HOW UNIVERSAL ARE THE 3–DIMENSIONAL CORRELATION PATTERNS?*

K. Fiałkowski

Institute of Physics, Jagellonian University
Reymonta 4, 30-059 Cracow, Poland

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The dependence on bin volume of the averaged scaled factorial moments is analyzed for various data. It is confirmed that for 3-dimensional bins the term, depending power-like on the number of division (inverse average bin volume), seems to determine the shape of data. The spread of the fit parameter values due to various corrections and fitting procedure is larger than the observed differences between various processes.

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1. Introduction

It has been noted recently that the scaled factorial moments averaged over 3-dimensional bins in the momentum space show a remarkably universal dependence on the average bin volume (number of bins) [1]. The preliminary data were presented at Ringberg workshop [2] for wide range of processes: μp [3], π/Kp [4], pAu, OAu and SAu [5] collisions. In all the cases the Ochs [6] and/or Białas–Gaździcki [7] variables were used to remove the effects of non-flat single particle distributions. According to the previous suggestions [8], one expected that the use of 3-dimensional bins will remove the distortions due to dimensional projections and reveal possibly the "intermittent" behaviour suggested by Białas and Peschanski [9]

$$\ln F_q = \phi_q \ln \left(M^3\right) + a_q \,, \tag{1}$$

up to the highest available values of bin number M^3 (smallest bin volumes). Let us remind here that the saturation of an increase of F_q observed in most cases for small one-dimensional bins was suggested to result simply from the

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dimensional projection effects [8]. One expected also much higher values of the intermittency slopes ϕ_q for 3-dimensional case.

However, all the data presented have shown an "overcompensation": the downward curvature seen in most of the $\ln F_q$ vs $\ln M$ plots for one--dimensional bins was replaced by an upward curvature in 3-dimensional data. Fitting the intermittency slopes ϕ_q became almost meaningless, as the fitted values depended very strongly on the M range chosen.

This apparently unexpected result becomes more understandable, if one associates the power-like dependence of moments on the bin size with singularities of correlation function for vanishing differences of moments [10, 1]. One expects then the dependence (1) not for the scaled factorial moments F_q, but rather for the scaled correlation integrals (factorial cumulants) or, even more precisely, for their short-range part. This results *e.g.* for F_2 in the following form replacing (1)

$$F_2 = 1 + c_{\mathrm{L}} + c_s \left(M^3\right)^{\phi_2} , \tag{2}$$

where long- and short-range constants c_{L} and c_s are expected to be much smaller than 1. Obviously, for very small bins (high M values) the last term should dominate and (1) should approximate well the data, but this is not yet the case for the available range of M.

The form (2) was found [1] to describe qualitatively well all the preliminary data [3–5] with surprisingly universal values of c_s (around 0.05) and ϕ_2 (around 0.5). Due to the preliminary character of these data, however, no detailed fit was attempted and no quantitative estimate of differences between the data from different processes was given.

In this note we supplement the results discussed above with the quantitative results of fits performed for different processes, different versions of the data and with different fitting procedures. We will show that there seems to be indeed the universality of correlation patterns for different processes, as the differences observed are not bigger than the variations induced by introducing some corrections and by changing the fitting procedure.

2. Fitting the data

We will discuss here (with few exceptions) the data, which are either preliminary, or even discarded in favour of "better" new versions, corrected for some effects. Thus our results should not be quoted as the valid parameterizations of real data, but rather as an exercise in estimating the spread of parameter values, and their sensitivity to the corrections and choice of fitting procedure. We minimize first X^2 (by MINUIT program) with respect to the parameterization (2). It has been noted, however, that the F_2

values measured for different average bin size are not independent [11]. In fact, the correlation matrices for scaled factorial moments in one dimension — rapidity or pseudorapidity — were found there to have practically all elements quite close to 1. Thus instead of minimizing the standard expression for X^2

$$X^2 = \sum \frac{\left(F_2^{\rm exp}(M) - F_2^{\rm th}(M)\right)^2}{\left(dF_2^{\rm exp}(M)\right)^2}, \tag{3}$$

one should minimize rather

$$X^2_{\rm corr} = \sum\sum \left(F_2^{\rm exp}(M) - F_2^{\rm th}(M)\right) \left(C^{-1}\right)_{MN} \left(F_2^{\rm exp}(N) - F_2^{\rm th}(N)\right), \tag{4}$$

where C is the covariance matrix given by

$$C_{MN} = \langle F_2^{\rm exp}(M) F_2^{\rm exp}(N)\rangle - F_2^{\rm exp}(M) F_2^{\rm exp}(N). \tag{5}$$

The bracket $\langle\cdots\rangle$ denotes the average over events, obviously present (but not marked explicitly) in all the F_q moments in formulae above.

For one dimension and two parameter fit to the form (1) the differences between the results of two procedures were found to be not very serious [11]: the values of slope parameters ϕ_q were practically the same, although the value of X^2 was usually much higher and the error of parameter value much smaller when the correlations were taken into account. More surprising were the changes of the intercept values a_q, which have led often to the apparently "bad" fits, where all or almost all data points were left on one side of the fit line. However, the values of a_q were anyway not of great interest, having no clear interpretation in most of the models.

Here we shall compare the results of "uncorrelated" and "correlated" fits to the formula (2) in the few cases when the covariance matrices (unofficial and preliminary) were available.

Let us take first the first "official" published data for the μp collisions, given separately for "all charged", "positive only", and "negative only" hadrons [12]. Neglecting the correlations between data points we find the values of fit parameters as presented in the upper half of Table I. In all the three cases the fit is acceptable and the values of c_s and ϕ_2 agree within errors. The differences in the values of c_L seem to reflect just the charge conservation in full phase-space. Thus there seems to be no clear difference between the data for "like charge" and "unlike charge" pairs, apparently contradicting the suggestion of the dominant role of Bose–Einstein interference in the observed increase of moments.

However, this contradiction disappeared completely in the later version of the same data [13]. In this version the $(+-)$ pairs with $Q = M_{12} - m_1 - m_2$ below 1 MeV were rejected (M_{12} is the invariant mass of hadron pair). Such

TABLE I

Values of the parameters in the fit of formula (2)

Data		c_L	c_s	ϕ_2	X^2/ND
μp charged	[12]	−0.04(2)	0.062(12)	0.311(23)	17.5/14
μp +ve	[12]	−0.23(3)	0.060(16)	0.347(33)	15.1/14
μp −ve	[12]	−0.21(4)	0.074(23)	0.348(39)	5.6/14
μp charged	[13]	0.00(1)	0.047(07)	0.308(19)	35.0/16
μp −ve	[13]	−0.13(1)	0.038(06)	0.423(24)	13.6/14
μp +−	[13]	−0.30	0.41	0.041	15.0/14
μp charged	[3]	0.00(1)	0.035(05)	0.416(17)	11.8/14
μp (corr.)	[3]	0.02(1)	0.028(04)	0.440(19)	27.0/14
π/Kp	[4]	0.13(1)	0.026(08)	0.377(47)	8.2/7
pAu	[5]	0.24(3)	0.011(07)	0.500(76)	5.4/7
pAu (corr.)	[5]	0.14(4)	0.071(26)	0.256(56)	15.0/7
OAu	[5]	0.02(2)	0.011(04)	0.492(42)	5.1/8
OAu (corr.)	[5]	−0.01(2)	0.030(07)	0.360(38)	14.0/8
SAu	[5]	−0.14(2)	0.137(18)	0.195(15)	22.0/7

a cut was justified by a Lund Monte Carlo simulation, which suggested that a large majority of rejected pairs are not $\pi^+\pi^-$, but non-identified e^+e^- photon-conversion pairs coming from π^0 decays.

In the new version the data for "like charge" pairs look practically the same as before, although the fit parameter values are somehow changed, mainly due to the addition of extra point for $M = 1$ (full phase-space considered). However, the conclusions about unlike-charge pairs change completely.

In the former version of the data [12] the F_2 moment for unlike-charge pairs was not directly calculated, since in principle the Ochs prescription requires to choose always bins with equal average multiplicities, and this cannot be done simultaneously for positive and negative particles (they have different shapes of single particle distributions). However, it was checked that selecting bins with equal positive, negative or all charged average multiplicities one does not change the F_2 values even by one standard deviation. Thus in the new version [13] the F_2 values for "unlike charge" pairs were calculated using Ochs variables for all charged particles. The result of fitting these data to formula (2) is striking: the ϕ_2 value is by an order of magnitude smaller than for other data (error is not quoted for unstable fit).

Thus the authors conclude that in their data "the observed intermittent behaviour is due to Bose–Einstein interference" — it seems to appear for "like charge" pairs only.

Let us note that these results, if confirmed, would present a beautiful example of nature's malice (Murphy's laws): the contamination from an electromagnetic process produced a term with practically identical dependence on number of bins as the effect we were investigating!

Surprisingly enough, the fitted value of ϕ_2 for all charged pairs did not change significantly after cut, but the fit quality deteriorated seriously (X^2/ND changed from 17.5/14 to 35.0/16). It would be extremely interesting to investigate to what extent the other data are contaminated by γ conversion pairs and how they would change when the suspicious Q range is cut out.

It is instructive to compare the preliminary version of the same data presented in Ringberg workshop [3] with the results presented above. The fit parameters to the old version differ from those to the version [12], as seen in Table I, by few standard deviations in both c_s and ϕ_2. One may note also a strong negative correlation between these two parameters. The corrections resulting in the final form of data came from the rejection of some very close pairs after more restrictive demands on tracks and events included in the analysis. Still, all these corrections were much less important than the Q — cut described above.

Taking into account the correlations between the data points in the old version of data did not change them within two standard deviations, but made the difference with the new version even more pronounced, as seen from the next line of Table I.

We should stress here once more that the fits to the discarded preliminary version of data are done here by purpose, since our aim is not to present the most reliable values of fit parameters, but to estimate their spread. Thus in the following we use also the unpublished preliminary versions of other data presented at Ringberg. They are: *i)* π/Kp data [4] and *ii)* pAu, OAu and SAu data [5], the first two also including correlations between data points. The results are shown in the lower part of Table I. Let us summarize here their most important features:

- The SAu data, for which the fit is worst, differ very strongly from all others. In particular, the c_s value, which seemed to decrease weakly with increasing complexity of beam and target, is here by an order of magnitude larger than in pAu and OAu case. We feel that the statistics is not sufficient to treat this result too seriously, at least not until the final version of data is published.
- The pAu and OAu data yield the c_s values smaller than in μp and πp cases. However, when the correlations between the data points are

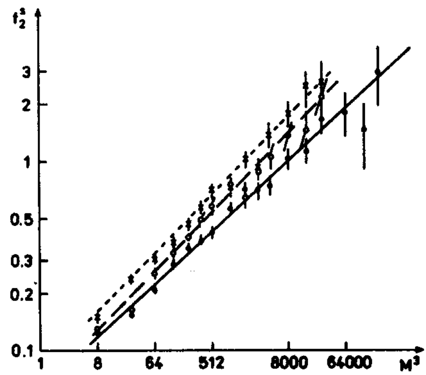

Fig. 1. Short-range part of the average second scaled factorial cumulant $f_2^s \equiv f_2 - c_L \equiv F_2 - 1c_L$ as the function of number of bins M^3 for the μp data of Ref. [12]. •, o and × denote the data for all charged, positive and negative hadrons, respectively. ——, — — —, and - - - lines show the corresponding fits to formula (2).

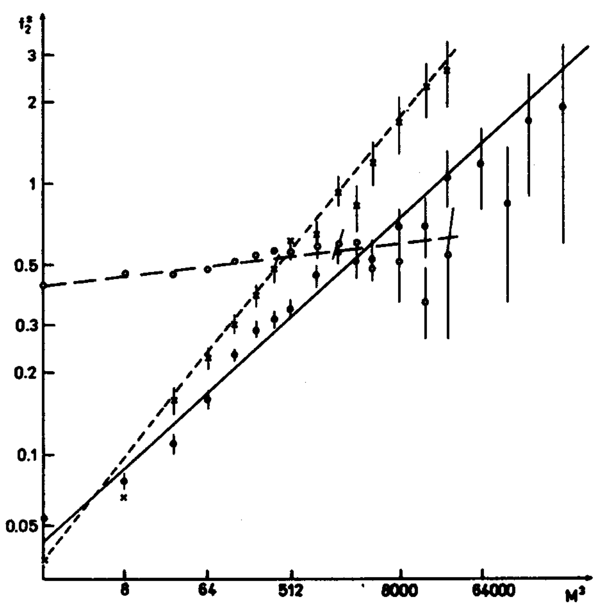

Fig. 2. Short-range part of the average second factorial cumulant $f_2^s \equiv f_2 - c_L \equiv F_2 - 1c_L$ as the function of number of bins M^3 for the μp data of Ref. [13]. •, o and × denote the data for all charged, +− pairs and negative hadrons, respectively. ——, — — —, and - - - lines show the corresponding fits to formula (2).

taken into account, this trend is reversed. In fact, if we do not take into account the SAu data, we find that the largest and smallest value of c_s found in all the considered processes comes from the same data (pAu collisions). The change introduced by taking the correlations into account is here bigger than the observed differences between various processes.

To complete our discussion, let us note that the preliminary data for μp/d collisions from the E665 experiment fitted with correlations taken into account give the parameter values in the range of those quoted in Table I [14].

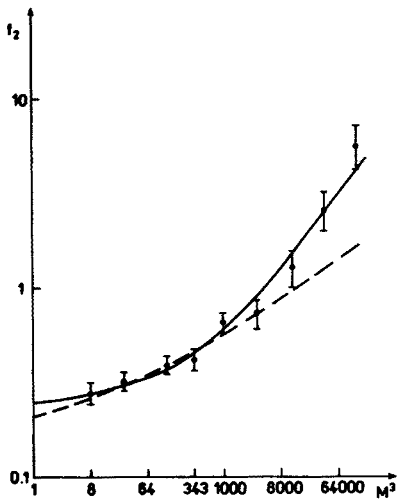

Fig. 3. f_2 vs M^3 for the preliminary pAu data from Ref. [5]. ——, and – – – show the fits to formula (2) with correlations neglected and taken into account, respectively.

Some of the results presented above are illustrated in Figs 1 and 2. In Fig. 1 we see that the three sets of first "official" μp data indeed differ little and show clear power dependence after subtracting a constant term. In contrast, in Fig. 2 we see a dramatic difference between the new data for "like" and "unlike" charge pairs. In Fig. 3 we see that including the correlations between the data points fitted gives here also the "counterintuitive" result, although less drastic than in one-dimensional case: the fit seems to ignore some data points.

3. Conclusions

We have performed the fits to the power-like form for the data describing the dependence of average second scaled factorial moment F_2 on the number of 3-dimensional phase-space bins. We have used not only the published data, but also the preliminary or discarded versions of data to estimate the spread of results due to the corrections and different fitting procedures. We have found that this spread is bigger than the differences between the parameter values for different processes. In particular, cutting out the range contaminated presumably by the γ conversion e^+e^- pairs we remove practically the power-like term from the data [13]. It remains to be seen if this effect is as strong in the other data. Taking into account the correlations between the F_2 values for different bin numbers may also change significantly the results.

Let us stress here that although the fits with correlations taken into account may be regarded as better justified, they should by no means be treated as the final answer. For long-tailed multiplicity distributions appearing in the small phase-space bins the values of errors for moments are not really fully reliable and even the use of X^2 itself is questionable. In any case, the models which want to be regarded as quantitative, should reproduce not just the values of fit parameters from formula (2), or even the values of F_2 together with covariance matrix, but simply the multiplicity distributions for any size of phase-space bins.

Our analysis is just the simplified fit showing three things:

i) that the data seem to reflect the power-like singularity in two-particle correlation function,

ii) that the exponent of this singularity seems to be independent of the type of collision from which the hadrons originate,

iii) that the singularity in unlike charge pairs may be "faked" by the contamination from γ conversion e^+e^- pairs, and the real effect seems to appear for like charges only (Bose–Einstein effect?).

In our opinion these facts, if confirmed, may well change our present understanding of multiple production processes.

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Note added in proof: After completing this manuscript I received a copy of talk by P. Seyboth "Recent Results from Experiment NA35" presented in a "Hadron Structure '92" Symposium at Stara Lesna, CSFR. It is shown there that removing e^+e^- pairs by a cut similar to that used in NA9 data one gets identical results: a power-like increasing term vanishes from the F_2 for unlike-charge pairs. Since similar results were also reported in Santiago de Compostella by UA1 group, one can say that the last of our conclusions seems to gain very good experimental basis.

REFERENCES

[1] K. Fiałkowski, *Universal Intermittency Slopes?*, Munich preprint MPI-Ph/91-53, contribution in Ref. [2]; *Phys. Lett.* **B272**, 139 (1991).
[2] R. Hwa, W. Ochs, N. Schmitz, *Proceedings of Ringberg Workshop on Multiparticle Production Fluctuations and Fractal Structure*, World Scientific, 1992.
[3] N. Schmitz, contribution in Ref. [2].
[4] W. Kittel, contribution in Ref. [2].
[5] I. Derado, contribution in Ref. [2].
[6] W. Ochs, Munich preprint MPI-PAE/P Th 63/90; *Z. Phys.* **C50**, 339 (1991).
[7] A. Białas, W.Gaździcki, *Phys. Lett.* **B252**, 483 (1990).
[8] W. Ochs, *Phys. Lett.* **B247**, 101 (1990); A. Białas, J. Seixas, *Phys. Lett.* **B250**, 161 (1990).
[9] A. Białas, R. Peschanski, *Nucl. Phys.* **B273**, 703 (1986).
[10] K. Fiałkowski, *Acta Phys. Pol.* **B22**, 397 (1991); *Intermittency in Multiple Production and the Singularities of Correlation Functions*, contribution in Ref. [2].
[11] B. Wosiek, *Acta Phys. Pol.* **B21**, 1021 (1990).
[12] I. Derado *et al.*, *A Study of Fluctuations and Correlations in Deep-inelastic Muon-Nucleon Scattering at 280 GeV*, Munich preprint MPI-PhE/91-08.
[13] I. Derado, G. Jancso, N. Schmitz, *Two-Particle Correlations and the Origin of Intermittency in Muon-Nucleon interactions*, Munich preprint MPI-PhE/92-07.
[14] J. Figiel, private communication.