$$
$$T_2 = -\tfrac{i}{2}\Omega^{\mu\nu\alpha\bar{\rho}}\bar{\psi}_\nu\gamma^5\gamma_\mu[D_\alpha, D_\rho]\varepsilon,$$
$$T_3 = -\tfrac{i}{4}[\Omega^{\mu\nu\alpha\bar{\rho}}\bar{e}^\sigma_k - 2\Omega^{\mu\nu\alpha\bar{\sigma}}\bar{e}^\rho_k]\bar{\psi}_\nu\gamma^5\gamma_\mu D_\alpha\psi_\rho\delta\bar{e}^k_\sigma,$$
$$T_4 = -fe^\nu_k(\bar{\varepsilon}\gamma^k\psi_\nu),$$
$$T_5 = -\tfrac{i}{2}f^{\mu\alpha\rho}\bar{\varepsilon}\gamma^5\gamma_\mu D_\alpha\psi_\rho. \tag{4.11}$$

Now the term T_1 can be written as:

$$T_1 = \sum_{i=1}^{3} A_1 + \text{ a total derivative} \tag{4.12}$$

with

$$A_1 = \tfrac{i}{2}\Omega^{\mu\nu\alpha\bar{\rho}}[\bar{\varepsilon}\gamma^5(D_\alpha\gamma_\mu)(D_\rho\psi_\nu) - \bar{\varepsilon}\gamma^5(D_\rho\gamma_\mu)(D_\alpha\psi_\nu),$$
$$A_2 = -\tfrac{i}{2}\Omega^{\mu\nu\alpha\bar{\rho}}\bar{\varepsilon}\gamma^5\gamma_\mu[D_\rho, D_\alpha]\psi_\nu,$$
$$A_3 = \tfrac{i}{2}[\partial_\alpha\Omega^{\mu\nu\alpha\bar{\rho}}\bar{\varepsilon}\gamma^5\gamma_\mu D_\rho\psi_\nu - \partial_\rho\Omega^{\mu\nu\alpha\bar{\rho}}\bar{\varepsilon}\gamma^5\gamma_\mu D_\alpha\psi_\nu]. \tag{4.13}$$

By noticing that

$$[D_\alpha, D_\rho] = \tfrac{1}{2}(R_{\alpha\rho})_{ab}\sigma^{ab} \tag{4.14}$$

and

$$\{\sigma_{ab}, \gamma_c\} = \eta_{abcd}\gamma^5\gamma^d \tag{4.15}$$

(with $\gamma^5 \equiv \gamma^0\gamma^1\gamma^2\gamma^3$; $\eta^{0123} = -\eta_{0123} = 1$) and using the Fierz rearrangement:

$$\bar{\varepsilon}\gamma^5\gamma_\mu\sigma_{ab}\psi_\nu = -\bar{\psi}_\nu\gamma^5\sigma_{ab}\gamma_\mu\varepsilon$$

we obtain (with $(\gamma^5)^2 = -1$):

$$A_2 + T_2 = +\tfrac{i}{4}\Omega^{\mu\nu\alpha\bar{\rho}}\eta_{abcd}e^c_\mu(\bar{\psi}_\nu\gamma^d\varepsilon)(R_{\alpha\rho})^{ab}. \tag{4.16}$$

Now, simplifying the expression $\Omega^{\mu\nu\alpha\bar{\rho}}\eta_{abcd}e^c_\mu$ and using the property:

$$(R_{\alpha\rho})^{ab} = -(R_{\rho\alpha})^{ab} = -(R_{\alpha\rho})^{ba} \tag{4.17}$$

one gets after a straightforward calculation:

$$A_2 + T_2 = O_1 + O_2, \tag{4.18}$$

where

$$O_1 = \tfrac{i}{4}(e\bar{e})^{1/2}(\bar{\varepsilon}\gamma^k\psi_\nu)[e^\nu_k R - 2e^{\nu a}\bar{e}^{\rho b}e^\alpha_k(R_{\alpha\rho})_{ab}] \tag{4.19}$$

and

$$O_2 = \tfrac{i}{4}(e\bar{e})^{1/2}(\bar{\varepsilon}\gamma^k\psi_\nu)[e^\nu_k R - 2\bar{e}^\rho_k e^{\nu b} e^{\alpha a}(R_{\alpha\rho})_{ab}]\,. \tag{4.20}$$

It is to be noted that:

$$\delta\mathcal{L}^{(1)}_{\mathrm{g}} = -O_1\,. \tag{4.21}$$

Now, let us take the variation of $\frac{1}{2}\delta\mathcal{L}_\varphi$. It turns out that if we choose:

$$f = \tfrac{i}{8}(ee)^{1/2}R\,, \tag{4.22}$$

$$f^{\mu\alpha\rho} = 2\left[\Omega^{\sigma\rho[\nu\tilde{\alpha}]}(D_\nu e^c_\sigma)e^\mu_c + \partial_\nu \Omega^{\mu\rho[\nu\tilde{\alpha}]}\right], \tag{4.23}$$

$$\delta\bar{e}^k_\sigma = \bar{\psi}_\mu Z^{\mu k}_\sigma \varepsilon\,, \tag{4.24}$$

and define

$$Z^{\mu k}_\sigma = g_{(\alpha\rho)}\varphi^\rho\gamma^k\gamma^\alpha\delta^\mu_\sigma\,, \tag{4.25}$$

(where "$[\nu\alpha]$" means antisymmetrization in the indices ν and α, and φ^ρ is a shorthand notation of $\partial^\rho\varphi$) and in order that the variation of the total Lagrangian vanishes, one has to have:

$$\delta\varphi^\nu = X_{\beta\mu\alpha\rho}\bar{\psi}_\sigma Y^{\mu\beta\alpha\rho\sigma}\gamma^\nu\varepsilon + G\varphi^\nu\,, \tag{4.26}$$

where

$$X_{\beta\mu\alpha\rho} = \tfrac{i}{4}\bar{\psi}_\beta\gamma^5\gamma_\mu D_\alpha\psi_\rho\,,$$
$$Y^{\mu\beta\alpha\rho\sigma} = (e\bar{e})^{-1/2}(\Omega^{\mu\beta\alpha\bar{\rho}}\bar{\gamma}^\sigma - 2\Omega^{\mu\beta\alpha\bar{\sigma}}\bar{\gamma}^\rho) \tag{4.27}$$

and

$$G = -\tfrac{1}{8}\left[2\bar{e}^\rho_i + \gamma^{(\mu\rho)}e_{\mu i}\right]g_{(\alpha\nu)}\varphi^\alpha(\bar{\psi}_\rho\gamma^i\gamma^\nu\varepsilon)$$
$$-\tfrac{i}{8}\left[2e^\rho_i + \gamma^{(\mu\rho)}\bar{e}_{\mu i}\right](\bar{\varepsilon}\gamma^i\psi_\rho) \tag{4.28}$$

$\gamma^{(\mu\rho)}$ is the inverse tensor of $g_{(\mu\nu)}$. (Note that it differs from the symmetric part $g^{(\mu\nu)}$ of the tensor $g^{\mu\nu}$). Thus, we have shown that:

$$\delta\mathcal{L}^{(1)}_{\mathrm{g}} + \delta\mathcal{L}^{(1)}_\psi + \tfrac{1}{2}\delta\mathcal{L}_\varphi = 0$$

and consequently the invariance of the Lagrangian density[2]:

$$\mathcal{L}_{\mathrm{g}} + \mathcal{L}_\psi + \mathcal{L}_\varphi\,.$$

[2] Similar equation holds for the complex hyperbolic conjugate of $\delta\mathcal{L}^{(1)}_{\mathrm{g}} + \delta\mathcal{L}^{(1)}_\psi + \frac{1}{2}\delta\mathcal{L}_\varphi$.

Now, to get the expression of the gravitino (fermionic partner of the graviton) and skewino (fermionic partner of the skewon) as well as the SUSY transformations of the graviton and skewon, one has to notice that under SUSY transformations, the graviton $G_{\mu\nu} \equiv g_{(\mu\nu)}$ and the skewon $\Phi \equiv \varepsilon^{\mu\nu} g_{[\mu\nu]}$ has to transform, respectively, as [15]:

$$\delta G_{\mu\nu} = i\bar{\varepsilon}[\Gamma_\mu \chi_\nu + \Gamma_\nu \chi_\mu] \tag{4.29}$$

and

$$\delta \Phi = \bar{\varepsilon} S\,, \tag{4.30}$$

where the pure imaginary hyperbolic complex antisymmetric tensor $\varepsilon^{\mu\nu}$ is defined by:

$$\varepsilon^{\mu\nu} = \begin{cases} +1 & \text{if } \mu < \nu\,, \\ -1 & \text{if } \mu > \nu\,, \\ 0 & \text{if } \mu = \nu\,. \end{cases} \tag{4.31}$$

χ_μ and S are, respectively, the gravitino and skewino fields. Here Γ_μ means $E^a_\mu \gamma_a$, where E^a_μ is the real vierbein, defined by [7]:

$$g_{(\mu\nu)} = E^a_\mu E^b_\nu \eta_{ab}\,.$$

Using the transformation laws of e^a_μ and $\tilde{e}^a_\mu$ one can show easily that:

$$\delta G_{\mu\nu} = \bar{\psi}_\alpha V^\alpha_{(\mu\nu)} \varepsilon \tag{4.32}$$

and

$$\delta \Phi = j \varepsilon^{\mu\nu} \bar{\psi}_\alpha V^\alpha_{\mu\nu} \varepsilon \qquad (j^2 = 1)\,. \tag{4.33}$$

Equations (4.32) and (4.33) imply that:

$$\bar{\chi}_\mu = \tfrac{i}{8} \bar{\psi}_\alpha V^\alpha_{\mu\nu} \Gamma^\nu \tag{4.34}$$

and

$$\bar{S} = j \varepsilon^{\mu\nu} \bar{\psi}_\alpha V^\alpha_{\mu\nu} \qquad (j^2 = 1)\,, \tag{4.35}$$

where

$$V^\alpha_{\mu\nu} = e_{\mu a} Z^{\alpha a}_\nu - i \delta^\alpha_\mu \tilde{\gamma}_\nu\,. \tag{4.36}$$

Notice that the $N = 1$, $d = 4$ NGT supersymmetric laws are a sort of a generalization of those of ordinary $N = 1$, $d = 4$ supergravity (SUGRA). This is due essentially to the fact that the vierbein e^a_μ does not represent only the graviton, but rather a mixed state (graviton and skewon). Similarly, the hyperbolic complex spinorial field ψ_μ does not describe just the gravitino field.

5. Conclusions

From our above described simple model, one can deduce that the structure of $N = 1$, $d = 4$ NSUGRA is totally different from that of SUGRA. As mentioned before, this is due essentially to the fact that in the proposed Lagrangian, the symmetric and antisymmetric parts of the tensor $g_{\mu\nu}$ are mixed. Moreover, the hypercomplex spinorial field ψ_μ is a mixed state of the gravitino, skewino and the supersymmetric partner of the real scalar field φ.

This suggests that the ultraviolet behaviour of the present model might be different from that of ordinary SUGRA, including possibility of renormalizability (or finiteness) of the theory.

Moreover, and because of its structure, NSUGRA coupled to nongravitational gauge theory may lead to a phenomenologically interesting new class of models. In this case, and contrary to the popular belief that the gravitational effects should be negligible at low energies, NSUGRA phenomena act as the trigger for the breakdown of the gravitational gauge theory symmetry. Then, one will have a dynamical unification of the NSUGRA nongravitational gauge theory phenomena. More details are under investigations.

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