# TESTING THE QUARTIC GAUGE COUPLINGS IN $Z^0W^+W^-$ PRODUCTION AT $e^+e^-$ COLLIDERS*

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The production of $Z^0W^+W^-$ gauge bosons at future $e^+e^-$ colliders with c.m. energy 500–2000 GeV can provide the testing ground for quartic gauge couplings. For this purpose we examine the cross sections for given helicities of the final state gauge bosons and analyse the role of beam polarizations. The Higgs boson effect is also discussed.

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## 1.

In spite of a number of critical experimental tests that the standard model (SM) of electroweak interactions [1] has passed in last few years, several aspects of the model, in particular the gauge boson self couplings and the existence of the Higgs boson, still await an experimental verification. The production of gauge bosons in the final state at future colliders should provide such crucial tests of the electroweak theory. Numerous studies showed that triple gauge boson couplings WWZ and $WW\gamma$ can be probed by measuring $e^+e^- \to W^+W^-$ at LEP II, ep $\to$ eWX at DESY HERA, and pp $\to$ WWX, WZX, $W\gamma X$ at LHC and SSC [3]. Because the behaviour of the production cross sections is crucially determined by strong gauge cancellations between processes with gauge boson and fermion exchanges, any deviations from the standard model couplings would be amplified and would lead to the anomalous magnitude and behaviour of the cross sections.

A particularly sensitive probes of the SM can be provided by studies of helicity properties of the produced gauge bosons. The helicity pattern of the single W production in pp collisions is quite different from that in

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WW pair production in $e^+e^-$ annihilation [4]. In pp collisions the $W^-$ is produced by a fusion of d and ū quarks of the incoming protons. Because of the (V-A) nature of the W coupling to fermions the $W^-$ is produced by a left-handed d quark and a right-handed ū antiquark. By angular momentum conservation the $W^-$ will therefore have either helicity $\lambda = -1$ if produced along the d direction or $\lambda = +1$ if along the ū direction. Similarly, the $e^+e^- \to W^+W^-$ process also takes place if the helicities of the incoming leptons are opposite. That is $\sigma_{++} = \sigma_{--} = 0$, where subscripts denote helicities $\pm 1$ of incoming leptons. In fact the $\sigma_{-+}$ cross section for the initial left-handed electron and right-handed positron dominates over the other cross section $\sigma_{+-}$. Since the differential cross section for $e^+e^- \to W^+W^-$ is strongly peaked in the forward direction the dominant helicity configuration will be either $(+1, 0)$ or $(0, -1)$ for $W^+W^-$, respectively. This demonstrates that the $e^+e^-$ machines allow to study longitudinally polarized W bosons which are very interesting because they are related to the W-mass generation mechanism and thus offer a possibility to probe the Higgs sector of the SM.

In last few years the triple gauge boson production processes [5–8] and WW production in $\gamma\gamma$ collisions [9] have also attracted attention because future colliders with high energies and luminosities may allow for their experimental studies. Such studies are interesting for several reasons. They allow for further independent tests of the standard model, the quartic WWZZ, WWZ$\gamma$ and WW$\gamma\gamma$ vertices can be probed and the Higgs boson plays an important role in WW and ZZ channels. These processes may also constitute a background to possible signals of the new physics. The numerical analyses so far have mainly focused on questions related to the presence or absence of the Higgs boson in the WW and ZZ channels. Here we try to assess to what extend the full *helicity* structure of the production amplitudes can be tested experimentally assuming that all three gauge bosons $Z^0W^+W^-$ are observed. We discuss contributions of helicity amplitudes to the total cross section, the effect of the Higgs boson and the sensitivity to the quartic gauge-boson coupling. We also examine the question of beam polarization.

## 2.

The matrix elements for triple gauge boson production which appear in the literature have been calculated using two different methods: helicity-amplitude [6] and spinor inner-product [7] techniques. Both methods are amplitude-level type which are particularly convenient for calculations involving many Feynman diagrams. In the present work the helicity technique has been used.

There are 20 Feynman diagrams at the tree level contributing to the process

$$e^-(p_1,\lambda_1)+e^+(p_2,\lambda_2)\to Z^0(q_1,\alpha_1)+W^-(q_2,\alpha_2)+W^+(q_3,\alpha_3) \quad (1)$$

in an arbitrary gauge. Here $p$ and $q$ denote momenta and $\lambda$ and $\alpha$ helicities. The helicity technique developed in Ref. [10] allows to express a tree-level amplitude in terms of strings of two component spinors and 2×2 matrices of the form

$$\chi^{\dagger}_{-\lambda_2}(p_2)[a_1a_2a_3\ldots]_\tau\chi_{\lambda_1}(p_1)\,, \quad (2)$$

where $\tau=\pm$ and the helicity eigenstate spinors are as follows

$$\chi_+(p_1)=\begin{bmatrix}1\\0\end{bmatrix},\quad \chi_-(p_1)=\begin{bmatrix}0\\1\end{bmatrix},$$
$$\chi_+(p_2)=\begin{bmatrix}0\\1\end{bmatrix},\quad \chi_-(p_2)=\begin{bmatrix}-1\\0\end{bmatrix}. \quad (3)$$

The square bracket in Eq. (2) is defined as

$$[a_1a_2a_3\ldots]_\tau=a_1^\tau a_2^{-\tau}a_3^\tau\cdots\,, \quad (4)$$

where $a^\tau=a_\mu\sigma^\mu_\tau$ and $\sigma^\mu_\pm=(1,\pm\boldsymbol{\sigma})$ is expressed in terms of the unit matrix 1 and 2×2 Pauli matrices $\boldsymbol{\sigma}$. The four-vectors $a_\mu$ can be built from particle four-momenta, gauge-boson polarization vectors or fermion strings with an uncontracted Lorentz index.

The helicity amplitude for the $Z^0W^+W^-$ production can be written in the form

$$\mathcal{M}(\lambda_1,\lambda_2,\alpha_1,\alpha_2,\alpha_3)=i\sqrt{s}\,\chi^{\dagger}_{-\lambda_2}(p_2)\,\mathcal{R}(\lambda_1,\lambda_2,\alpha_1,\alpha_2,\alpha_3)\,\chi_{\lambda_1}(p_1)\,. \quad (5)$$

The details and explicit results for the reduced helicity amplitudes $\mathcal{R}$ can be found in Ref. [6]. However, the expression for the diagram with two triple-gauge boson vertices given there (Fig. 1d of Ref. [6]) is correct only in the Feynman gauge. We introduce a slightly more explicit notation for the vector boson V with Lorentz index $\delta$ coupled to two vector bosons with momenta and polarization vectors $q_i$, $\epsilon_i^\nu$ and $q_j$, $\epsilon_j^\mu$ as [8]

$$\Gamma_V^\delta(q_i,\epsilon_i;q_j,\epsilon_j)=\frac{g^{\delta\beta}+(1-\xi)\frac{q_{ij}^\delta q_{ij}^\beta}{\xi q_{ij}^2-M_V^2}}{q_{ij}^2-M_V^2}\epsilon_i^\nu\epsilon_j^\mu$$
$$\times[(q_j-q_i)_\beta g_{\mu\nu}+(2q_i+q_j)_\mu g_{\nu\beta}-(2q_j+q_i)_\nu g_{\mu\beta}]\,, \quad (6)$$

where $q_{ij} = q_i + q_j$ and $\xi$ is a gauge fixing parameter of the V propagator. Then the reduced helicity amplitude for this diagram can be written as follows

$$\begin{aligned}\mathcal{R}^{(d)} =& g^3 \cos\theta_W \delta_{\lambda_1,-\lambda_2} \\ &\times \Big( g^e_{\lambda_2} \left( [\Gamma_Z(q_3,\epsilon_3;q_{21},\Gamma_W(q_2,\epsilon_2;q_1,\epsilon_1))]_{\lambda_1} \right. \\ &\left. + [\Gamma_Z(q_{13},\Gamma_W(q_1,\epsilon_1;q_3,\epsilon_3);q_2,\epsilon_2)]_{\lambda_1} \right) \\ &+ x_W Q_e \left( [\Gamma_\gamma(q_3,\epsilon_3;q_{21},\Gamma_W(q_2,\epsilon_2;q_1,\epsilon_1))]_{\lambda_1} \right. \\ &\left. + [\Gamma_\gamma(q_{13},\Gamma_W(q_1,\epsilon_1;q_3,\epsilon_3);q_2,\epsilon_2)]_{\lambda_1} \right) \Big) , \end{aligned} \tag{7}$$

where $g = 2^{5/4} G_F^{1/2} M_W$, $g^i_\lambda = T^i_3 - \lambda x_W Q^i$, $x_W = \sin^2\theta_W$, $T^e_3 = -0.5$, $Q^e = -1$.

## 3.

In principle, one can separately study the cross section for transversally (T) and longitudinally (L) polarized gauge bosons by using the energy spectrum of the decay products as an analyser [11]. The Monte Carlo studies on the possibility of measuring the W polarization performed in Ref. [12] gave encouraging results. Therefore in what follows we assume that all three gauge bosons are identified and their helicities are also measured.

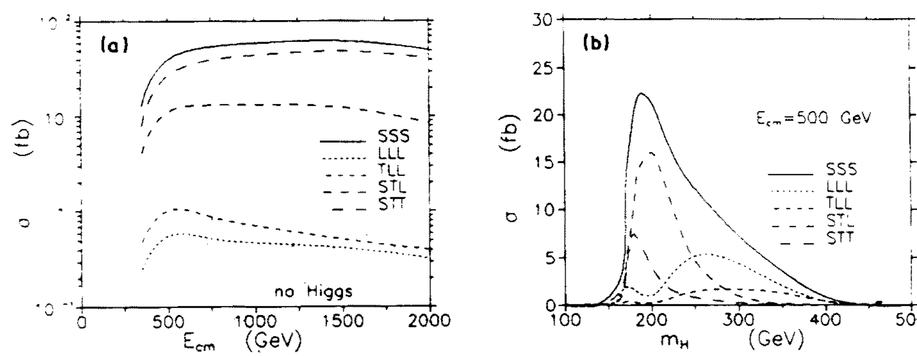


Fig. 1. Results for the cross sections in the standard model. a — The cross sections for the production of $Z^0W^+W^-$ as functions of $E_{cm}$. The Higgs boson effect is not included. b — The SM Higgs boson contribution to the helicity cross sections *vs* Higgs boson mass for $E_{cm} = 500$ GeV. The labeling of curves is as follows: L-longitudinal, T-transversal, S-sum over helicities. The first letter refers to the $Z^0$ boson, the second to $W^-$ and the last one to $W^+$. SLT stands for the sum SLT+STL.

The energy dependence of the production cross section is shown in Fig. 1a. We also show the cross sections for producing gauge bosons with transversal (T), longitudinal (L) or summed (S) over polarizations. Letters refer to Z, W, W, respectively. For example TLL means the Z boson with transversal and WW with longitudinal polarizations and STL - sum over Z boson polarization and one W longitudinally and the other W transversally polarized. At a 500 GeV $e^+e^-$ collider the total cross section for ZWW production is 40 fb which at assumed integrated luminosity of 20 $fb^{-1}$ would give 800 events per year. At 1000 GeV the cross section rises to 62 fb. As we can see in the figure, the process with transversal W polarization dominates in the whole energy range. Processes with longitudinal polarizations contribute less than 1 fb to the total cross section. The above numbers do not include the possible enhancement from a physical Higgs boson. The Higgs contribution (shown in Fig. 1b as a function of $m_{\rm H}$) is important only when $2m_{\rm W} < m_{\rm H} < E_{cm} - m_{\rm Z}$. It is largest close to the threshold where we find that its magnitude is very sensitive to the assumed Higgs width and the STT and STL configurations are the most important ones. On the other hand, for $m_{\rm H} \geq 250$ GeV the process with longitudinally polarized gauge bosons dominates.

The $Z^0W^+W^-$ process allows one to probe experimentally possible anomalous self interactions of gauge bosons. By the time the new colliders will come into operation the triple self-couplings will be tested at LEP II. Therefore, we assume the standard model results for these couplings in assessing the sensitivity to the quartic WWZZ and WWZ$\gamma$ couplings. Unlike the WWZ and WW$\gamma$, the possible anomalous quartic vertices have not been systematically analyzed. Before doing a full analysis it may be interesting to check first the sensitivity of various processes to changes of the strength of the coupling. In this note we simply modify the quartic couplings $g_4$ by a fudge factor $\rho$, *i.e.* $g_4 = \rho g_4^{\rm SM}$, where $g_4^{\rm SM}$ is a SM result. If such drastic modifications do not change significantly the expected cross sections, a more refined analysis is unnecessary.

In Fig. 2a we display the results for the cross sections when the strength $g_4$ of quartic couplings WWZZ and WWZ$\gamma$ is modified by an overall factor $\rho = 0.9$. Up to $E_{cm} \sim 500$ GeV the results are not affected significantly. For example, $\sigma_{\rm SSS}(500\ {\rm GeV}) = 41$ fb. At higher $e^+e^-$ energies, however, the deviations from the SM predictions are clearly seen. It is the processes with longitudinal polarizations that are responsible for the rapid growth of the cross section. Turning the quartic coupling off ($\rho = 0$, Fig. 2b) changes the picture dramatically. Already at 500 GeV the total cross section $\sigma_{\rm SSS} = 140$ fb and grows with energy very fast, for instance $\sigma_{\rm SSS} = 2440$ fb at $E_{cm} = 1000$ GeV. Only STT polarization cross section is not sensitive to $g_4$.

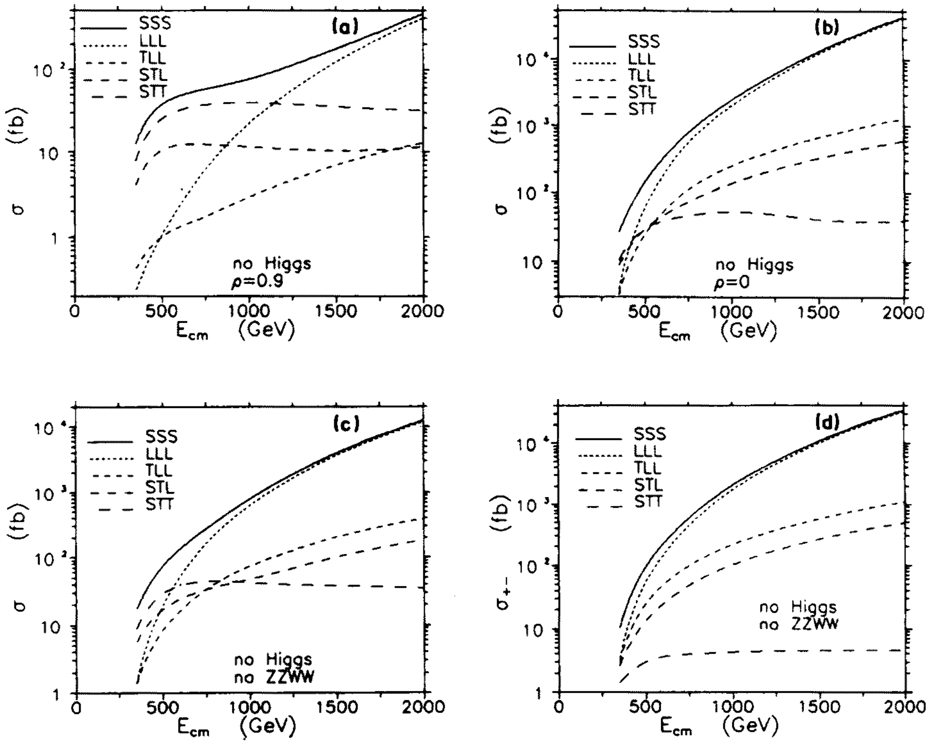


Fig. 2. Results for the cross sections with modified quartic couplings. a — the cross sections for unpolarized beams for $\rho = 0.9$, b — the cross sections for unpolarized beams for $\rho = 0$, c — The cross sections for unpolarized beams with the WWZ$\gamma$ coupling as predicted in the standard model and neglecting WWZZ, d — The cross sections $\sigma_{+-}$ for incoming right-handed electron and left-handed positron with the WWZ$\gamma$ coupling as predicted in the standard model and neglecting WWZZ. The Higgs boson effect is not included and the labeling is the same as in Fig. 1.

Another way to test the standard model is to use beams with definite helicities. Because of the vector or axial-vector interactions, scattering occurs only if the helicities of the initial leptons are opposite, *i.e.* for $(\lambda_1\lambda_2) = (-+)$ or $(+-)$. For the ZWW process we find that $\sigma_{+-} \ll \sigma_{-+}$, *i.e.* the situation is similar to that in the WW pair production in $e^+e^-$ collisions. This can be understood as follows. At high energies the $Z^0$ mass becomes unimportant and thus instead of $\gamma$ and $Z^0$ exchanges in the s channel one can think in terms of the original $W_3$ and the hypercharge gauge boson B. The $W_3$ exchange contributes only to $(-+)$ and the B, which could contribute to $\sigma_{+-}$, does not couple to WW. Taking $g_4 = \rho\, g_4^{SM}$ does not affect the above argument. Therefore, the rate in the interesting $(+-)$ state is discouragingly small.

An interesting situation arises however if the WWZ$\gamma$ and WWZZ couplings deviate differently from the SM predictions. Then the photon and $Z^0$ exchanges do not conspire to the $W_3$ exchange and the $\sigma_{+-}$ can be strongly enhanced. This is illustrated in Fig. 2c and Fig. 2d where the WWZZ coupling is turned off and WWZ$\gamma$ is taken as predicted in the standard model.

To summarize, we find that the $e^+e^- \to Z^0W^+W^-$ process can be a valuable source of information on the gauge structure of the theory. It will provide an independent test of the triple gauge boson couplings and possibly of the Higgs sector. It is particularly suitable to probe the quartic couplings. At $E_{cm} = 1000$ GeV deviations of the order 10% from $g_4^{SM}$ can be clearly seen. We find that the process with longitudinal polarizations of gauge bosons is the most sensitive to any deviations from the standard model. On the other hand, polarized beams will not provide a substantial gain in sensitivity, unless the quartic couplings WWZ$\gamma$ and WWZZ deviate differently from the SM predictions.

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