ANALYSIS OF SPATIAL CORRELATIONS IN CHAOTIC SYSTEMS*

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The method for estimation of the nonlinear interrelation in chaotic systems is described. Influence of choice of variables on the interrelation is discussed. Results of the analysis of a simple numerical model and semiconductor experiment are presented.

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1. Introduction

Basic ideas of nonlinear dynamics are put forward to explain complicated behaviour in various fields of research. Relevant experiments can be classified arbitrarily into three classes. First, there are simple, well controlled laboratory experiments, usually dealing with electronic circuits or mechanics. In this case the underlying equations can be easily written and sometimes an experiment is even carried out to model some known equations. Complete control of the system and low level of intrinsic random noise enable us to observe nearly all effects predicted by theory. Spatially extended systems in the regime of homogeneous distribution of parameters can be considered as the second class. For this regime, partial differential equations that are required to model the spatially extended system, often can be reduced to balance equations. A small number of degrees of freedom involved in the dynamics and strong nonlinearity lead to the appearance of various nonlinear effects, such as different routes to chaos, various types of intermittency, evolution of chaos. These effects have been observed in semiconductor experiments [1–5], and corresponding dimensions, entropies and Liapunov exponents have been estimated for the chaotic regimes. However, results obtained are often not so clear as compared to the first type experiments due to impossibility to control precisely system parameters and due to higher level of intrinsic noise.

Experiments with complicated spatio–temporal behaviour belong to the third class. For this most interesting case a large number of degrees of freedom is essential and the low-dimensional dynamics can manifest itself only locally. In numerical investigations, spatially extended systems are frequently represented as collections of coupled nonlinear subsystems. Possibility of such a reduction depends on spatial correlations, including nonlinear ones, in the experiment.

Our work is aiming to develop methods for estimation of nonlinear interaction. In the next Section a method of estimating nonlinear interrelation from experimental time series is presented [6], and advantages of the method in comparison with other methods are discussed. A simple model of two coupled nonlinear systems is considered as an example in Section 3. The problem of the best choice of variables is discussed in Section 4. In Section 5 we present results of the analysis for a semiconductor experiment.

2. Estimation of interrelation

A method for estimation of nonlinear interrelation proposed in Ref. [6] is based on the dynamical nature of considered observables $x(t)$ and $y(t)$. Underlying ideas have been formulated by Pacard *et al.* [7] and have been used to estimate the number of degrees of freedom [8]. The m-dimensional point $x_i^m \equiv \{x(t_i),\ x(t_i+\tau),\ldots,\ x[t_1+(m-1)\tau]\}$ in the reconstructed phase space (τ is delay time of the reconstruction) completely characterizes state of dynamical system for a high embedding dimension. Sufficient value of m is defined by Taken's theorem as $m \geq 2d+1$, where d is the dimension of the attractor. If both observables x and y are generated by the same system, small distance between points $|y_i^m - y_j^m| \sim \varepsilon$ in the reconstructed phase space of variable $y(t)$ is expected, when $|x_i^m - x_j^m| < \varepsilon$. For the independent observables x and y we expect $|y_i^m - y_j^m|$ to be constant. The mean conditional dispersion

$$\sigma_{xy}^m(\varepsilon) = \left\{ \frac{\Sigma |y_i^m - y_j^m|^2 \Theta(\varepsilon - |x_i^m - x_j^m|)}{\Sigma \Theta(\varepsilon - |x_i^m - x_j^m|)} \right\}^{1/2} \tag{1}$$

should be calculated taking into account all points of the time series. Here $\Theta(x)$ is Heaviside function. Presence of interrelation can be detected from the dependence of σ_{xy}^m on ε. Thus observables x and y are interrelated if the conditional dispersion σ_{xy}^m decreases with the decrease of ε, and are independent if σ_{xy}^m does not depend on ε. To estimate a magnitude of the interrelation let us consider the following dynamical equations for variables x and y:

$$\begin{aligned} \frac{dx^m}{dt} &= f(x^m) + \gamma_x F(x^m, y^m)\,, \\ \frac{dy^m}{dt} &= g(y^m) + \gamma_y G(x^m, y^m)\,. \end{aligned} \tag{2}$$

Here indices m point out the vector form of the equations. Functions f and g define subsystem dynamics and the second terms are responsible for the interrelation. When coupling parameters γ_x and γ_y differ, essentially nonsymmetric interrelation takes place. Having in mind Eqs (2) we suppose that more precise coincidences between points are needed to find weaker coupling. Dependence of the conditional dispersion σ_{xy}^m on ε should appear only for ε smaller than some ε_0. It can be expected that magnitude of ε_0 depends on the magnitude of the coupling and

$$K_{xy} = \lim_{m\to\infty} \frac{\varepsilon_0^m}{\varepsilon_{\max}^m} \tag{3}$$

can be chosen as an interrelation parameter. Parameter $\varepsilon_{\max}^m$ characterizes the size of the attractor in m-dimensional space. Definition of conditional dispersion (2) is not symmetric for the interchange of the observables x

and y. Therefore the introduced interrelation parameter K_{xy} is also not symmetric, $K_{xy} \neq K_{yx}$, and shows nonsymmetry of coupling.

The relation between ε_0 and coupling parameters γ, existence of the limit (3), and the possibility to find out nonsymmetry of coupling were proved numerically for coupled Henon maps in a previous paper [6]. In Section 3 we shall present results for another simple model. Here we would like to point out advantages of the suggested method. 1° — this is the only method to find out nonsymmetry of interrelation. 2° — it takes into account nonlinearity of interrelation and has statistics $\sim N^2$ (N is the number of points in the time series), when statistics of the usual correlator is $\sim N$. 3° — formula for calculation of conditional dispersion is very simple and similar to the formula for calculation of correlation integral [9]. Numerical efforts in both cases are similar, but estimation of interrelation is easier due to necessity to find only knee in the curve $\sigma_{xy}(\varepsilon)$ instead of the power law. Calculation of the mutual information [10], which also allows us to estimate nonlinear interrelation, is significantly more complicated.

3. Simple model

As an example of the most simple coupled system we investigated two independent chaotic signals passing through coupled linear filters:

$$\begin{aligned} x_{n+1} &= a_x x_n + \gamma_x y_n + h_n^x , \\ y_{n+1} &= a_y y_n + \gamma_y x_n + h_n^y . \end{aligned} \tag{4}$$

Independent chaotic time series h^x and h^y have been generated by the same Henon map with essentially different initial conditions. Calculations of the conditional dispersion σ_{xy}^m have been performed for $a_x = a_y = 0.5$ using time series of $N = 5000$ points.

Dependence of σ_{xy} on ε for different embedding dimensions m is shown in Fig. 1a for coupling parameter $\gamma = \gamma_x = \gamma_y = 0.1$. As it can be seen from the figure, all curves tend to an asymptotic one and a limiting value of ε_0^m can be estimated for $m \geq 6$. Numerical values of ε_0^m have been determined from the condition $\sigma_{xy}^m(\varepsilon_0^m) = 0.9\max(\sigma_{xy}^m$. In Fig. 1b the values are shown by arrows for different magnitudes of symmetric coupling. Correlation between coupling parameter γ and introduced interrelation parameter K_{xy} is shown in Fig. 1c. We obtained almost linear dependence, which can be caused by our choice of model, but in general, it can be more complicated. Possibility to detect nonsymmetry of interrelation is demonstrated in Fig. 1d. As it can be seen, curves $\sigma_{xy}(\varepsilon)$ and $\sigma_{yx}(\varepsilon)$ differ essentially for nonsymmetric coupling $\gamma_x = 0.1$, $\gamma_y = 0.01$.

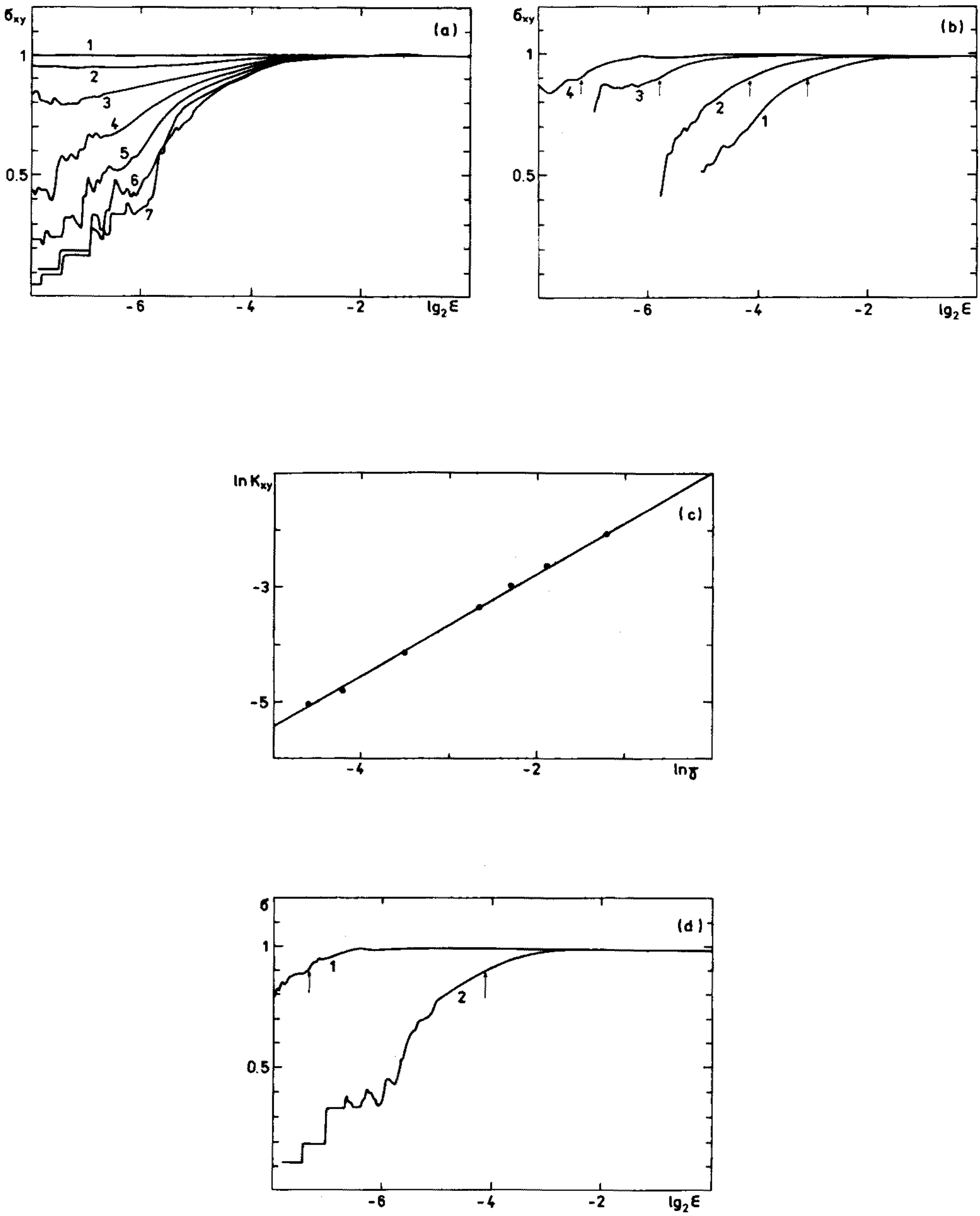

Fig. 1. Conditional dispersion σ_{xy} as a function of ε for model system: (a) — for various embedding dimensions m, denoted by numbers ($\gamma_x = \gamma_y = 0.1$); (b) — for $m = 7$ and various symmetric coupling parameters $\gamma = \gamma_x = \gamma_y$: *1* — $\gamma = 0.3$, *2* — $\gamma = 0.1$, *3* — $\gamma = 0.03$, *4* — $\gamma = 0.01$; (d) *1* — σ_{xy} and *2* — σ_{yx} for nonsymmetric coupling $\gamma_x = 0.1$, $\gamma_y = 0.01$. Curve in (c) shows dependence of interrelation parameter K_{xy} on coupling γ.

The difference between interrelation parameters K_{xy} and K_{yx} calculated from these curves points out nonsymmetry of interrelation.

4. Choice of variables

As we have mentioned in the Introduction, representation of a complicated spatially extended system as a collection of coupled subsystems is very attractive, especially when coupling is weak. This method is widely used for linear systems, and the concept of phonons in solid state theory is perhaps the simplest example. In practice localized weakly coupled regions can be found experimentally estimating interrelation between different points of the system. However, estimation of interrelation can reveal localized subsystems only when time series are recorded directly within the investigated system. In some experiments such direct measurements are impossible. For example in biology EEG can be recorded only on the surface of the head. In such cases experimental time series seem to be the combined effect of the intrinsic signals, and interrelation between recorded time series is expected to be strong. Reconstruction of intrinsic signals by transformation of variables is needed to detect weakly coupled regions in the system, if they exist.

The situation described above can be formulated mathematically in the following way: Spatially extended dynamical system can be described by a large number of ordinary differential equations or, as a limit, by partial a differential equation. Weakly coupled regions in physical systems correspond to the subsystems of equations with weak coupling between them. However, the coupling strength depends on the choice of variables and can be reduced by a proper choice. The method to eliminate linear correlation is well known and is used in singular value analysis [9]. Finding the general transformation of variables which minimizes nonlinear interrelation seems to be hardly solvable problem, especially from experimental data. Below we shall consider the simplest case of two signals. Moreover, we will search only for the best linear transformation minimizing nonlinear interrelation.

Let us consider two normalized time series $x(t)$ and $y(t)$ ($\langle x\rangle = \langle y\rangle = 0$, $\langle x^2\rangle = \langle y^2\rangle = 1$, here $\langle\cdots\rangle$ is time average). We suppose that interrelation between $x(t)$ and $y(t)$, estimated by the above described method, is strong. Our aim is to find parameters of the linear transformation

$$u(t) = A[x(t) + \alpha y(t)]\,,$$
$$v(t) = B[\beta x(t) + y(t)]\,, \tag{5}$$

which minimize nonlinear interrelation between the new variables $u(t)$ and $v(t)$. Constants A and B are defined by normalization conditions for new variables. Straightforward minimization, however, is complicated due to numerical effort required. More rational way is to fulfill condition for linear correlator $\langle uv \rangle = 0$, which leads to the following relation

$$\beta = -\frac{\langle xy \rangle + \alpha}{1 + \alpha \langle xy \rangle}. \tag{6}$$

Thus, only one independent parameter α remains.

As an example, we shall consider again the model (4) of coupled systems. Let us assume that only linear transformation of the original signals

$$\begin{aligned} x'(t) &= ax(t) + by(t), \\ y'(t) &= cx(t) + dy(t) \end{aligned} \tag{7}$$

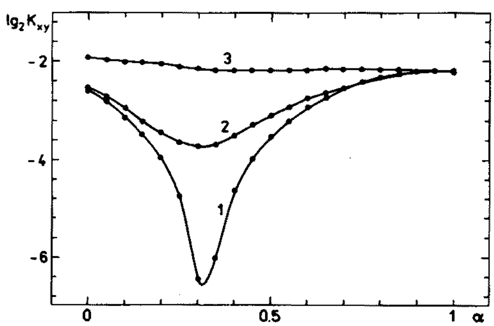

Fig. 2. Dependence of interrelation K_{xy} on minimization parameter α for model system with various magnitude of coupling *1* — $\gamma = 0.01$, *2* — $\gamma = 0.1$, *3* — $\gamma = 0.3$.

is observed. In numerical calculations we have used $a = d = 1$, $b = 0.3$, and $c = 0.5$. Applying the above described procedure we have calculated dependence of the interrelation parameter K_{xy} on α for various values of symmetric coupling parameter γ. As it ca be seen from Fig. 2, this dependence has a sharp minima for small coupling. The transformation (7) only slightly influences the interrelation for the strong coupling. Value of α corresponding to the minima in K_{xy} curve along with β determined by relation (6), defines linear transformation (5) inverse to (7). Therefore, the linear transformation minimizing nonlinear interrelation enables us to reconstruct original variables for the considered simple model.

5. Experiment

Experimental system consisted of monocrystalline p-doped germanium, electrically driven into the low-temperature avalanche breakdown *via* impurity impact ionization. The sample geometry and the electronic measuring configuration are schematically shown in Fig. 3. The sample dimensions were $0.25 \times 0.25\text{mm}^3$. Ohmic contacts were properly arranged on one of the two largest surfaces as indicated in Fig. 3. Bias voltage V_0 was applied to a series combination of sample and load resistor R_L ($R_L = 100\Omega$). A d.c. magnetic field B perpendicular to the broad surfaces was applied by a superconducting solenoid surrounding the semiconductor sample. The potentials V_1 and V_2 were detected by means of the inner probe contacts (of about 0.2mm diameter). During the experiment, the semiconductor sample was kept at liquid-helium temperature and carefully protected against external electromagnetic irradiation. Further details of the experimental techniques can be found in [12].

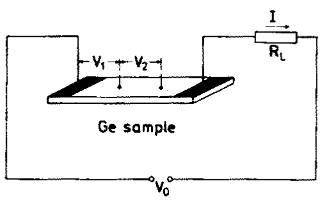

Fig. 3. Scheme of the experimental setup.

Time series of two voltage signals $V_1(t)$ and $V_2(t)$ were analyzed numerically with respect to their chaotic properties. Results of calculations of fractal dimension and Liapunov exponents have already been reported elsewhere [13]. Recently, the evidence of complicated spatial organization has been found. Namely, existence of local oscillation centers and interaction between them, leading to quasi-periodical behaviour, has been shown experimentally [14]. The analysis of nonlinear correlation between spatially separated regions has been presented in [15].

Parameter of nonlinear interrelation between signals $V_1(t) \equiv x(t)$ and $V_2(t) \equiv y(t)$ is calculated in the situation when the system evolves from the chaotic state to hyperchaotic with the increase of magnetic field [4]. As is seen from Fig. 4 interrelation decreases with the evolution of chaos. Decrease of interrelation along with the increase of nonsymmetry demonstrates complicated spatial behaviour of the system in the case of transition from chaotic to hyperchaotic state. The signals from other points of sample are needed for more detailed analysis of correlation and localization of generation of chaotic oscillations.

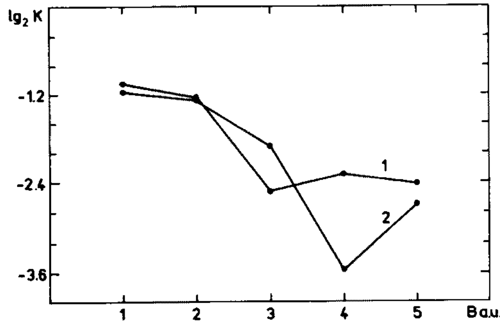

Fig. 4. Evolution of interrelation parameters: *1* — K_{xy} and *2* — K_{yx} for transition from chaotic state to hyperchaotic in semiconductor experiment.

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