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SCALING ANALYSIS FOR CHAOTIC IONIZATION OF EXCITED HYDROGEN ATOMS IN MICROWAVE FIELD*

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Scaling analysis of the classical and quantum dynamics of the hydrogen atom in a monochromatic field is presented. Some scaling relations and functional dependencies for the classical and quantum processes are revealed and discussed.

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Since a highly excited hydrogen atom in a monochromatic field is one of the simplest real non-linear system with the stochastic behaviour, the experimental and theoretical studies of the microwave ionization of highly excited atoms provide a unique opportunity to explore an important paradigm for the quantum manifestation of a classical chaos [1, 2]. That is why at present a great attention is focused on investigations of the dynamics of Rydberg electrons in a strong microwave field. Experimental studies of the microwave ionization have been carried out for excited hydrogen atoms with principal quantum numbers up to $n_0 = 90$ [3]. Observations of excitation and ionization rates provide evidence for stochastic behaviour of a weakly bound electron: the ionization of Rydberg atoms exhibits a threshold dependence on the electric field amplitude and appears as a diffusion-like process.

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For such high quantum numbers one might expect the classical theory is to be a good approximation for the dynamics of the system. However, for a high relative frequency $s_0 = \omega n_0^3$ of the microwave field (with ω being the microwave frequency in atomic units), the classical chaotic diffusion is suppressed of a quantum interference effect [1] and ionization appears as a kinetical delocalization process. Later [4] another conditions for the quantum ionization have been introduced.

In this report we present the exact scaling for the description of the hydrogen atom in a monochromatic field and investigate the functional dependencies of the classical and quantum processes.

The Hamiltonian of the hydrogen atom in a linearly polarized microwave field of the frequency ω and field strength F in atomic units has the form

$$H = \frac{p^2}{2} - \frac{1}{r} + zF\cos\omega\tilde{t}\,. \tag{1}$$

Introducing the scale transformation [5]

$$t = \omega\tilde{t}, \quad r_s = \omega^{2/3}r, \quad p_s = \frac{p}{\omega^{1/3}}, \quad F_s = \frac{F}{\omega^{4/3}} \tag{2}$$

we find

$$H = \omega^{2/3}H_s, \quad H_s = \frac{p_s^2}{2} - \frac{1}{r_s} + z_sF_s\cos t\,. \tag{3}$$

Thus, the classical motion of the electron with the definite scaled energy $E_s = E/\omega^{2/3}$ depends only on the scaled field strength F_s.

Analogously, the scaled time-dependent Schrödinger equation can be expressed as

$$i\omega^{1/3}\frac{\partial\chi}{\partial t} = H_s\chi\,, \tag{4}$$

$$H_s = \frac{\omega^{2/3}}{2}\Big[-\frac{1}{r_s^2}\frac{\partial}{\partial r_s}\Big(r_s^2\frac{\partial}{\partial r_s}\Big) + \frac{l(l+1)}{r_s^2}\Big] - \frac{1}{r_s} + z_sF_s\cos t\,. \tag{5}$$

The scaled energy spectrum of the unperturbed scaled hydrogen atom is

$$E_s = -\frac{1}{2\omega^{2/3}n^2} = -\frac{1}{2s^{2/3}}\,, \tag{6}$$

where $s = \omega/(-2E)^{3/2}$ is the ratio of the microwave frequency ω to the Kepler orbital frequency $\Omega = (-2E)^{3/2}$.

It follows from equations (4) and (5) that the motion of the quantum hydrogen atom in a monochromatic field is governed in addition to the scaled field strength by the scaled Planck's constant $\hbar_s = \omega^{1/3}$ (in a.u.). The increase of the scaled Planck's constant with increasing frequency ω indicates the rise of the quantum properties of the system in the high frequency region.

The scale transformation also reduce the number of parameters of the problem, simplify analysis and comparison of classical and quantum dynamical processes and reveal the functional dependencies of the different processes and approximations. The detailed scaled analysis of the dynamics of the simplified one-dimensional model, mostly based on the classical and quantum Kepler maps [1, 6–10], is presented in the paper [5]. Here we want only to mention some conclusions.

The classical motion depends only on the scaled field strength $F_s = F/\omega^{4/3}$ while the quantum dynamics depends, in addition, on the scaled Planck's constant $\hbar_s = \omega^{1/3}$. However, the transition amplitudes between the states of the hydrogen atom as well as between the photonic states [1, 10] also depend only on one parameter — quantum scaled field strength $F_q = F/\omega^{5/3}$. Thus, the quantum scaled threshold field strength for the microwave quenching of Rydberg atoms in a definite state is some function of the relative frequency s, *i.e.*, $F_q^{\text{quen}}(s)$. One commonly introduces the relative field strength $F_0 \equiv n_0^4 F = F_q s_0^{4/3} \omega^{1/3}$ [1–4, 8]. Therefore, the relative threshold quenching field strength may be expressed as $F_0^{\text{quen}} = s_0^{4/3} \omega^{1/3} F_q^{\text{quen}}(s)$. We see that F_0^{quen} depends not only on the relative frequency s_0, but also on the absolute field frequency ω. This is in agreement with the experimental results [3]. The quantum diffusive ionization is a result of a kinetical delocalization of the quantum suppression of a chaotic diffusion and depends on the both scaled parameters: the field strength and the Planck's constant. The similar scaling analysis may be applied also for the investigation of atoms in a superintense laser field.

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