# THE CLASSIFICATION OF HOMOGENEOUS AND SYMMETRIC CELLULAR AUTOMATA* **

DANUTA MAKOWIEC

Institute of Theoretical Physics and Astrophysics, Gdańsk University,
Wita Stwosza 57, 80-952 Gdańsk, Poland

The review of author's results of computer simulation on homogeneous and symmetric with respect to up-down symmetry, two dimensional cellular automata is given. The classification problem of the cellular automata is revised *via* the distribution function of the probability to find any neighbourhood on a lattice. The intrinsic structure of a rule has been introduced to explain the results obtained.

PACS numbers: 05.40.+j, 05.50.+q

## 1. Introduction

Cellular automata (CA) are the simplest models of nonlinear dynamical systems. They attract the scientific attention becauce their evolution can be observed in computer step-by-step simulations. There exist some distinct approaches and many applications of the model [1,2 and references quoted therein]. The purpose of this poster is to present one of the possible ways of understanding the dynamics of CA. The poster sumarizes the results of the author's papers on homogeneous and symmetric CA [3,4,5].

Generally, in CA each site of a large lattice $[\sigma] = \{\sigma_i\}$ carries one spin $\sigma_i$, pointing either up $= 1$ or down $= 0$. The orientation of any spin at time $t+1$ is determinded completely by its neighbourhood at time $t$. In this paper I consider a square lattice with dimension $L$ and only nearest - neighbours interactions are taken into account. Thus, the state of any $\sigma_i$ at time $t+1$ is fixed exactly by the following neighbourhood:

* Presented at the IV Symposium on Statistical Physics, Zakopane, Poland, September 19–29, 1991.

** Work partially supported by Polish Ministry of National Education.

$\Theta_i(t) = (E_i(t), N_i(t), W_i(t), S_i(t)),$

$$\begin{array}{ccccc} * & \dots & N_i & \dots & * \\ \vdots & & \vdots & & \vdots \\ W_i & \dots & \sigma_i & \dots & E_i \\ \vdots & & \vdots & & \vdots \\ * & \dots & S_i & \dots & * \end{array} \tag{1.1}$$

in the following sense:

$$\sigma_i(t+1) = r(\Theta_i(t)), \tag{1.2}$$

where $r \in R$ is a rule which gives a method of finding the new spin value.

Let me restrict the set of rules $R$ to the rules which do not depend on a lattice site (*homogeneous CA*). Since there are the great numbers of both possible initial states of a lattice and homogeneous rules, the examination of CA are aimed at dividing the huge problem into smaller parts. The observation that after many time steps the evolution of CA stabilizes, is the easiest way of the selection. The Wolfram-type classifications [1,2] group different CA according to the final patterns. The number of CA attracted to the same pattern gives the next possibility to go through the problem of the classification, [6]. The mean field theory approach to the classification has been proposed also, [7].

The rules leading to stabilization of CA can be called *dissipative.* The stabilization of CA can occur in one of the following ways:

- the whole pattern is shifted in one direction; a rule works as a translation by 1 lattice site. This can be called *fixed point stabilization.*
- the whole pattern is shifted in one direction together with flipping all spins; a rule works as as the composition of a translation and a spin value conjugation. This is the - *oscillating fixed point* class.
- the spin configurations are repeated periodically and during one time period the pattern is shifted by a spatial period in one direction; a rule works as a periodic transformation which after some (often $L/2$, or $L$ ) steps combine to the translation - *limit cycle.*

Let me consider a subclass of rules, $r_s \in R_S$, symmetric with respect to up-down symmetry:

$$r_s(-\Theta_i(t)) = 1 - r_s(\Theta_i(t)) \tag{1.3}$$

where $-\Theta_i(t) = (1 - E_i(t), 1 - N_i(t), 1 - W_i(t), 1 - S_i(t))$.

Notice that for any rule $r_s$ there exists in $R_S$ an *Anti-Rule*, $r_s^A$ defined by the following relation: $r_s^A(\Theta_i(t)) = 1 - r_s(\Theta_i(t))$, and any CA $[\sigma^A]$ governed by $r_s^A$ evolves in such a way that after each 2 steps both patterns $[\sigma]$ and $[\sigma^A]$ are the same. Hence, if a rule stabilizes the system as a fixed point, then the corresponding *Anti-Rule* will lead the system to the oscilating fixed point, and reversally (see [4] for details).

In the whole set $2^{16} = 65\,536$ of all homogeneous rules on a square lattice, the percentages for rules stabilizing as a fixed or oscillating point (classes 0 to 4 by [2]) are: 5.7 and 2.8, respectively. The class of symmetric rules contains of $2^8 = 256$ ones, which stands only for 0.39% of all rules. But among them 136 rules always reach the stabilization and there is a suspicion that only because of the short time of observation the stabilization of further 48 ones does not always appear.

There exist four patterns that play the significant role among all final states obtained in the evolution of symmetric cellular automata. These are as follows:

$$\text{cluster-board} \qquad \begin{matrix} 0000 \\ 0000 \\ 0000 \\ 0000 \end{matrix} \quad \text{or} \quad \begin{matrix} 1111 \\ 1111 \\ 1111 \\ 1111 \end{matrix} \tag{1.4}$$

$$\text{chess-board} \qquad \begin{matrix} 0101 \\ 1010 \\ 0101 \\ 1010 \end{matrix} \tag{1.5}$$

$$\text{line-board} \qquad \begin{matrix} 0000 \\ 1111 \\ 0000 \\ 1111 \end{matrix} \quad \text{or} \quad \begin{matrix} 1010 \\ 1010 \\ 1010 \\ 1010 \end{matrix} \tag{1.6}$$

$$\text{pair-board} \qquad \begin{matrix} 0011 \\ 0110 \\ 1100 \\ 1001 \end{matrix} \quad \text{or} \quad \begin{matrix} 0011 \\ 1001 \\ 1100 \\ 0110 \end{matrix} \tag{1.7}$$

Their meaning follows from the fact that all of them are stable points of the evolution equation (1.2). It means that if one considers any symmetric rule $r_s \in R_S$ and the lattice $[\sigma_0]$ is in one of (1.4), (1.5), (1.6) or (1.7) shapes, then $[\sigma_0] \equiv r_s[\sigma_0]$ where the equivalence is up to a translation or a translation + conjugation. But starting from any random state of the lattice only a few rules lead to the one of above patterns. The above given patterns have one common feature: at least two neighbourhoods result in building them. Hence rules can be uniquely characterized by the function which point out the resulting neighbourhoods.

This paper is to give the computer results of simmulations of CA with all symmetric rules. In Section 2 one can find the comparision between the old description of CA by the functions known as *Magnetization* and *Activity*, and the new one which is based on the distribution of neighbourhoods. In Section 3 there is given an explanation to the obtained results. Because of the unique correspondence between stabilizing rules and their distributions, our propositions can be considered as powerful tool in examinations of CA.

## 2. Simulation results

There are possible 16 different configurations of four nearest - neighbours on the square lattice. They can be divided with respect to up-down symmetry into 8 pairs. Since we consider symmetric rules only, we can take into account one representative of each pair. Fig. 1 presents 8 configurations considered. One can assign a figure $n$ ranging from 7 to 0 to the configuration as indicated in Fig. 1.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | | | 0 | | | 1 | | | 0 | | | 0 | |
| 0 | | 0 | 0 | | 1 | 0 | | 0 | 1 | | 0 | 0 | | 0 |
| | 0 | | | 0 | | | 0 | | | 0 | | | 1 | |
| | (7) | | | (6) | | | (5) | | | (4) | | | (3) | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | | | 0 | | | 0 | |
| 0 | | 1 | 1 | | 1 | 0 | | 1 |
| | 0 | | | 0 | | | 1 | |
| | (2) | | | (1) | | | (0) | |

Fig. 1. Configurations and their numbers.

Any rule can be defined as the sequence of up and down spin states which are taken by the central spin if the corresponding configuration surrounds the spin. So, each symmetric rule $r_s$ can be identify by the following number:

$$r_s = \sum_{n=0}^{7} \sigma_i(n)\, 2^n. \tag{2.1}$$

Notice that an *Anti-Rule* number, $r_s^A$, has the property $r_s^A = 255 - r_s$.

The property (1.3) allows us to decrease the total number of considered rules by half. Moreover, symmetries of the lattice (rotational and mirror symetries), reduce it to 24 ones. All 24 representantatives considered are listed in the first column of Table I. The first 12 rules of the table always stabilize CA. The next 5 sometimes can stabilize the system but for the last 7 rules stabilization of CA never occurs [4].

One can characterize the final lattice state by the following functions [3,4]:

*Magnetization* - the number of spins being in up state. This is the mean field characterization of CA [6].

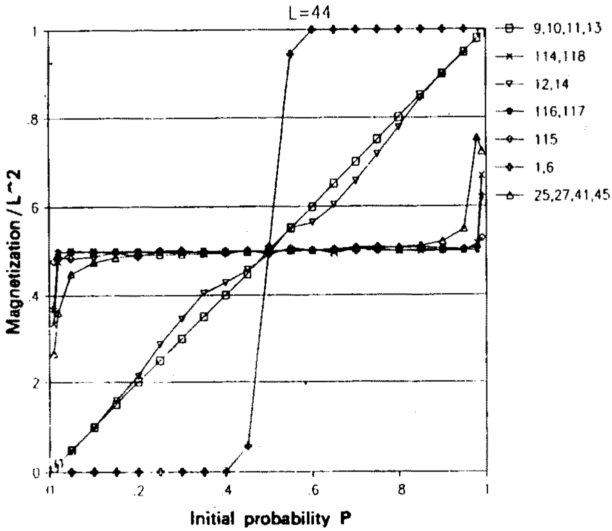


**Fig. 2.**

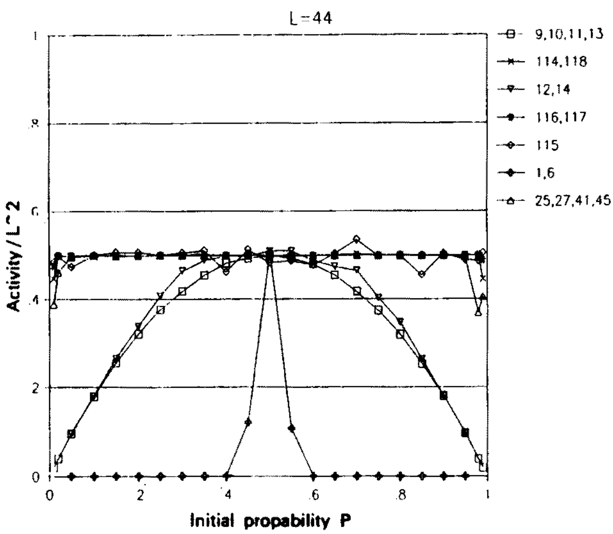


**Fig. 3.**

*Activity* - the number of spins changing theirs states in the last time step.
They both can depend on $P$ — the probability of a single spin to point up in the initial random state. In Figs 2,3 the $P$-dependence is presented for stabilizing rules. *Magnetization* and *Activity* of non-stabilizing rules are independent of $P$. The presence or not of the dependency on $P$ divide the

TABLE I

The distributions of configurations $7, \cdots, 0$ in final patterns for all symmetric rules. $L = 44$ and $P = 0.5$, both probabilities and STD-errors are expressed in per cent. The number of CA represented by each rule is given in the last column.

| Conf.No Rule No | $p(7)$ | $p(6)$ | $p(5)$ | $p(4)$ | Numbers of rules |
|---|---|---|---|---|---|
| 1 | 47.9 ± 25.0 | 0.23 ± 0.26 | 0.18 ± 0.45 | 0.22 ± 0.26 | 8 |
| 6 | 47.0 ± 21.0 | 1.37 ± 0.93 | 0.01 ± 0.04 | 1.37 ± 0.93 | 8 |
| 9 | 6.3 ± 0.60 | 6.2 ± 0.37 | 6.2 ± 0.44 | 6.3 ± 0.37 | 8 |
| 10 | 12.8 ± 1.6 | 5.9 ± 0.40 | 6.0 ± 0.41 | 5.9 ± 0.34 | 16 |
| 11 | 8.9 ± 0.96 | 4.8 ± 0.41 | 8.8 ± 0.39 | 4.8 ± 0.30 | 8 |
| 13 | 7.0 ± 0.58 | 8.9 ± 0.42 | 4.7 ± 0.34 | 9.0 ± 0.48 | 16 |
| 25 | 0.78 ± 1.1 | 0.28 ± 0.59 | 0.25 ± 0.22 | 0.28 ± 0.59 | 16 |
| 27 | 1.4 ± 1.0 | 0.03 ± 0.08 | 0.80 ± 0.69 | 0.03 ± 0.08 | 16 |
| 41 | 0.00 ± 0.01 | 0.22 ± 0.21 | 0.05 ± 0.23 | 0.23 ± 0.22 | 8 |
| 45 | 0.00 ± 0.00 | 1.2 ± 1.4 | 0.00 ± 0.00 | 1.2 ± 1.4 | 8 |
| 114 | 0.12 ± 0.56 | 6.2 ± 0.65 | 6.1 ± 0.42 | 6.2 ± 0.68 | 16 |
| 118 | 0.00 ± 0.00 | 4.7 ± 0.37 | 8.9 ± 0.50 | 4.7 ± 0.47 | 8 |
| 12 | 11.9 ± 2.9 | 11.9 ± 2.1 | 0.53 ± 0.21 | 11.9 ± 2.1 | 8 |
| 14 | 30.5 ± 12.0 | 6.3 ± 1.3 | 3.5 ± 1.4 | 6.2 ± 1.3 | 8 |
| 115 | 0.02 ± 0.04 | 5.9 ± 1.9 | 3.3 ± 0.73 | 5.9 ± 1.9 | 8 |
| 116 | 0.56 ± 0.17 | 0.51 ± 0.14 | 12.2 ± 0.8 | 0.54 ± 0.18 | 8 |
| 117 | 0.12 ± 0.15 | 3.3 ± 0.8 | 6.0 ± 0.8 | 3.4 ± 0.9 | 16 |
| 42 | 9.7 ± 1.9 | 7.7 ± 0.67 | 4.7 ± 0.50 | 7.6 ± 0.62 | 8 |
| 43 | 5.9 ± 0.63 | 6.0 ± 0.47 | 5.9 ± 0.59 | 6.0 ± 0.33 | 8 |
| 49 | 5.5 ± 0.89 | 7.7 ± 0.51 | 4.7 ± 0.38 | 7.6 ± 0.44 | 16 |
| 51 | 6.2 ± 0.9 | 6.3 ± 0.5 | 6.0 ± 0.3 | 6.2 ± 0.5 | 16 |
| 113 | 6.4 ± 0.81 | 6.3 ± 0.45 | 6.3 ± 0.52 | 6.3 ± 0.55 | 8 |
| 121 | 5.0 ± 0.67 | 4.7 ± 0.37 | 7.3 ± 0.50 | 4.6 ± 0.44 | 8 |
| 124 | 6.2 ± 0.60 | 6.2 ± 0.48 | 6.2 ± 0.33 | 6.3 ± 0.42 | 8 |

TABLE I (continued)

| Conf. No Rule No | $p(3)$ | $p(2)$ | $p(1)$ | $p(0)$ | Number of rules |
|---|---|---|---|---|---|
| 1 | 0.18 ± 0.45 | 0.63 ± 0.60 | 0.00 ± 0.02 | 0.68 ± 0.69 | 8 |
| 6 | 0.01 ± 0.03 | 0.08 ± 0.27 | 0.00 ± 0.00 | 0.09 ± 0.29 | 8 |
| 9 | 6.3 ± 0.38 | 6.4 ± 0.57 | 6.3 ± 0.55 | 6.2 ± 0.54 | 8 |
| 10 | 6.0 ± 0.33 | 12.8 ± 0.83 | 0.00 ± 0.00 | 0.00 ± 0.00 | 16 |
| 11 | 8.9 ± 0.42 | 7.2 ± 0.51 | 0.00 ± 0.00 | 6.8 ± 0.56 | 8 |
| 13 | 4.7 ± 0.36 | 0.00 ± 0.00 | 6.9 ± 0.39 | 8.8 ± 0.53 | 16 |
| 25 | 0.25 ± 0.22 | 47.5 ± 1.6 | 0.66 ± 0.60 | 0.00 ± 0.01 | 16 |

TABLE I (continued)

| Conf. No<br>Rule No | $p(3)$ | $p(2)$ | $p(1)$ | $p(0)$ | Number of rules |
|---|---|---|---|---|---|
| 27 | $0.80 \pm 0.69$ | $47.5 \pm 1.6$ | $0.34 \pm 0.65$ | $0.00 \pm 0.00$ | 16 |
| 41 | $0.05 \pm 0.23$ | $0.71 \pm 0.56$ | $47.8 \pm 1.6$ | $0.81 \pm 0.62$ | 8 |
| 45 | $0.01 \pm 0.02$ | $0.28 \pm 1.0$ | $47.7 \pm 2.1$ | $0.05 \pm 0.23$ | 8 |
| 114 | $6.1 \pm 0.30$ | $0.11 \pm 0.55$ | $12.5 \pm 1.1$ | $12.6 \pm 1.1$ | 16 |
| 118 | $8.8 \pm 0.54$ | $7.0 \pm 0.60$ | $8.8 \pm 0.54$ | $7.0 \pm 0.36$ | 8 |
| 12 | $0.53 \pm 0.21$ | $0.52 \pm 0.16$ | $12.3 \pm 2.4$ | $0.52 \pm 0.16$ | 8 |
| 14 | $3.5 \pm 1.4$ | $0.12 \pm 0.14$ | $0.02 \pm 0.04$ | $0.10 \pm 0.12$ | 8 |
| 115 | $3.3 \pm 0.71$ | $0.12 \pm 0.12$ | $30.9 \pm 2.7$ | $0.13 \pm 0.13$ | 8 |
| 116 | $12.2 \pm 0.7$ | $11.7 \pm 1.5$ | $0.53 \pm 0.16$ | $11.8 \pm 1.3$ | 8 |
| 117 | $6.1 \pm 0.8$ | $30.7 \pm 1.9$ | $0.15 \pm 0.13$ | $0.04 \pm 0.08$ | 16 |
| 42 | $4.7 \pm 0.34$ | $5.3 \pm 0.52$ | $4.9 \pm 0.52$ | $5.4 \pm 0.53$ | 8 |
| 43 | $6.0 \pm 0.47$ | $6.3 \pm 0.55$ | $6.5 \pm 0.37$ | $6.2 \pm 0.58$ | 8 |
| 49 | $4.7 \pm 0.39$ | $9.3 \pm 1.1$ | $5.4 \pm 0.45$ | $5.5 \pm 0.37$ | 16 |
| 51 | $6.1 \pm 0.4$ | $6.5 \pm 0.5$ | $6.0 \pm 0.5$ | $6.1 \pm 0.6$ | 16 |
| 113 | $6.3 \pm 0.47$ | $6.1 \pm 0.67$ | $6.3 \pm 0.44$ | $6.3 \pm 0.57$ | 8 |
| 121 | $7.4 \pm 0.58$ | $5.3 \pm 0.77$ | $9.8 \pm 1.4$ | $5.2 \pm 0.36$ | 8 |
| 124 | $6.1 \pm 0.37$ | $6.2 \pm 0.56$ | $6.3 \pm 0.38$ | $6.3 \pm 0.45$ | 8 |

whole set of symmetric rules. But the above defined functions do not answer the questions about leading or not to the stabilization of CA.

The results of computer simulations on pattern distribution together with their *STD-errors* are collected in Table I (see [5] for the description of the computer experiments). One can see that there is a unique correspondence between the distribution of configurations in final patterns and dissipative rules. Because of the range of values of distribution functions, it is easy to perform a general division of the set of dissipative rules into two subsets: *rules leading to the distributions of neighbourhoods with sharp peaks:* 1, 6, 25, 27, 41, 45;
*rules leading to the distributions of neighbourhoods with strong zeros:* 9, 10, 11, 13, 114, 118, 12, 14, 115, 116, 117.
The role of rule 9 will be explained in the next section.

Since *STD- errors* are small, this characterization can be extended to the whole set of symmetric rules. The general difference between the stabilizing and non- stabilizing cases consists in the fact that there are neither peaks nor zeros in the distributions of non- stable automata.

One can check in computer simmulations that the properties of the sharp peaks or the strong zeros are independent of $P$ (see [5]).

Notice, that *Magnetization* can be easy expressed by the probabilities

$p^r_P(i)$ of finding the $i$ neighbourhood in the final pattern:

$$M([\sigma]) = \frac{L^2}{4} \sum_{i=0}^{15} a_i p^r_P(i), \tag{2.2}$$

where $a_i$ is the number of up states in the $i$ configuration, $i = 0, \cdots, 15$. Since the values of probabilities can depend on $P$, the index $P$ is introduced to notation of $p^r_P(i)$. However, in case $P = 0.5$ the probabilities of symmetric with respect to up-down symmetry configurations are the same.

## 3. Analysis of stabilities

A rule $r$, viewed as a function over configurations, is a function of one variable and its domain consists of 16 elements. Because of the spin values taken by neighbours, the 8 elements of the domain of the symmetric rules can be grouped as follows:

$$\boldsymbol{A} = \{7\}, \qquad \boldsymbol{B} = \{6, \ldots, 3\}, \qquad \boldsymbol{C} = \{2, 1, 0\}. \tag{3.1}$$

Hence, any $r_s$ can be defined as a mapping from the union of the separate subdomains: $\boldsymbol{A}$, $\boldsymbol{B}$and $\boldsymbol{C}$to the set of "actions" which is also divided into three parts, respectively to the actions on the subdomains:

$$r_s : \boldsymbol{A} \cup \boldsymbol{B} \cup \boldsymbol{C} \quad \rightarrow \quad \left( \begin{pmatrix} F \\ FA \end{pmatrix}, \begin{pmatrix} FF \\ AF \\ S \\ AS \\ \vdots \\ S-E \\ S-N \\ \vdots \end{pmatrix}, \begin{pmatrix} s \\ as \\ \vdots \end{pmatrix} \right) \tag{3.2}$$

Notation introduced above can be easily explained by the following observations.

There exist two possible actions $\sigma_i(t+1)$ of a spin $\sigma_i$ in time $t+1$ in the case when its neighborhood $\Theta_i(t)$ in time $t$ forms an $\boldsymbol{A}$ configuration: one consists in following neighbours in a *Ferro* way,

$$\sigma_i(t+1) = 0 \quad \text{if} \quad \Theta_i(t) \in \boldsymbol{A}; \tag{3.3}$$

and the second one in *Anti-Ferro* way,

$$\sigma_i(t+1) = 1 \quad \text{if} \quad \Theta_i(t) \in \boldsymbol{A}; \tag{3.4}$$

Let us denote them $F$ and $FA$, respectively. Notice, that $r_s$ with property (3.4) is called *Anti-Rule* in the previous Section .

The set of actions over $B$ subdomain, $B$-actions, is the greatest one because the number of $B$ elements is the biggest one. All $16 = 2^4$ $B$-actions can be defined as:

- clustering: 2-*actions*
  $FF$: $\sigma_i(t+1) = 0$ if $\Theta_i(t) \in B$;
  $AF$: $\sigma_i(t+1) = 1$ if $\Theta_i(t) \in B$;
- shifts: 8-*actions*
  $S$: $\sigma_i(t+1) = S_i(t)$ if $\Theta_i(t) \in B$;
  $E, N, W$: as above and the letter denotes the neighbour which is shifted;
  $AS$: $\sigma_i(t+1) = 1 - S_i(t)$ if $\Theta_i(t) \in \mathrm{B}$;
  $AE, AN, AW$: as above and letters denote both the spin value conjugation and the neighbour which is shifted;
- alternative shifts: 6-*actions*
  $S - E$: $\sigma_i(t+1) = \max(S_i(t), E_i(t))$ if $\Theta_i(t) \in B$;
  $S-N, S-W, E-W, E-N, N-W$: as above with active neighbours denoted by the letters.

The last part of $r_s$ is responsible for the rule action when neighbourhood of a spin is $C$-type, $C$-action. One can see that it has to be one from below listed shifts:

$s$: $\sigma_i(t+1) = S_i(t)$ if $\Theta_i(t) \in C$;
$e, n, w$: as above and the letter denotes the neighbour which is shifted;
$as$: $\sigma_i(t+1) = 1 - S_i(t)$ if $\Theta_i(t) \in C$;
$ae, an, aw$: as above and letters denote both the spin value conjugation and the neighbour which is shifted.

The relations between the numbers of the rules (2.3) and actions (3.2) as presented in Table I are given as

$$
\begin{array}{l}
1 = (F, FF, s) \quad 6 = (F, FF, as) \\
9 = (F, S, s) \quad 10 = (F, S, w) \quad 11 = (F, S, an) \quad 13 = (F, S, aw) \\
25 = (F, W-S, s)\ 27 = (F, W-S, an)\ 41 = (F, S-N, s)\ 45 = (F, S-N, aw) \\
114 = (F, AS, w) \quad 118 = (F, AS, as) \quad 12 = (F, S, n) \quad 14 = (F, S, as) \\
115 = (F, AS, an) \quad 116 = (F, AS, n) \quad 117 = (F, AS, aw) \\
42 = (F, S-N, w)\ 43 = (F, S-N, an)\ 49 = (F, W-N, s)\ 51 = (F, W-N, an) \\
113 = (F, AS, s) \quad 121 = (F, AF, s) \quad 124 = (F, AF, as)
\end{array} \tag{3.5}
$$

The actions of any rule on the separated parts of domain are clearly seen from Eqs (3.5). When one considers a lattice where spins with $A$, $B$

or $C$ neighbourhoods are separated from each other, then one can observe the evolution as a product of the actions:

$$r(A \cup B \cup C) = r(A) \times r(B) \times r(C). \tag{3.6}$$

On a lattice the only way to see separated actions is to create a lattice with all neighbourhoods belonging to one subdomain. One can check that patterns (1.4) – (1.7) fulfill this condition. On random initial states the elements from different subdomains are randomly mixed. If $P = 0.5$ then the half of configurations belongs to $B$-subdomain, and therefore any action on $C$ elements is essential for the whole evolution of automata. This action dominates over other partial movements in the following sense: *automata reach the stabilization almost always on a pattern where a rule can work as a translation in the direction which agree with $B$-action.* This occurs almost always because there are two exeptions: when there exists a strong contradiction between $B$ and $C$-action (rules $(F, S, n)$ and $(F, AS, n)$) and when $B$-action does not determine any direction (rules $(F, FF, .)$-type). In the first case the evolution has the property that all changes made during one period combine to the translation by the number of the period lenght in the direction fixed by $B$-action. In the second case, the fixed point stabilization rarely is not a translation in one direction but there are a few parts in a final pattern shifted in different directions, [6].

The role of $B$-neighbourhood is easy to see when $B$-action is a shift, see Table II.

Rules $9 = (F, S, s)$ and its *Anti-Rule* - $246 = (FA, AS, as)$ play a special role. One can see that rule 9 is a translation from South, and then rule 246 is a composition of a translation from South and a spin conjugation, and any initial pattern is its own attractor. The evolution with a rule $r_s$ which has the same $B$-action but acts differently on $C$-subdomain, can be described as the elimination of neighbourhoods on which the rule $r_s$ differs from one of the pair 9 and 246. The smaller number of differences determines towards which evolution the rule $r_s$ leads. When the same numbers of differences appear, case 113=(F,AS,s), the stabilization cannot be reached. The conflict in the directions of shifts over $B$ and $C$ subdomains is also reflected in the increase of the length of time needed to reach the stabilization.

There is a reaction coming from $C$-action on $B$-subdomain, also. According to this process one obtains the same probabilities to find $B$ configurations having the same vertical $(6, 4)$ and horizontal $(5, 3)$ neighbours. The strongest interaction is observed when the shifts have opposite directions, as in rules: $12 = (F, S, n)$, $116 = (F, AS, n)$. The favoured pair is the one in which $B$-actions $N$ and $S$ or $N$ and $AS$ agree. (see Fig. 4) Other $C$-shifts: $w, e, s$, do not change the initial distribution of $B$-neighbourhoods. They are neutral. The influence of the *an* $C$-action can be explained in the same

TABLE II

**Differences between the rules and the property of a rule of leading to the stabilization. There are given results of the lenght of time needed by CA to reach the stabilization and the type of the observed stabilizations in case L=44, P=0.5 and time of observation do not exceed 400 steps.**

| Rule number and actions | Odd config. to SHIFT | Odd config. to SHIFT+CONJ. | $\langle T \rangle$ to stabilise | Probability and limit |
|---|---|---|---|---|
| $9 = (F, S, s)$ | none | $7, \cdots, 0$ | $\langle T \rangle = 0$ | 1 -point |
| $11 = (F, S, an)$ | 1 | $7, \cdots, 2, 0$ | $\langle T \rangle = 6$ | 1 -point |
| $13 = (F, S, aw)$ | 2 | $7, \cdots, 3, 1, 0$ | $\langle T \rangle = 6$ | 1 -point |
| $10 = (F, S, w)$ | 1,0 | $7, \cdots, 2$ | $\langle T \rangle = 11$ | 1 -point |
| $12 = (F, S, n)$ | 2,0 | $7, \cdots, 3, 1$ | $\langle T \rangle = 50$ | 1 - $\frac{\text{point}}{\text{period}}$ |
| $14 = (F, S, as)$ | 2,1,0 | $7, \cdots, 3$ | $\langle T \rangle = 206$ | 0.5 -point |
| $113 = (F, AS, s)$ | 6,5,4,3 | 7,2,1,0 | never | 0 |
| $115 = (F, AS, an)$ | $6, \cdots, 3, 1$ | 7,2,0 | $\langle T \rangle = 250$ | 0.26 -oscil. |
| $117 = (F, AS, aw)$ | $6, \cdots, 3, 2$ | 7,1,0 | $\langle T \rangle = 206$ | 0.53 -oscil. |
| $116 = (F, AS, n)$ | $6, \cdots, 3, 2, 0$ | 7,1 | $\langle T \rangle = 50$ | 1 - $\frac{\text{oscil.}}{\text{period}}$ |
| $114 = (F, AS, e)$ | $6, \cdots, 3, 1, 0$ | 7,2 | $\langle T \rangle = 12$ | 1 -oscil. |
| $118 = (F, AS, as)$ | $6, \cdots, 0$ | 7 | $\langle T \rangle = 7$ | 1 -oscil. |
| $246 = (FA, AS, as)$ | $7, \cdots, 0$ | none | $\langle T \rangle = 0$ | 1 -oscil. |

way as for $n$, by comparison of $B$-actions $S$ with $AN$, and $AS$ with $AN$ (see Fig. 5). The rest of $C$-actions: $as, ae, aw$ favour the other pair than $an$ action does.

If the action over $B$ is not of the shift-type, the evolution does not only mean elimination of some $A$ or $C$-neighbourhoods but it also works on converting $B$-neighbourhoods into one from the list (1.4) – (1.7). Such double effect needs more time than the destroying process. But the stabilization at the one of the patterns from (1.4) – (1.7) is observed only if $C$-action does not oppose $B$-action. It means that rules: $25 = (F, W - S, s)$, $27 = (F, W - S, an)$, $41 = (F, S - N, s)$, $45 = (F, S - N, aw)$ always lead to stabilization (in time shorter than 100 steps if $L = 44$) as a translation with one from directions given by $B$-action. On the other hand, rules: $42 = (F, S - N, w)$, $43 = (F, S - N, an)$, $49 = (F, W - N, s)$, $51 = (F, W - N, an)$ never cause stabilization in my experiments. Notice, that stabilization is reached on the pattern where there are only such $C$-neighbourhoods which have the property $W_i = S_i$ in case of rules 25, 26 and $S_i = N_i$ in cases 41, 45.

The action $FF$ over $B$-subdomain destroys $B$-neighbourhoods, independently of $C$-shift, and also converts them into $A$ neighbourhoods. In this case $B$-action does not fix the translation direction and therefore the

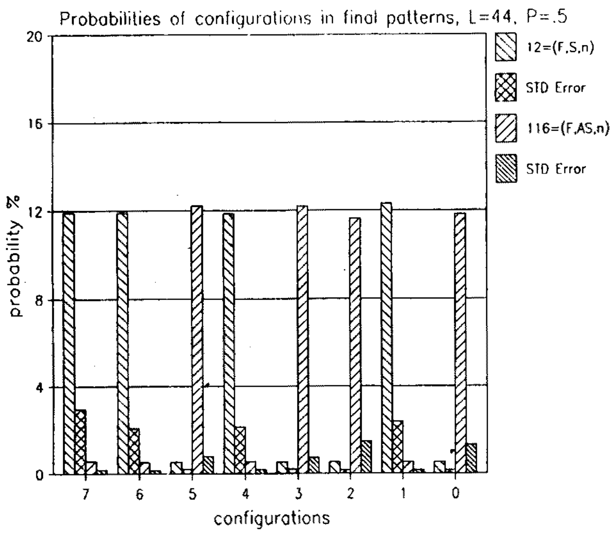


**Fig. 4.**

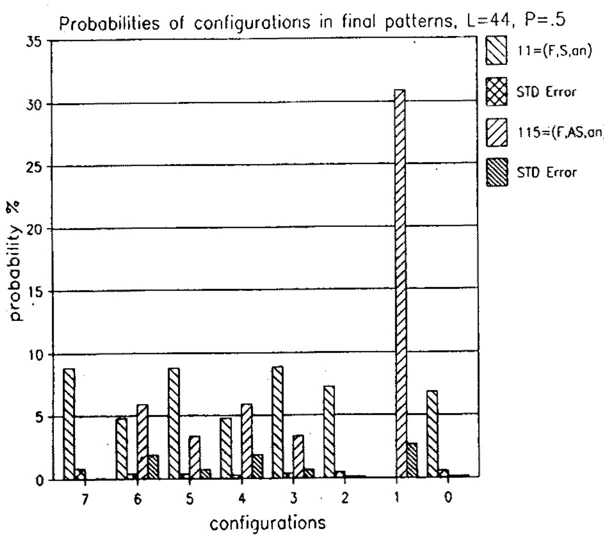


**Fig. 5.**

direction depends on $C$-action but it is not fixed, [6,7]. The rule with $B$-action $AF$ as a opposed to $FF$ cannot give a stable solution.

## 4. Conclusions

The previous approaches to the CA classification were either phenomenological in the sense that do not touch the dynamical problems [1,2], or not sufficiently sensitive to reflect these problems [7]. Hence, they do not give the answer to the question about the nature of CA. One can bridge the gap between properties of the final CA states and the rules via neighbourhood distributions. The attracting patterns attainable by the automata are firmly determinded in this picture if the initial states of automata are taken randomly. The distribution functions as the macroscopic functions can give answers to some global questions (for example it can be used to compute *Magnetization* (2.2)). Moreover, they also give some new hints for concerning the problem of classification of cellular automata. The final patterns can be viewed not only as a mixture of zeros and ones but as well defined structures which are conserved in time.

Since the properties of non-stabilizing rules expressed by the distributions of neighbourhoods are also firmly determinded, this approach can be powerful in this case, either.

The introduced structure of the rules via the discussed actions makes it possible to give a satisfactory explanation not only of the obtained neighbourhoods distributions in the final patterns but also allows elucidation of such properties of CA as: reaching or not the stabilization, the lenght of time needed to stabilize the system and even type of stabilization.

The author thanks the Organizers of the IV Conference on Statistical Physics, Zakopane 1991, for the invitation.

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