SKEWONS AND GRAVITONS

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A gravitational theory based on the generalised affine geometry is presented. It contains an antisymmetric torsion potential in addition to the metric. The coupling to matter is derived by the minimal coupling principle, implying that the metric couples to a symmetrised energy-momentum tensor, and the antisymmetric field couples to the canonical spin tensor of matter. The coupling constant for both cases is Newton's constant, G. Phenomenological consequences of the linearised theory for quantum processes at the tree level are explored.

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1. Introduction

Invariance groups of continuous tranformations of fields produce conserved tensors. In gauge theories there are specific interactions associated with the conserved tensors. While Einstein's General Theory of Relativity provides an interaction with energy as a source, there is no interaction coming from spin-angular momentum, even though angular momentum is conserved. Moreover, Einstein's theory of gravitation requires the energy-momentum tensor to be symmetric, this being the generalisation of the symmetry of the stress-energy tensor in Newtonian mechanics. Planck [1] argued that this symmetry also holds in field theory. But Planck's considerations were based on the assumption that the velocity of transport of field energy is proportional to the local field momentum. In general this is not true.

Weyssenhoff and Raabe [2] and Papapetrou [3] showed that the energy-momentum tensor of a "perfect gas" of spinning particles necessarily is non-symmetric if the particles must be considered as point particles. This is also the case in field theory where the canonical stress-energy tensor of the Dirac field is non-symmetric [4-7].

Hence, one might argue that Einstein's theory therefore has two shortcomings — there is no interaction which couples to spin, and there is the dilemma that the Einstein-(Christoffel) tensor is symmetric but the energy-momentum tensor is non-symmetric. The standard solution is to assume that the *metric* energy-momentum tensor — which is obtained by variations with respect to the metric and therefore itself automatically symmetric — is the true gravitational source. The other alternative is to generalise the theory of gravitation so as to support a possible non-symmetric source. The first suggestions in this directions were made by Cartan [8-11] who argued that the proper geometry of space-time is non-Riemannian and that spin couples to the antisymmetric part of the connection.

With the advent of a gauge theoretic approach to gravity [12-15] the ideas of Cartan were revived. This lead to the Einstein–Cartan theory [13,16-25], which has many attractive features. It is most appealing that the problems of spin and of symmetry of energy-momentum are unified and resolved in the Riemann–Cartan framework. Here spin-angular momentum conservation law[1] relates a divergence of the spin tensor with the antisymmetric part of the energy-momentum tensor. A main problem with this theory is the non-propagating nature of torsion. This problem may be mended, by introducing higher order invariants in the action integral, only at the cost of 151 additional free parameters [26]. Admittedly the number of free constants is reduced considerably by the requirement of no ghosts and no tachyons [27], but one should as far as possible refrain from introducing new fundamental constants.

Here we modify the Einstein–Cartan theory by introducing an antisymmetric torsion potential (see Appendix B for references to earlier theories of this type). The torsion potential has a geometrical meaning in the framework of a generalised affine geometry (Appendix A), which from a gauge theoretic point of view may be more natural as a model of space-time [28] than the conventional affine geometry. Because of the geometric motivation of the theory, its structure is unique once we demand the field equations to be second order differential equations, and since the torsion field is derived from a potential we also obtain a propagating torsion field without introducing new fundamental constants.

The derivation of the field equations is given in Appendix B, and a discussion of their strength in Appendix C. In Section 2 the proper source of gravity is found by the minimal coupling principle. Then in Section 3 the theory is quantised, and the interaction of matter with gravitons and skewons is discussed.

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[1] The spin and the energy conservation laws correspond to the first and the second Bianchi identities, respectively

2. The proper source of gravity

2.1. Heuristic arguments

The metric energy-momentum tensor is the source of gravity in the standard formulation of General Relativity. At a fundamental level energy-momentum is related to space-time translations. Because orbital motion can be reduced to a series of infinitesimal translations, it is not surprising that orbital angular momentum plays a rôle in this theory. Thus, by virtue of the Ricci identity, Einstein's General Relativity contains an orbital angular momentum conservation law [29]. Moreover the theory predicts specific rotation induced phenomena such as the Lense–Thirring effect [30-33].

But in addition to energy and orbital angular momentum, matter may possess spin, a quantity which is connected with local rotational symmetry of space-time. In Einstein's General Relativity spin has no direct gravitational effect[2], but being dependent upon a space-time symmetry, one should expect spin — or something intimately related to spin — to appear as a source of gravity.

Curvature may be generated by a nonuniform stretching of space-time, caused by an active local translation of points $x + dx$ away from x. The metric field holds all information about this space-time stretching because it determines the distances from all points x to $x + dx$. Similarly one could think of local torques making reference frames rotate when paralell transported. This would appear as space-time torsion.

Mass is known to give rise to metric curvature. By analogue one expects spin or some related torque to produce torsion.

2.2. Noether's tensors of space-time symmetries

Noether's theorem [36] states that if an action is invariant under a continuous group of transformations of the fields, the corresponding Lagrangean determines a conserved tensor.

When applied to the symmetry of translation, Noether's theorem yields a conserved energy-momentum tensor, whereas the rotational symmetry implies a conserved angular momentum tensor [37]. The canonical energy-momentum tensor has the form

$$\Sigma^{\alpha\beta} \equiv \frac{\partial L}{\partial(\partial_\alpha \psi_A)} \partial^\beta \psi_A - g^{\alpha\beta} L, \tag{1}$$

[2] In General Relativity spin has a very weak gravitational effect: It acts on the polarisation of weak gravitational waves [34]. There is also an effect coming from the gravitational spin-orbit interaction [35].

where the field ψ carries the summation index A to encompass multicomponent fields. In general this energy-momentum tensor is asymmetric in α and β. The conserved angular momentum tensor is

$$M^{\alpha}{}_{\beta\gamma} \equiv x_\beta \Sigma_\gamma{}^{\alpha} - x_\gamma \Sigma_\beta{}^{\alpha} + S^{\alpha}{}_{\beta\gamma}. \tag{2}$$

$L^{\alpha}{}_{\beta\gamma} \equiv 2x_{[\beta}\Sigma_{\gamma]}{}^{\alpha}$ may be identified with the *orbital* angular momentum of the field[3], and $S^{\alpha}{}_{\beta\gamma}$ represents the *spin* density tensor. Following the established practice in Einstein–Cartan theory [25], we will use Hehl's spin tensor $t^{\alpha}{}_{\beta\gamma}$ which is *half* the conventional one. Then Tetrode's law [4] takes the form

$$t^{\alpha}{}_{\beta\gamma,\alpha} = \Sigma_{[\beta\gamma]}. \tag{3}$$

Hence, the antisymmetric part of the energy-momentum tensor has a natural interpretation as a torque: The divergence of the spin tensor equals the applied torque. It is sometimes argued that the field torque necessarily must be zero: The torque, τ, on a cube of dimension L, has the form

$$\tau^z = (\Sigma^{xy} - \Sigma^{yx})L^3.$$

Now, if we let the length L of the cube shrink, the torque decreases as L^3, while the moment of inertia decreases as L^5. According to Misner, Thorne and Wheeler [38], this would *"set an arbitrarily small cube into arbitrarily great angular acceleration — which is absurd."* Central to this argument is the idea that angular momentum is related only to *rotation* of a volume. But spin angular momentum is something different, something which cannot be obtained from orbital angular momentum by a limiting proceedure where the orbital radius tends to zero. If such a notion of intrinsic angular momentum is accepted, an asymmetric energy-momentum tensor arises naturally as a way of transforming spin angular momentum to orbital angular momentum and *vice versa.*

While the nature of the conserved Noether tensors associated with the symmetries of space-time strongly suggests a non-symmetric theory of gravitation, this is not the only possibility: Belinfante [39] and Rosenfeld [40] have shown that in general it is possible to construct a symmetric conserved energy-tensor from the canonical energy- and spin angular-momentum tensors. This combined symmetric energy-momentum tensor could be the source of a symmetric theory of gravitation. Then the gravitational effect of the canonical spin tensor is only to compensate the antisymmetric part of the canonical energy-momentum tensor, and therefore eliminate the need

[3] Expressions with indices in the parentheses ( ) and [ ] are to be symmetrised and antisymmetrised, respectively.

for an antisymmetric gravitational field in addition to the metric. We do, however, feel that this construction is less direct than the non-symmetric approach, which is based on the idea that Noether's tensors of space-time symmetries are truly fundamental and themselves the proper sources of gravity.

2.3. Spinor-gravity minimal coupling

In a local gauge theory of a group of symmetry G, one introduces $4m$ gauge potentials ${A_\mu}^b$ in the gauge-covariant derivative operator $\nabla_\mu \psi = \partial_\mu \psi - T_b {A_\mu}^b \psi$ where the m quantities T_b are generators of the group G as acting on the field ψ. When G is the Poincaré group, it is possible to identify the gauge-covariant derivative with the covariant derivative of a Riemann–Cartan geometry [25]. In addition to metric curvature this geometry has torsion. For a spinor field interacting with gravity, the generators T_b are represented by the spin matrices $\sigma_{\alpha\beta} = (1/4)\gamma_{[\alpha}\gamma_{\beta]}$. Thus the covariant derivatives of a spinor field and its conjugate are [41-45]

$$\left.\begin{aligned} \nabla_\alpha \psi &= \partial_\alpha \psi + ({\Lambda^\mu}_{\beta\alpha} {\sigma_\mu}^\beta + {K^\mu}_{\beta\alpha} {\sigma_\mu}^\beta)\psi \\ \nabla_\alpha \bar{\psi} &= \partial_\alpha \bar{\psi} - \bar{\psi}({\Lambda^\mu}_{\beta\alpha} {\sigma_\mu}^\beta + {K^\mu}_{\beta\alpha} {\sigma_\mu}^\beta) \end{aligned}\right\} \tag{4}$$

where the total connection ${\Gamma^\mu}_{\beta\alpha}$ has been split into a sum of a Riemannian connection ${\Lambda^\mu}_{\beta\alpha}$ and the contortion tensor ${K^\mu}_{\beta\alpha}$.

At this stage it should be stressed that gravitation has no recognized gauge version [46], although several gauge theory constructions have been proposed [12,13,47,48]. The problems stem from the fact that the Riemannian part of the connection (the Christoffel– or Fock–Ivanenko-symbols) are *determined* by the metric potentials. Accordingly this part of the connection cannot play the rôle of a fundamental field like in other gauge theories. As opposed to the Einstein–Cartan theory, here we assume that even the contortion part of the connection is a derived quantity defined in terms of the R^4-part of the connection of the underlying generalised affine geometry [28].

Motivated by the equivalence principle [49,50] and by the minimal coupling prescription for gauge fields as introduced by Gell–Mann [51], we assume that gravitation couples to matter only through the substitutions $\eta_{\mu\nu} \to g_{\mu\nu} = {e_\mu}^{\hat{\mu}} {e_\nu}^{\hat{\nu}} \eta_{\hat{\mu}\hat{\nu}}$ and $\partial_\alpha \to \nabla_\alpha$ in the matter field Lagrange function[4]. In addition there is an overall factor of $\sqrt{-g} = e$ in the action integrand which comes from the invariant volume element. Thus the

[4] Greek indices with a hat denotes an (pseudo)-orthonomal frame, and those without a hat denotes a general frame. Latin indices refer to holonomic coordinates.

Dirac Lagrange function of Minkowski (M_4) space-time[5]

$$L(M_4)_{\rm Dirac} = -\frac{i}{2}\bar{\psi}\gamma^{\alpha}\overleftrightarrow{\partial}_{\alpha}\psi - m\bar{\psi}\psi \tag{5}$$

changes to[6]

$$L(U_4)_{\rm Dirac} = -\frac{i}{2}e\bar{\psi}\gamma^{\hat{\alpha}}\overleftrightarrow{\partial}_{i}\psi e^{i}{}_{\hat{\alpha}} - em\bar{\psi}\psi - \frac{i}{2}e\Gamma^{\hat{\mu}}{}_{\hat{\beta}\hat{\alpha}}t^{\hat{\alpha}}{}_{\hat{\mu}}{}^{\hat{\beta}}, \tag{6}$$

where

$$t^{\hat{\alpha}\hat{\beta}\hat{\gamma}} = \frac{i}{2}\bar{\psi}\big(\gamma^{\hat{\alpha}}\sigma^{\hat{\beta}\hat{\gamma}} + \sigma^{\hat{\beta}\hat{\gamma}}\gamma^{\hat{\alpha}}\big)\psi = \frac{i}{4}\bar{\psi}\gamma^{[\hat{\alpha}}\gamma^{\hat{\beta}}\gamma^{\hat{\gamma}]}\psi \tag{7}$$

is identified as the canonical spin tensor. Decomposing the connection in a Riemannian and a contortion part, we may write the last term of (6) as

$$-e\Gamma^{\hat{\mu}}{}_{\hat{\beta}\hat{\alpha}}t^{\hat{\alpha}}{}_{\hat{\mu}}{}^{\hat{\beta}} = -e\big(\Lambda^{\hat{\mu}}{}_{\hat{\beta}\hat{\alpha}} + K^{\hat{\mu}}{}_{\hat{\beta}\hat{\alpha}}\big)t^{\hat{\alpha}}{}_{\hat{\mu}}{}^{\hat{\beta}}. \tag{8}$$

Expressing the Riemannian part of the connection, in terms of the structure coefficients

$$\Lambda^{\hat{\mu}}{}_{\hat{\beta}\hat{\alpha}} = \frac{1}{2}\big(C_{\hat{\beta}}{}^{\hat{\mu}}{}_{\hat{\alpha}} + C_{\hat{\alpha}}{}^{\hat{\mu}}{}_{\hat{\beta}} - C^{\hat{\mu}}{}_{\hat{\beta}\hat{\alpha}}\big) \tag{9}$$

with

$$C^{\hat{\mu}}{}_{\hat{\alpha}\hat{\beta}} = e^{i}{}_{\hat{\alpha}}e^{j}{}_{\hat{\beta},i}e_{j}{}^{\hat{\mu}} - e^{j}{}_{\hat{\beta}}e^{i}{}_{\hat{\alpha},j}e_{i}{}^{\hat{\mu}}, \tag{10}$$

we get

$$\frac{\partial L(U_4)_{\rm Dirac}}{e\partial e^{i}{}_{\hat{\alpha},j}} = -\big(t^{\hat{\alpha}j}{}_{i} + t_{i}{}^{j\hat{\alpha}} - t^{j\hat{\alpha}}{}_{i}\big). \tag{11}$$

The graviton source is

$$S^{\hat{\alpha}}{}_{i} \equiv \frac{\delta L(U_4)_{\rm Dirac}}{e\delta e^{i}{}_{\hat{\alpha}}} = \frac{1}{e}\Big(\frac{\partial L(U_4)_{\rm Dirac}}{\partial e^{i}{}_{\hat{\alpha}}} - \partial_j\frac{\partial L(U_4)_{\rm Dirac}}{\partial e^{i}{}_{\hat{\alpha},j}}\Big). \tag{12}$$

Here we consider only first order perturbations of Minkowski space-time, expressed by the metric perturbations $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. Hence, we neglect terms in $\delta L(U_4)/\delta e^{i}{}_{\hat{\alpha}}$ containing the connection. In this approximation the expressions (11) and (12) lead to

$$S^{\hat{\alpha}}{}_{i} = \Sigma^{\hat{\alpha}}{}_{i} + \partial_j\big(t^{\hat{\alpha}j}{}_{i} + t_{i}{}^{j\hat{\alpha}} - t^{j\hat{\alpha}}{}_{i}\big), \tag{13}$$

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[5] We use the sign convention $\eta_{\mu\nu} = (-1, 1, 1, 1)$.

[6] Note that spinors can only be treated self-consistently by using orthonormal frames.

where

$$\Sigma_{\mu\nu} \equiv -\frac{i}{2}\bar{\psi}\gamma_\mu \overleftrightarrow{\partial}_\nu \psi + \frac{1}{2}\eta_{\mu\nu}(\bar{\psi} i\gamma^\alpha \overleftrightarrow{\partial}_\alpha \psi + 2m\bar{\psi}\psi) \tag{14}$$

is the canonical energy-momentum tensor.

$S^{\mu\nu}$ is the Belinfante–Rosenfeld [39,40] energy-momentum tensor. Owing to the identity $\Sigma^{[\mu\nu]} = \partial_j t^{j\mu\nu}$, this tensor is automatically *symmetric*. In the case of a Dirac field the graviton source is simply the symmetric part of the canonical energy-tensor $S^{\mu\nu} = \Sigma^{(\mu\nu)}$. Recalling that $e^i{}_\mu e_{i\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ we get $\delta h_{\mu\nu} = 2e_{i(\nu}\delta e^i{}_{\mu)}$, so that the Dirac–Einstein interaction term may be written as

$$L_{\text{Dirac-Einstein}} = \frac{1}{2}h_{\mu\nu}\Sigma^{(\mu\nu)}. \tag{15}$$

The non-Riemannian part of the connection gives the interaction term

$$L_{\text{Dirac-torsion}} = -K_{\mu\nu\alpha}t^{\alpha\mu\nu}. \tag{16}$$

Because the spin-tensor $t^{\alpha\beta\gamma}$ is totally antisymmetric, only the antisymmetric part of the contortion interacts with the Dirac field. Using that[7]

$$K_{[\alpha\beta\gamma]} = -\frac{1}{2}T_{[\alpha\beta\gamma]} = \frac{3}{2}s_{[\alpha\beta|\gamma]}, \tag{17}$$

and neglecting self-interaction of the antisymmetric field by replacing the covariant derivative with the partial derivative, the Dirac-torsion interaction term takes the form

$$L_{\text{Dirac-torsion}} = -\frac{3}{2}s_{\mu\nu,\alpha}t^{\alpha\mu\nu}. \tag{18}$$

This is the fundamental expression for the fermion-torsion interaction. Here we have used that $t^{\alpha\beta\gamma}$ is totally antisymmetric, so it is not necessary to antisymmetrise the torsion potential. Then adding the the total divergence $\partial_\alpha(3/2 s_{\mu\nu}t^{\alpha\mu\nu})$, the Dirac-Gravity coupling takes the form

$$L_{\text{Dirac-Gravity}} = \frac{1}{2}h_{\mu\nu}\Sigma^{(\mu\nu)} + \frac{3}{2}s_{\mu\nu}t^{\alpha\mu\nu}{}_{,\alpha}. \tag{19}$$

The spin-torsion coupling may also be written in terms of an axial vector current. Because the spin-tensor is totally antisymmetric, we may express it as an axial vector

$$J^\beta \equiv \frac{1}{6}\eta^{\beta\mu\nu\rho}t_{\mu\nu\rho}. \tag{20}$$

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[7] Here $T_{\alpha\beta\gamma} \equiv 2\Gamma_{\alpha[\gamma\beta]} - C_{\alpha\beta\gamma}$ is the torsion tensor. $K_{\alpha\beta\gamma} \equiv \frac{1}{2}(T_{\beta\alpha\gamma} + T_{\gamma\alpha\beta} - T_{\alpha\beta\gamma})$ is the contortion tensor. The skewon field $s_{\mu\nu}$ is the antisymmetric torsion potential, see also Appendix B.

This current is not conserved, $J^{\beta}{}_{,\beta} \neq 0$. In terms of the axial vector spin-current the skewon interaction is

$$L_{\text{Dirac-Gravity}} = -\frac{3}{2}\eta^{\alpha\beta\gamma\delta} J_{\beta}\partial_{\alpha}s_{\gamma\delta}. \tag{21}$$

As was pointed out by Ogievetskiĭ and Polubarinov [52], this interaction shows a pecularity of the antisymmetric field. While the free field behaves like a scalar, having only one physical degree of freedom, in interactions it carries all three polarisation states of spin 1. The point is that a *virtual* skewon does not have zero mass, and therefore it acquires the additional states with helicity ± 1, associated with a massive antisymmetric tensor field. Then by the definition of the spin of an interacting field, the interacting skewon field has spin 1.

On the mass shell we may use the angular momentum conservation law $t^{\alpha\mu\nu}{}_{,\alpha} = \Sigma^{[\mu\nu]}$, and write the linearised Dirac-Gravity interaction Lagrangean (19) as

$$L_{\text{Dirac-Gravity}} = \frac{1}{2}h_{\mu\nu}\Sigma^{(\mu\nu)} + \frac{3}{2}s_{\mu\nu}\Sigma^{[\mu\nu]}. \tag{22}$$

The minimal coupling prescription in a space-time with an antisymmetric torsion potential leads to the conclusion that the proper sources of gravity are the Noether energy-tensor and the divergence of the Noether spin tensor.

The symmetric Belinfante–Rosenfeld combination is coupled to gravitons, and the divergence of the Noether spin tensor is coupled to the torsion potential. For a Dirac field on the mass shell, these sources reduce to the symmetric and antisymmetric parts of the canonical energy-momentum tensor. This type coupling was first proposed by Papapetrou [3].

2.4. Photons and torsion

While minimal coupling lead to a satisfying result for Dirac fields, a direct replacement of ∂_μ with ∇_μ in the definition of the electromagnetic field tensor implies a photon-torsion coupling which would break the $U(1)$ gauge invariance. If, however, the field tensor is defined in terms of the exterior derivative of the potential, a cancellation of torsion terms occurs which means that photons do not couple to torsion [53]. From a gauge theoretic point of view [48] this procedure finds a natural justfication, since in this framework the gauge fields (like the electromagnetic potential A_μ) are treated as Poincaré scalars which should not couple to the connection. The decoupling of gauge bosons from torsion also appears naturally in the Kaluza–Klein scheme [54]. Here the gauge bosons are space-time projections of higher-dimensional *metric* exitations which are not expected to produce torsion.

3. Non-Symmetric Quantum Gravity

3.1. Quantised General Relativity

Quantisation of General Relativity has been the subject of much investigation [55-63], but when coupled to matter the theory is not renormalizable [64-69]. Despite these problems useful results can be obtained. Indeed in connection with the phenomenological Lagrangian approach to current algebra it was found that the tree-graph approximation to quantum field theory reproduces the classical field [70,71]. Explicite calculations [72,73] have verified that the quantum-gravity tree-graph contribution correctly reproduces the classical Schwarzschild solution. Thus macroscopically the theory gives predictions which are in excellent agreement with experimental data [74]. Papini and Valluri [75] have given a detailed review of the quantum theory of gravitons.

3.2. Skewons and gravitons — propagators and vertex rules

In previous Sections we have given theoretical motivation for a skew symmetric component of the gravitational field. Because these considerations are based on the spin concept, a microscopic quantum phenomenon, it is natural to look for consequences of the theory in a quantised theory.

The linearised field equations are (see Appendix B)

$$\Box\,\bar{h}_{\mu\nu} = -16\pi G\Sigma_{(\mu\nu)}, \qquad \bar{h}^{\mu\nu}{}_{,\nu} = 0, \tag{23}$$

$$\Box\,s_{\mu\nu} = -16\pi G t^{\alpha}{}_{\mu\nu,\alpha}, \qquad s^{\mu\nu}{}_{,\nu} = 0. \tag{24}$$

In the harmonic gauge these field equations, can be inverted as

$$\bar{h}_{\mu\nu} = \frac{16\pi G}{k^2}\Sigma_{(\mu\nu)} \quad \text{and} \quad s_{\mu\nu} = \frac{16\pi G}{k^2}t^{\alpha}{}_{\mu\nu,\alpha}. \tag{25}$$

Taking the trace invers of the first equation and rewriting the right hand sides by use of projection tensors, we get

$$h_{\mu\nu} = \frac{16\pi G}{k^2}P_{\mu\nu\alpha\beta}\Sigma^{\alpha\beta} \quad \text{and} \quad s_{\mu\nu} = \frac{16\pi G}{k^2}N_{\mu\nu\alpha\beta}t^{\sigma\alpha\beta}{}_{,\sigma}. \tag{26}$$

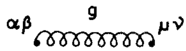
The projection tensors are

$$P_{\mu\nu\alpha\beta} \equiv \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta} - \eta_{\mu\nu}\eta_{\alpha\beta}) \tag{27}$$

and

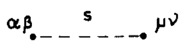
$$N_{\mu\nu\alpha\beta} \equiv \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} - \eta_{\nu\alpha}\eta_{\mu\beta}). \tag{28}$$

In view of the couplings (19) and the above inverted field equations, we may write down the following Feynman rules



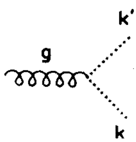
$$-\frac{iP_{\mu\nu\alpha\beta}}{k^2 + i\epsilon}, \tag{29}$$

Fig. 1.



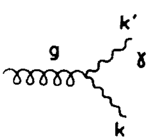
$$-\frac{iN_{\mu\nu\alpha\beta}}{k^2 + i\epsilon}, \tag{30}$$

Fig. 2.



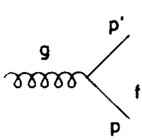
$$-\frac{1}{2}i\lambda\{k'_\mu k_\nu + k_\mu k'_\nu - \eta_{\mu\nu}(k \cdot k' + m^2)\}, \tag{31}$$

Fig. 3.



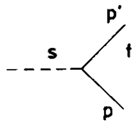
$$\begin{aligned} -\frac{1}{2}i\lambda\{k'_\alpha(k_\nu\eta_{\mu\beta} + k_\mu\eta_{\nu\beta}) + k_\beta(k'_\nu\eta_{\mu\alpha} + k'_\mu\eta_{\nu\alpha}) \\ -\eta_{\alpha\beta}(k'_\mu k_\nu + k_\mu k'_\nu) + \eta_{\mu\nu}(k \cdot k'\eta_{\alpha\beta} - k'_\alpha k_\beta) \\ -k \cdot k'(\eta_{\mu\beta}\eta_{\nu\alpha} + \eta_{\mu\alpha}\eta_{\nu\beta})\}, \end{aligned} \tag{32}$$

Fig. 4.



$$\begin{aligned} -\frac{1}{8}i\lambda\{\gamma_\mu(p_\nu + p'_\nu) + \gamma_\nu(p_\mu + p'_\mu) \\ -2\eta_{\mu\nu}[\gamma^\alpha(p_\alpha + p'_\alpha) + 2m]\}, \end{aligned} \tag{33}$$

Fig. 5.



$$-\frac{1}{8}i\lambda\gamma_{[\alpha}\gamma_\mu\gamma_{\nu]}(p'^\alpha - p^\alpha). \tag{34}$$

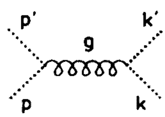
Fig. 6.

On the mass shell the last vertex rule may be written as

$$-\frac{1}{8}i\lambda\{\gamma_\mu(p_\nu + p'_\nu) - \gamma_\nu(p_\mu + p'_\mu)\}. \tag{35}$$

3.3. Static limit

We now consider the problem of gravitational interaction between two massive scalars. Since scalars do not couple to the skewon field, only the graviton exchange process is relevant. Using the graviton propagator (29) and the vertex rule (31) in the static approximation $k^\alpha = k'^\alpha = (M, 0)$ and $p^\alpha = p'^\alpha = (m, 0)$, we find the matrix element



$$\mathcal{M}(q) = \frac{i\lambda^2}{2q^2}m^2M^2. \tag{36}$$

Fig. 7.

Using that $q^2 \approx |\vec{q}\,|^2$, the classical Newtonian potential is recognized as

$$V(\vec{r}) = \frac{i}{4mM}\mathcal{M}(\vec{r}) \tag{37}$$

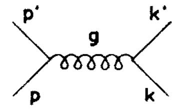
where

$$\mathcal{M}(\vec{r}) = \frac{1}{(2\pi)^3}\int \mathcal{M}(\vec{q})e^{i\vec{q}\cdot\vec{r}}d^3\vec{q}. \tag{38}$$

This gives

$$V(\vec{r}) = -\frac{\lambda^2 mM}{32\pi\,|\,\vec{r}\,|} = -\frac{GmM}{|\,\vec{r}\,|} \quad \text{with} \quad \lambda^2 = 32\pi G. \tag{39}$$

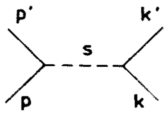
For the gravitational interaction between two fermions, we also have to consider exchange of skewons. But using the static approximation, *i.e.* neglecting the space-momentum, we find



$$\mathcal{M}_1(q) = \frac{i\lambda^2}{8q^2} mM\bar{u}(p')\gamma_0 u(p)\bar{u}(k')\gamma_0 u(k) \quad (40)$$

Fig. 8.

and



$$\mathcal{M}_2(q) = 0. \quad (41)$$

Fig. 9.

This is equivalent to the Newtonian force found for scalars. Hence, the skewon interaction does not change the gravitational force in the static limit.

3.4. Scattering of fermions by graviton and skewon exchange

The exact expression for the matrix element owing to graviton exchange is

$$\mathcal{M}_g = \frac{i\lambda^2}{32q^2}\{a(K\cdot P)(j(P)\cdot j(K)) + b(K\cdot j(P))(P\cdot j(K)) - c(P\cdot j(P))(K\cdot j(K))\}, \quad (42)$$

where $K = k + k'$, $P = p + p'$, and $j^\mu(P) = \bar{u}(p')\gamma^\mu u(p)$, $j^\mu(K) = \bar{u}(k')\gamma^\mu u(k)$ are the Dirac vector currents at the interaction vertices. (p, p', k, and k' are the particle momenta, as given in Fig. 8). The coefficients a, b, and c are given by $a = b = c = 1$ for graviton exchange. The last term $\sim c$ does not contribute in the limit $m \to 0$, owing to the Dirac equation. For skewon exchange, the amplitude $\mathcal{M}_s$ can be written in the same form as (42), with $a = -b = 1, c = 0$. Using the fermion-skewon coupling in the form (34), a more interesting expression for $\mathcal{M}_s$ is obtained:

$$\mathcal{M}_s = \frac{i\lambda^2}{16}\left(\eta_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) j_A^\mu(P) j_A^\nu(K)\,, \quad (43)$$

where $j_A^\mu(P) = \bar{u}(p')\gamma^\mu\gamma_5 u(p)$ is the axial current. In the limit $m \to 0$, $q\cdot j_A(P) \to 0$, which means that skewon exchange is effectively a point (local) interaction in this limit. More specific, $q \cdot j_A(P) = 2m\bar{u}(p')\gamma_5 u(p)$ where the scalar density $\bar{u}'\gamma_5 u$ vanishes in the static limit. Thus (43) is in agreement with (41).

The modifications of the gravitational interactions between the fermions are found by adding $\mathcal{M}_{\rm g}$ and $\mathcal{M}_{\rm s}$. In the limit of vanishing fermion mass we have explicitely calculated the quantity

$$S_{(\rm s+g)} = \sum_{\rm Pol.} |\mathcal{M}_{\rm g} + \mathcal{M}_{\rm s}|^2, \tag{44}$$

which determines the cross section for scattering of two fermions ($f_1 f_2 \to f_1 f_2$) and annihilation of two fermions ($f_1 \bar{f}_1 \to f_2 \bar{f}_2$). The full expression for $S_{(\rm s+g)}$ is given in Appendix D, Eq. (D1). For scattering angles $\theta \to 0$, $\theta = \pi/2$, and $\theta = \pi$, we obtain the following modifications due to torsion

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)^{(\rm g+s)}}{\left(\frac{d\sigma}{d\Omega}\right)^{\rm g}} = \begin{cases} 1; & \theta \to 0 \\ 180/149; & \theta = \pi/2 \\ 4; & \theta = \pi. \end{cases} \tag{45}$$

For annihilation the total cross section can be calculated, and we obtain

$$\sigma^{(\rm g+s)}_{\rm Ann.} = \frac{8}{3}\sigma^{(g)}_{\rm Ann.} \; . \tag{46}$$

We have also considered the interference with the total gravitation amplitude $\mathcal{M}_{(\rm g+s)}$ with the one photon exchange amplitude. The unpolarised square of the total amplitude is then

$$S_{(\gamma+\rm g+s)} = \sum_{\rm Pol.} |\mathcal{M}_\gamma + \mathcal{M}_{\rm g} + \mathcal{M}_{\rm s}|^2 = S_\gamma + S_{(\rm g+s)} + S_{\rm Int}, \tag{47}$$

where S_γ is given in textbooks in QED, $S_{(\rm g+s)}$ is given in (44) and (D1), and the interference between the one photon exchange amplitude and our extended gravity is given in (D2). The skewon interaction modifies the result obtained by pure graviton exchange. But of course $S_{\rm Int}$ is only interesting numerically for energies rather close to the Planck energy $M_{\rm Pl}c^2$. To make predictions one should use the effective couplings for extremely high energies. This is known in pure QED. But $\alpha_{\rm em}$ and the cross section itself are modified also by electroweak interactions, and furthermore by Grand Unified (GUT) electroweak and strong interactions, if they exist. In the end we should also have a consistent theory of particle physics including gravity. This is unfortunately still lacking. Of course the processes considered in this subsection have occured in the very early universe, but because of the mentioned shortcomings the given results are rather academic.

One could also consider scattering of a fermion in the fields of macroscopic objects. From (43) we see that skewon exchange is essentially a dipole-dipole interaction, because the space part of the axial fermionic current is a spin density. Using the relation between the spin and the magnetic moment, the skewon will couple to the magnetization of a macroscopic object, for instance of a neutron star. Thus we can write down amplitudes for scattering of a charged fermion in the field of a magnetized neutron star due to skewon and photon exchange. But the ratio between the skewon and photon exchange amplitudes are roughly of order $Gm_{\rm N}q/(\hbar\alpha_{\rm em})$, which is extremely tiny even for a momentum transfer $q \sim M_{\rm Pl}c$. ($m_{\rm N}$ is the neutron mass). If the scattered fermion is a neutrino with a magnetic moment $e\mu_\nu/(2m_e)$ (m_e being the electron mass) the ratio will be of order $Gm_{\rm N}m_e/(\hbar\alpha_{\rm em}\mu_\nu)$, which is still extremely tiny, for the small neutrino magnetic moment predicted by the electroweak standard model [76] $\mu_\nu \sim 10^{-19}[m_\nu/(1\ {\rm eV})]$.

3.5. Photoproduction of graviton and skewon

The processes γf $\to$ fg, and their crossed versions $f\bar{\rm f} \to \gamma g$ have been considered in the literature [58,75], and the corresponding skewon interaction was considered by Neville [77]. The contributing diagrams to lowest order are given in Fig. 10.

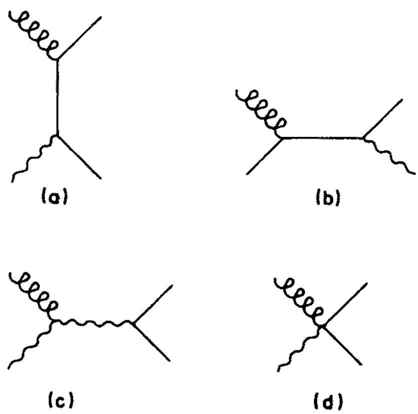


Fig. 10. Diagrams for γf $\to$ fg

The $\gamma\gamma$g coupling can be found from the energy momentum tensor of the Maxwell field

$$T^{\rm M}_{\mu\nu} = \frac{1}{4}\eta_{\mu\nu}F^2 - F_\mu{}^\rho F_{\nu\rho}. \tag{48}$$

The diagram (c) is gauge invariant by itself with respect to QED because $T^{\rm M}_{\mu\nu}$ involves only the electromagnetic field tensor. The contact term (d) is obtained by using the replacement $\partial_\mu \to \partial_\mu + ieA_\mu$ in the fermion energy-momentum tensor (14). The sum of (a)-(d) is found to be gauge invariant with respect to QED (*i.e.* the sum is zero when the replacement $\epsilon(\gamma) \to p_\gamma$ is performed) and similarly with respect to gravity. The unpolarised square of the total amplitude (a)-(d),

$$S^{\gamma\rm f\to fg} = \sum_{\rm Pol.} \mid \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c + \mathcal{M}_d \mid^2 , \tag{49}$$

which is given in (D3), can also be used to find the cross section for $\rm f\bar{f} \to \gamma g$. In the last case we have found the cross section in the center of mass (CM) frame in the limit $m \to 0$:

$$(\frac{d\sigma}{d\Omega})_{\rm CM} = \frac{3}{4}\alpha_{\rm em}G\hbar(1+\cos^2\theta), \tag{50}$$

where θ is the scattering angle. This is a factor of 3 more than in Ref. [75].

For $\gamma\rm f \to sf$ or $\rm f\bar{f} \to \gamma s$, the diagram (c) does not exist because torsion does not couple to electromagnetism [25,53]. Using the coupling (34) derived from (18), there is no contact term like (d) because the skewon is electrically neutral. One could also use the coupling (35) derived from (22) in (a) and (b). Then there will be a contact term (d), but the result for $S^{\gamma\rm f\to sf}$, given in (D4), is unchanged. It should be emphasized that $S^{\gamma\rm f\to sf}$, vanishes in the limit $m \to 0$, reflecting the fact that emission of a skewon flips the fermion spin[8]. To order m^2 the cross section for $\rm f\bar{f} \to \gamma s$ is

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \alpha_{\rm em}G\hbar\frac{4m^2}{E^2\sin^2\theta}. \tag{51}$$

3.6. Bremsstrahlung of g *and* s

We have considered *Bremsstrahlung* of g and s from a fermion scattered in an external electromagnetic field. These processes are similar to $\gamma\rm f \to fg$ and $\gamma\rm f \to fs$, except that now the photon is virtual. Considering soft graviton emission we find an amplitude

$$\mathcal{M}^{\rm f\to fg}_{\rm Br} \sim \frac{e\sqrt{G}}{k^2}\epsilon^\sigma(\gamma^*)\bar{u}(p')\gamma_\sigma u(p)\left(\frac{p_\mu p_\nu}{p\cdot q} - \frac{p'_\mu p'_\nu}{p'\cdot q}\right)\epsilon^{\mu\nu}(g)^* , \tag{52}$$

[8] This is a quantum analogue to the spin-precession which follows from the classical equations of motion of a spinning particle in a Riemann–Cartan background [78,79].

which is similar to the result of soft photon *Bremsstrahlung.* ($\epsilon^\sigma(\gamma^*)/k^2$ represents the Fourier tranform of the external field). Especially, the infrared divergence for $q \to 0$ is present. The result for $m \to 0$ is given in (D5).

For soft emission of a skewon the amplitude is

$$\mathcal{M}_{\rm Br}^{\rm f\to fs} \sim \frac{e\sqrt{G}}{k^2}\epsilon^\sigma(\gamma^*)\bar{u}(p')\gamma_\sigma\gamma_5 u(p)\Big(\frac{p_\rho}{p\cdot q} - \frac{p'_\rho}{p'\cdot q}\Big)\eta^{\mu\nu\rho\lambda}q_\lambda\epsilon_{\mu\nu}(s)^* \ , \quad (53)$$

which has no infrared divergence as $q \to 0$. Thus soft *Bremsstrahlung* of skewons (even from astrophysical objects as energetic as supernovae) will be extremely weak compared to that of gravitons. Moreover, the unpolarised squared amplitude goes to zero for $m \to 0$, which can be seen from the expression given in the (D6).

4. Conclusion

In this article we have presented a version of gravity including torsion as a propagating field, the skewon. As shown in Section 2.3, the coupling of torsion to matter is obtained through a gauge principle. As a result, torsion is coupling to the well known spin tensor obtained from Noether's theorem for Lorentz invariance. Thus one obtains a completeness of the interactions in nature in the sense that the well known Noether quantities couple to some field: The currents couple to gauge fields, the energy-momentum tensor couples to the curvature (graviton) field, — and the spin tensor to the torsion (skewon) field. We considered the aestetically most attractive model where the torsion coupling function $\rho \equiv 1$, see Appendix A. This implies that the skewon couples to spin with the same coupling constant as the graviton couples to energy-momentum.

We have explored some consequences of the inclusion of the propagating torsion field at microscopic (*i.e.* elementary particle) level. Torsion obviously modifies the gravitational forces between fermions, and emission of skewons certainly will occur. But to observe significant effects due to exchange or emission of skewons seems hopeless because the effects are extremely tiny and will be overwhelmed by electromagnetic (and other) interactions. Furthermore, we have shown that emission of skewons are rather unimportant compared to graviton emission — in the high energy-limit the relative probability is typically going as m^2/E^2, where m and E are the mass and energy of the radiating fermion, respectively.

We thank F. Ravndal and R. Stabell for useful comments.

Appendix A

Generalised affine geometry

It was noted by Trautman [80] that the metric tensor may be interpreted as a Higgs field breaking the symmetry from $GL(4,R)$ to the homogeneous Lorentz group $O(1,3)$. However, it is generally recognized that the total symmetry group of a gauge theory of gravity should at least encompass the Poincaré group $P(4) = O(1,3) \circledS R^4$. It is therefore natural to consider the simplest extension of $GL(4,R)$ which contains $P(4)$ as a subgroup, as the gauge group of gravitation [28]. Thus one is lead to the group $A(4) = GL(4,R) \circledS R^4$, with a corresponding extension from the bundle of linear frames, $L(M)$, over the manifold, M, to the bundle of affine frames $A(M)$ as the principal bundle in which the connection is defined.

Points in $L(M)$ are linear frames. Hence, $u \in L(M)$ means that $u = (p, e_\alpha)$; u is the linear frame, $e_\alpha = e^i{}_\alpha \partial_i$, at the point, $p = \pi(u)$, where $\pi : L(M) \to M$ is a projection map. A point in $A(M)$ consists of an element of the bundle of linear frames, $u \in L(M)$, together with a vector v at $\pi(u)$. Thus $w \in A(M)$ means $w = (p, e_\alpha, v)$. Therefore the bundle of linear frames $L(M)$ is the subset of $A(M)$ consisting of $(p, e_\alpha, 0)$.

A connection on $A(M)$ consists of a $GL(4)$ part $\Omega^\mu{}_\nu$ and a R^4-part ϕ^μ. The first part defines a linear connection on $L(M)$ and the second a tensorial 1-form on $L(M)$. The fundamental structure of an affin geometry is given by basis forms $\boldsymbol{\omega}^\mu$ and basis vectors $\boldsymbol{e}_\nu$ and the action of a covariant derivative on these quantities. The forms and vectors are related through the fundamental contraction $\langle \boldsymbol{\omega}^\mu, \boldsymbol{e}_\nu \rangle = \delta^\mu{}_\nu$. A connection is defined by the covariant derivative operator $\nabla_{\boldsymbol{u}}$. Let f and g be scalar functions, $\boldsymbol{t}$, $\boldsymbol{u}$, and $\boldsymbol{v}$ vectors and $\boldsymbol{\omega}$ a 1-form. Then the covariant derivative has the following defining properties

$$\nabla_{\boldsymbol{u}} f = \partial_{\boldsymbol{u}} f = \langle \boldsymbol{d}f, \boldsymbol{u} \rangle, \tag{A1}$$

$$\nabla_{\boldsymbol{u}} f\boldsymbol{v} = f \nabla_{\boldsymbol{u}} \boldsymbol{v} + \boldsymbol{v} \nabla_{\boldsymbol{u}} f, \tag{A2}$$

$$\nabla_{f\boldsymbol{t}+g\boldsymbol{u}} \boldsymbol{v} = f \nabla_{\boldsymbol{t}} \boldsymbol{v} + g \nabla_{\boldsymbol{u}} \boldsymbol{v}, \tag{A3}$$

$$\nabla_{\boldsymbol{t}} (\boldsymbol{u} \otimes \boldsymbol{v}) = (\nabla_{\boldsymbol{t}} \boldsymbol{u}) \otimes \boldsymbol{v} + \boldsymbol{u} \otimes (\nabla_{\boldsymbol{t}} \boldsymbol{v}), \tag{A4}$$

$$\nabla_{\boldsymbol{u}} \langle \boldsymbol{\omega}, \boldsymbol{v} \rangle = \langle \nabla_{\boldsymbol{u}} \boldsymbol{\omega}, \boldsymbol{v} \rangle + \langle \boldsymbol{\omega}, \nabla_{\boldsymbol{u}} \boldsymbol{v} \rangle. \tag{A5}$$

Due to the Leibnitz rule for the covariant derivative of the contraction (A5), the action of the covariant derivative on both basis forms and basis vectors are specified by the 64-component symbol $\Gamma^\alpha{}_{\beta\gamma}$. Thus

$$\nabla_\nu \boldsymbol{e}_\mu = \boldsymbol{e}_\alpha \Gamma^\alpha{}_{\mu\nu} \quad \text{and} \quad \nabla_\nu \boldsymbol{\omega}^\mu = -\boldsymbol{\omega}^\alpha \Gamma^\mu{}_{\alpha\nu}. \tag{A6}$$

Defining the structure coefficients $C^{\alpha}{}_{\beta\gamma}$ by the exterior derivative of the basis forms

$$d\omega^{\alpha} = -\frac{1}{2} C^{\alpha}{}_{\beta\gamma} \omega^{\beta} \wedge \omega^{\gamma}, \qquad \text{(A7)}$$

torsion is given by

$$T^{\alpha}{}_{\beta\gamma} \equiv \Gamma^{\alpha}{}_{\gamma\beta} - \Gamma^{\alpha}{}_{\beta\gamma} - C^{\alpha}{}_{\beta\gamma}. \qquad \text{(A8)}$$

The metric tensor is defined as the scalar product

$$g_{\alpha\beta} \equiv e_{\alpha} \cdot e_{\alpha}. \qquad \text{(A9)}$$

The non-metricity tensor, $Q_{\alpha\beta\gamma}$, is defined as the covariant derivative of the metric

$$Q_{\alpha\beta\gamma} \equiv g_{\alpha\beta|\gamma} \equiv \nabla_{\gamma} g_{\alpha\beta}. \qquad \text{(A10)}$$

So far we have defined the basic structure of the Riemann–Cartan–Weyl geometry. The generalised affine structure is specified by an additional $\binom{1}{1}$-tensor $\phi^{\alpha}{}_{\beta}$ which defines the R^4-part of the generalised affine connection:

$$\boldsymbol{\phi}^{\mu} = \phi^{\mu}{}_{\alpha} \boldsymbol{\omega}^{\alpha}. \qquad \text{(A11)}$$

Just as the covariant exterior derivative of the basis form defines the torsion form,

$$\boldsymbol{T}^{\mu} \equiv \mathrm{D}\boldsymbol{\omega}^{\mu}, \qquad \text{(A12)}$$

one may define a generalised torsion form, $\boldsymbol{\Phi}^{\mu}$, by

$$\boldsymbol{\Phi}^{\mu} \equiv \mathrm{D}\boldsymbol{\phi}^{\mu}. \qquad \text{(A13)}$$

In general the R^4-valued 1-form $\boldsymbol{\phi}^{\mu}$ may be written [28]

$$\boldsymbol{\phi}^{\mu} = \rho \boldsymbol{\omega}^{\mu} + \boldsymbol{\tau}^{\mu}. \qquad \text{(A14)}$$

The scalar function ρ is called the torsion coupling function. In the literature one has normally choosen

$$\rho = 1 \quad \text{and} \quad \boldsymbol{\tau}^{\mu} = 0, \qquad \text{(A15)}$$

so that the R^4 potentials, $\boldsymbol{\phi}^{\mu}$, reduces to the basis forms, and the R^4 curvature, $\boldsymbol{\Phi}^{\mu}$, to the torsion 2-form, $\boldsymbol{T}^{\mu}$. This assumption appears to be too restrictive. A minimal generalisation would be[9]

$$\rho = 1 \quad \text{and} \quad \boldsymbol{\tau}^{\mu} \neq 0. \qquad \text{(A16)}$$

[9] A variable ρ behaves as a scalar potential for the torsion vector, which is identified as a Weyl gauge field [81].

It is now possible to identify the generalised affine connection with gauge potentials of gravitation. Thus the $GL(4)$ part $\boldsymbol{\Omega}^{\mu}{}_{\nu}$ corresponds to the 6 Lorentz rotational degrees of freedom, and $\boldsymbol{\phi}^{\mu}$ to the translational part. Note that both these terms are part of the generalised affine connection. In the conventional approach the translational potential $\boldsymbol{\omega}^{\mu}$ is only indirectly a part of the connection.

The corresponding gauge fields are the covariant exterior derivatives of the connection form, represented by the $GL(4)$ part $\boldsymbol{R}^{\mu}{}_{\nu} \equiv \mathbf{D}\boldsymbol{\Omega}^{\mu}{}_{\nu}$ and the R^4 part $\boldsymbol{\Phi}^{\mu} \equiv \mathbf{D}\boldsymbol{\phi}^{\mu}$. Geometrically $\boldsymbol{T}^{\mu}$ is directly related to the closure failure of parallelograms and therefore to a kind of *translation* [82]. In general the physical meaning of $\boldsymbol{\Phi}^{\mu}$ is less clear since it depends strongly on $\boldsymbol{\tau}^{\mu}$.

Among the generalised affine geometries there is one special class which generate linear connections on $L(M)$ such that autoparallels with respect to this connection are mapped into straight lines in the local affine space. This geometry is specified by an *invertible* $\phi^{\alpha}{}_{\beta}$, and a metric $g_{\alpha\beta}$ satisfying the generalised compatibility conditions [83]

$$Q_{\alpha\beta\gamma} = 0 \quad \text{and} \quad \phi_{\alpha\beta|\gamma} = \phi_{\alpha\rho}T^{\rho}{}_{\beta\gamma}. \tag{A17}$$

Zero non-metricity means that angles and lengths are preserved under parallel transport. The second condition implies that the generalised torsion form is very closely related to the space-time torsion

$$\boldsymbol{\Phi}^{\mu} = -\phi^{\mu}{}_{\nu}\boldsymbol{T}^{\nu}. \tag{A18}$$

The existence of an inverse $\hat{\phi}^{\alpha}{}_{\beta}$ of $\phi^{\alpha}{}_{\beta}$ is secured if

$$\phi^{\alpha}{}_{\beta} \equiv \exp\left(-a^{\alpha}{}_{\beta}\right). \tag{A19}$$

In this notation $\boldsymbol{\tau}^{\mu} = -a^{\mu}{}_{\nu}\boldsymbol{\omega}^{\nu}$ to first order in $a_{\mu\nu}$. If $a_{\alpha\beta}$ is an antisymmetric tensor, ϕ can be given a geometric interpretation as a rotational transformation of the frames. By allowing ϕ-transformations which include shear and dilatations, it is possible to construct a theory which takes into account traceless proper hypermomentum and intrinsic dilatation currents along with spin currents. These additional intrinsic currents have earlier been considered in connection with the so-called metric affine theory [84].

The second compatibility condition, which is identical to the one Einstein imposed on the fundamental tensor in the non-symmetric field theory [85,86], here takes the form of a defining relation for a torsion potential. Hence we find

$$a_{\alpha[\beta|\gamma]} = -T_{\alpha\beta\gamma} \quad \text{and} \quad a_{\alpha(\beta|\gamma)} = 0. \tag{A20}$$

Note that for an antisymmetric field $a_{\alpha\beta}$ these relations imply that $a_{\alpha\beta|\gamma} = a_{[\alpha\beta|\gamma]}$. Thus the torsion tensor is totally antisymmetric and autoparallels defined by the linear connection coincide with the geodesics.

Appendix B

Non-symmetric theory of gravitation

Having motivated the choice of underlying geometry, we have to choose an action integral. The simplest generalisation of the Einstein–Hilbert action is implied if we in the Einstein–Hilbert action simply replace the Christoffel connection with the more general connection with torsion. In this case the action is given by the Riemann–Cartan curvature scalar [87]

$$R(\Gamma) \simeq R(\{\ \}_g) - \frac{1}{4} a_{\alpha\beta|\gamma} a^{[\alpha\beta|\gamma]} + \lambda_{\alpha\beta\gamma} a^{\alpha\beta|\gamma}. \tag{B1}$$

Here the first term is the usual Einstein–Hilbert term, whereas the second, the totally antisymmetric torsion term, is of the Kalb–Ramond type [88], which is expected to appear in string-modified four-dimensional gravitation theory [89,90]. $\lambda_{\alpha\beta\gamma} \equiv \lambda_{\alpha(\beta\gamma)}$ is a Lagrange multiplier corresponding to the constraint $a_{\alpha(\beta|\gamma)} = 0$. Here we have given formal geometric arguments (straight lines of the affine space are mapped to autoparallels of the linear space) in favour of this constraint, but other theoretical considerations also lead to a totally antisymmetric torsion field [91]. *Post priori* this restriction is acceptable because Dirac particles can only interact with the totally antisymmetric part of torsion [25], and these particles seem to be fundamental building blocks of matter.

An asymmetric generalisation of the metric tensor was first introduced in the unified field theory of Einstein and Straus [85,86]. Instead of identifying its antisymmetric part with electromagnetism, Moffat has interpreted it as an *antisymmetric gravitational field* of a Non-symmetric Gravitational Theory [95,96]. Such fields have also been considered in connection with an invariance requirement under extended (gauge) transformations of the Poincaré group [47,92-94]. Finally antisymmetric torsion potentials have been introduced *ad hoc* in the Riemann–Cartan geometry to make torsion propagate in vacuum [97-101]. The geometric approach followed here has the advantage that the ratio of gravitational and inertial mass most naturally[10]

[10] Choosing a torsion coupling scalar $\rho \neq 1$ would lead to a different coupling of the torsion potential, but the value $\rho = 1$ is singled out because only in this case the ϕ-transformation (A11) is a pure rotation.

is the same as the ratio of gravitational and inertial spin. Hence, the coupling strength of torsion is the same as the Einstein–Newton coupling. Moreover, the structure of torsion is uniquely specified by the geometry.

The static spherically symmetric vacuum solution of this theory is equal to the Schwarzschild solution up to second order modulo an unobservable rescaling of gravitational mass [87]. Thus the theory passes all solar system tests [74].

As a redefinition of the fundamental torsion potential we introduce the antisymmetric field $s_{\alpha\beta}$ satisfying

$$3s_{[\alpha\beta|\gamma]} \equiv a_{\alpha\beta|\gamma} = -T_{\alpha\beta\gamma}, \tag{B2}$$

without any constraint on $s_{\alpha(\beta|\gamma)}$. Then we get

$$R(\Gamma) \simeq R(\{\,\}_g) - \frac{9}{4} s_{\alpha\beta|\gamma} s^{[\alpha\beta|\gamma]}. \tag{B3}$$

We now consider curvature and torsion perturbations of Minkowski space-time: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ where $| h_{\mu\nu} | \ll 1$ and $| s_{\mu\nu} | \ll 1$. Here $h_{\mu\nu}$ is a Riemannian metric perturbation (graviton), and $s_{\mu\nu}$ is an antisymmetric torsion potential field. In accordance with earlier practice [102-104] in connection with antisymmetric gravitational fields, we call the particle associated with the skew field the *skewon.*

For convenience we introduce the trace inverse of the graviton field

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h. \tag{B4}$$

This leads to the action

$$L_G = -\frac{1}{4}\bar{h}_{\nu\beta,\alpha}\bar{h}^{\nu\beta,\alpha} + \frac{1}{8}\bar{h}_{,\alpha}\bar{h}^{,\alpha} + \frac{1}{2}\bar{h}_{\mu\alpha}{}^{,\alpha}\bar{h}^{\mu\beta}{}_{,\beta} - \frac{9}{4}s_{\alpha\beta,\gamma}s^{[\alpha\beta,\gamma]}. \tag{B5}$$

This action is identical to the second order action [104] of AHG (Algebraically extended Hilbert Gravity) [105]. The full nonlinear theory represented by the action (B1) is, however, inequivalent to AHG. In the nonlinear regime the two theories have different couplings between the symmetric and antisymmetric fields. In the present theory this coupling is responsible for a rescaling of the effective Newtonian mass of the static spherically symmetric vacuum geometry [87]. Since the graviton-skewon terms are of minimum third order in the fields, the symmetric and antisymmetric fields decouple in the linearised theory. The terms depending on h and ∂h in the weak field action (B5) correspond to the Einstein–Hilbert action. It is invariant under the gauge transformation

$$h_{\mu\nu} \rightarrow h'_{\mu\nu} = h_{\mu\nu} + \partial_{(\mu}X_{\nu)}, \tag{B6}$$

whereas the Kalb–Ramond (torsion) field $T_{\alpha\beta\gamma} = -3s_{[\alpha\beta,\gamma]}$ is invariant under

$$s_{\mu\nu} \to s'_{\mu\nu} = s_{\mu\nu} + \partial_{[\mu}Y_{\nu]}. \tag{B7}$$

This gauge invariance has been associated with the interaction between closed strings [88]. With the sources found in Section 2, the classical linearised field equations take the simple form

$$\Box \bar{h}_{\mu\nu} = -16\pi G\Sigma_{(\mu\nu)}, \qquad \bar{h}^{\mu\nu}{}_{,\nu} = 0, \tag{B8}$$

$$\Box s_{\mu\nu} = -16\pi G t^{\alpha}{}_{\mu\nu,\alpha}, \qquad s^{\mu\nu}{}_{,\nu} = 0. \tag{B9}$$

These are the Papapetrou [3] field equations.

Appendix C

The strength of the field equations

In general a set of field equations does not determine the field completely. Einstein [86] introduced the concept of *strength* of field equations to give a quantitative measure of the number of free data consistent with the system of field equations.

For reviews of the strength concept see Mariwalla [106], Shutz [107] and Hoenselaers [108].

The concept of *strength* is of such generality that it is possible to compare vastly different systems. In four space-time dimensions it turns out that the complex Klein–Gordon field, Maxwell's field, the massless Dirac field, as well as the vacuum gravitational field of Einstein's General Relativity all satisfy field equations of the same strength [106]. Since the *strength* is directly related to the number of dynamical degrees of freedom [107], it is not surprising that the above mentioned massless field equations have the same strength.

Suppose that we have a set of fields satisfying a system of differential equations which are analytic in some neighbourhood of some point on a manifold (here we assume four dimensions). Then the field equations imply certain relations among the various coefficients of order n. By subtraction there remains a number $Z(n)$ of free coefficients. This number is a measure of *compatibility* and *strength* according to Einstein.

The number of coefficients in the nth order Taylor expansion in d dimensions is

$$\begin{bmatrix} d \\ n \end{bmatrix} = \binom{n+d-1}{n}. \tag{C1}$$

Subtracting the number of independent restrictions imposed by the field equations, one finds an expression for the number of free coefficients. When the system is invariant under gauge-transformations, one also has to subtract unphysical degrees of freedom. In general the expression for free Taylor coefficients will be of the form

$$Z(n) \equiv \begin{bmatrix} d \\ n \end{bmatrix} \left(Z_0 + \frac{Z_1}{n} + \frac{Z_2}{n^2} + \cdots \right). \tag{C2}$$

If $Z_0 \geq 0$ the system is called *absolutely compatible.* Then if $Z_0 = 0$, the parameter Z_1 is called the *coefficient of freedom.* The system is stronger the smaller the value of this integer.

One may also express $Z(n)$ as [107]

$$Z(n) \equiv \sum_{k=1}^{4} N_k \begin{bmatrix} k \\ n \end{bmatrix}. \tag{C3}$$

In this representation $N_4 \geq 0$ implies *absolute compatibility.* For all systems of equations normally used in physics $N_4 = 0$: There is no free function of four variables in the solution. N_3, which corresponds to the coefficient of freedom, Z_1, is the number of free variables on a three dimensional hypersurface. In both Maxwell's and Einstein's vacuum equations this number is four. This corresponds to the free choice of two variables and their time derivatives on a Cauchy surface.

The physical meaning of the one- and two-dimensional coefficients N_1 and N_2 is not clear. Consider now the free skewon field. In the field representation (here the physical measureable field is the torsion axial vector), there are 4 free fields. These are restricted by 7 first order field equations [88]. Among these equations there are 4 identities of second order. These identities are also not all independent because there is one third order identity of the identities.

This gives

$$Z(n)_{\text{Skewon}} = 4 \begin{bmatrix} 4 \\ n \end{bmatrix} - 7 \begin{bmatrix} 4 \\ n-1 \end{bmatrix} + 4 \begin{bmatrix} 4 \\ n-2 \end{bmatrix} - \begin{bmatrix} 4 \\ n-3 \end{bmatrix}. \tag{C4}$$

Using the basic relations [107]

$$\begin{bmatrix} d \\ n+1 \end{bmatrix} = \begin{bmatrix} d \\ n \end{bmatrix} + \begin{bmatrix} d-1 \\ n+1 \end{bmatrix} \tag{C5}$$

and

$$\begin{bmatrix} d \\ n-1 \end{bmatrix} = \begin{bmatrix} d \\ n \end{bmatrix} - \begin{bmatrix} d-1 \\ n \end{bmatrix} \tag{C6}$$

the function $Z(n)$ for the free skewon field may be written

$$Z(n)_{\text{Skewon}} = 2\begin{bmatrix}3\\n\end{bmatrix} + \begin{bmatrix}2\\n\end{bmatrix} + \begin{bmatrix}1\\n\end{bmatrix} = (n+2)^2. \tag{C7}$$

The result, $N_4 = 0$ and $N_3 = 2$, shows that the free skewon field has only one degree of freedom.

This result should be compared with the Maxwell and Einstein vacuum equations. In the physical field representation these theories give [107]

$$Z(n)_{\text{Maxwell}} = 4\begin{bmatrix}3\\n\end{bmatrix} + 2\begin{bmatrix}2\\n\end{bmatrix} = 2(n+1)(n+3), \tag{C8}$$

$$Z(n)_{\text{Einstein}} = 4\begin{bmatrix}3\\n\end{bmatrix} + 6\begin{bmatrix}2\\n\end{bmatrix} = 2(n+1)(n+5). \tag{C9}$$

All coefficients N_k are independent of the use of potential formulation, and therefore also the lower dimensional coefficients appear to contain some real information about the structure of these theories. The skewon field differs from the other two in having a nonzero one-dimensional coefficient.

An interesting alternative to the theory presented here is the *Poincaré gauge theory* (PGT). PGT is a U_4-theory obtained from a higher order curvature action with the torsion field regarded as a fundamental field [48,109]. Sué and Mielke [110] have calculated the strength of the general PGT vacuum equations. They found $Z_1 = 120$, showing that the field equations of PGT are much weaker than those of the present theory which has $Z_1 = 18$.

Appendix D

Squared amplitudes

The squared unpolarised fermion fermion amplitude is

$$S_{(g+s)} = \left(\frac{\lambda^2}{128t}\right)^2 \{a^2(2s+t)^2[(2s+t)^2+t^2] + 2ab(2s+t)^2[(2s+t)^2-t^2] + b^2[(2s+t)^2-t^2]^2\}\,, \tag{D1}$$

where $s=(p+k)^2$ and $t=(p-p')^2$ as usual. For graviton exchange only, $a=b=1$; for skewon exchange only, $a=-b=1$; and for graviton plus skewon exchange, $a=2, b=0$. Note that the expression (D1) for pure skewon exchange has no term $\sim t^{-2}$, in agreement with (43).

The interference between photon exchange and graviton plus skewon exchange is

$$S_{\text{Int}} = 2\mathcal{M}_\gamma\mathcal{M}^*_{(g+s)} = \frac{e^2\lambda^2}{64t^2}(2s+t)\{a[(2s+t)^2+t^2]+b[(2s+t)^2-t^2]\}, \tag{D2}$$

where a and b are defined as above.

For $\gamma\mathrm{f} \to \mathrm{fg}$ we find

$$S^{\gamma\mathrm{f}\to\mathrm{fg}} = -(\lambda e)^2 \Big\{ 4m^6 \left[\frac{1}{y} + \frac{1}{z}\right]^2 + m^2 \left[\frac{y}{z} + \frac{z}{y} - 8\right] + 3\left[y + z - 2\frac{yz}{y+z}\right]\Big\}, \qquad (D3)$$

where $z = (p+k)^2 - m^2$, and $y = (p-q)^2 - m^2$. The corresponding quantity for $\gamma\mathrm{f} \to \mathrm{fs}$ is

$$S^{\gamma\mathrm{f}\to\mathrm{fs}} = -\frac{(\lambda e)^2}{2} m^2 \left[\frac{y}{z} + \frac{z}{y} - 8\right]. \qquad (\mathrm{D4})$$

For graviton *Bremsstrahlung* from a fermion in an external field we obtain for $m \to 0$.

$$\begin{aligned} S_{\mathrm{Br}}^{\mathrm{f}\to\mathrm{fg}} = \left(\frac{\lambda e}{z+y}\right)^2 \{ & 2(p\cdot X)^2[13t^2 - 17ty - 3tz + 4y^2] \\ & + 2(p'\cdot X)^2[13t^2 - 17tz - 3ty + 4z^2] \\ & + 4(p\cdot X)(p'\cdot X)[10t^2 - 3ty - 3tz + 4yz] \\ & + \frac{1}{2}X^2[46t^3 - 72t^2(y+z) + 29t(y^2+z^2) \\ & - 3(y^3+z^3) - 11yz(y+z) + 60tyz]\}, \end{aligned} \qquad (\mathrm{D5})$$

where $t = k^2$ and $X = \epsilon(\gamma^*)/t$. Note that (D5) is symmetric under the replacements $y \leftrightarrow z$ & $(p\cdot X) \leftrightarrow (p'\cdot X)$. $t = k^2$ and $X = \epsilon(\gamma^*)/t$. The corresponding result for skewon *Bremsstrahlung* is to order m^2:

$$S_{\mathrm{Br}}^{\mathrm{f}\to\mathrm{fs}} = -\frac{(\lambda e m)^2}{4yz}\{4t[(p\cdot X)^2 - (p'\cdot X)^2] + X^2(y+z)^2\}. \qquad (\mathrm{D6})$$

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