SYMMETRY BREAKING EFFECTS IN S-WAVE CHANNELS

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Dedicated to Wiesław Czyż in honour of his 65th birthday

The effects of symmetry breaking in S-wave channels are discussed, attempting to illuminate the physics that distinguishes the S-wave channel. The importance of a centrifugal barrier in the other channels is emphasized. An overview of the origins of approximate symmetries, both among the light quarks and among the heavy quarks, is given in terms of quarks and their properties. Some new relations among weak decay matrix elements are obtained.

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In the Fall of 1962 I came to Stanford University, fresh from graduate school, and had the good fortune to share an office with Wiesław Czyż, already an established nuclear scientist. Using electron scattering to probe nucleon and nuclear structure was an exciting field of research at Stanford. Bob Hofstadter had recently received the Nobel Prize for his pioneering work exploring the nucleon elastic form factors and was, among other things, carrying out similar studies of the ^{3}He and ^{3}H system. The possibility of doing coincidence (e,e$'$p) experiments was being studied and Wiesław provided an excellent introduction for me to this approach for exploring nuclear structure, which proved very helpful later when Tom Griffy and I carried out several calculations in this area [1].

The Stanford Linear Accelerator Center, or project M (for monster), was growing fast and provided a very stimulating atmosphere for us theorists, although no one envisioned the dramatic results the planned deep inelastic scattering experiments would yield, particularly quarks. However, there was a curious fascination with the extension of the approximate SU(2) symmetry of isotopic spin to SU(3) by Gell-Mann [2, 3] and Ne'eman [4].

As a classification of the proliferating strongly interacting resonances being found in bubble chamber data, the representations of SU(3) seemed to work very well. However mass splittings of the order of 100 MeV, comparable to the widths of these resonances in many cases, were very difficult to reconcile at that time, when our prior prejudices were all based on isotopic spin symmetry. Particularly, difficult to understand was how both very narrow and very broad resonances could possibly belong to the same SU(3) multiplet and be described by wave functions related by an SU(3) rotation. SU(3) symmetry also implied various coupling constants were simply related by Clebsch–Gordan coefficients, making the large disparity in partial width difficult to understand. Gell-Mann captures the essence of the issues very nicely in the introductory section of his seminal unpublished report [2] of 1961: "In Section VIII we take up the vexed question of the broken symmetry, how badly it is broken, and how we might succeed in testing it." To exacerbate our bewilderment, the Gell-Mann–Okubo mass formula [5], a first order correction, worked remarkably well. Any real insight into what was happening had to await the quarks and quantum chromodynamics, as we shall discuss later, — our "vexed questions" were basically a consequence of not knowing either the elementary particles nor the nature of their fundamental interactions.

Undeterred by this lack of basic understanding, but with some knowledge of group theory, I reasoned that if the nucleons belonged to an SU(3) octet then nuclei, being composed of nucleons, should also belong to SU(3) multiplets and have analogs with strangeness. (My officemate, Wiesław, seemed unenthusiastic about this idea but was too kind to discourage me from exploring the possibility.) So, I proceeded to compute Kronecker products of octets and lots of Clebsch–Gordan coefficients. I reported on this work first at a conference at Ohio University [6], at Athens, Ohio, which was well attended as it followed the annual American Physical Society Meeting in Washington. After my talk in which I identified the hypernuclei as the SU(3) partners of nuclei, Dick Dalitz, who was the session chairman, stepped out of that role and gently but firmly explained that we were not ignorant of the structure of hypernuclei and they certainly did not look like the SU(3) partners of the corresponding nuclei. What I came to understand somewhat later is that for S-waves, as in the light nuclei I was considering, the symmetry is broken very badly.

To elaborate on this, let us consider the simplest nucleus, the deuteron. According to SU(3) considerations [7] this ${}^3S_1$ system belongs to the $\overline{10}$ representation of SU(3). The Gell-Mann–Okubo mass formula predicts the masses in this multiplet to be equally spaced. That is, the masses are related by

$$M = M_0 + M_1 Y \,, \tag{1}$$

where Y is the hypercharge. The strangeness-1 states are

$$|Y = 1,\ \mathrm{I} = \tfrac{1}{2},\ \mathrm{I}_3 = \tfrac{1}{2}\rangle = \frac{\sqrt{3}(\mathrm{p}\Lambda - \Lambda\mathrm{p}) + \Sigma^\circ\mathrm{p} - \mathrm{p}\Sigma^\circ + \sqrt{2}(\Sigma^+\mathrm{n} - \mathrm{n}\Sigma^+)}{\sqrt{12}}, \qquad (2a)$$

and

$$|Y = 1,\ \mathrm{I} = \tfrac{1}{2},\ \mathrm{I}_3 = -\tfrac{1}{2}\rangle = \frac{\sqrt{3}(\Lambda\mathrm{n} - \mathrm{n}\Lambda) + \Sigma^\circ\mathrm{n} - \mathrm{n}\Sigma^\circ + \sqrt{2}(\mathrm{p}\Sigma^- - \Sigma^-\mathrm{p})}{\sqrt{12}}. \qquad (2b)$$

The ΛN and ΣN thresholds are separated by about 75 MeV, which is quite large compared to the binding energy of the loosely bound deuteron. The strangeness-1 analog state could possibly appear as a bound state pole on the real axis below the ΛN threshold or as a resonance pole on either the unphysical sheet reached by passing through the branch cut between the ΛN and ΣN thresholds or the unphysical sheet reached by crossing the cut on the real axis above the ΣN threshold [8]. In any case the wave function at large distances in the ΛN and ΣN channels would look quite different — not exhibiting the SU(3) symmetry implied by Eq. (2). However, it still might be possible that the wave function at short distances, which is the important region for the computation of many matrix elements, is still SU(3) symmetric.

To explore this possibility the R-matrix formalism is convenient. Dividing space into an interior region ($r < r_0$), where r_0 is the range of the interactions, and an exterior region ($r > r_0$), the wave function and its first derivative must be matched at $r = r_0$. Then in the basis of SU(3) states the wave function satisfies

$$(R + iK^{-1})\psi = 0\,, \qquad (3)$$

where R is the derivative matrix and K is the wave number matrix. Assuming SU(3) symmetry in the interior region implies R is diagonal, but K will have comparable on-diagonal and off-diagonal matrix elements, in general, due to the rather different kinematics in each channel because the ΛN and ΣN thresholds are so widely separated. As a result ψ will not belong to a single SU(3) representation, the $\overline{10}$ in this case, but also will have rather large admixtures of other SU(3) representations.

This simple argument precludes a pure SU(3) symmetric interior wave function being matched to an exterior wave function, which is necessarily quite different due to the different kinematics in the various channels, at a single radius r_0, which is what the R-matrix formalism requires for S-waves. However, for the higher partial waves with $L \neq 0$ there is a centrifugal barrier and the interior region is effectively insulated from the exterior region by the width of the barrier. That is, there are two radii at which ψ and ψ' must match and these are separated by a centrifugal barrier, making the matching of both ψ and ψ' possible [9].

Presumably, this important effect in S-wave channels of the large mass differences, which separate the thresholds so widely, accounts for our failure to observe the SU(3) analog states of nuclei predicted in my naive, early attempt [6, 7] to extend SU(3) into nuclear physics. These same large SU(3) symmetry breaking effects are also evident in other S-wave systems; for example, most clearly in the meson–meson S-wave resonances [10].

The centrifugal barrier also offers an explanation of how the P-wave decuplet of $3/2^+$ meson–baryon resonances can exhibit SU(3) symmetry in spite of the large mass splittings within the multiplet. The equal spacing mass formula is obeyed to remarkable accuracy even though some of these resonances have widths comparable to the mass splittings. When Yang and I raised this problem in 1963 [11] we "simplified" the analysis by explicitly neglecting the centrifugal barrier. However, it is precisely the absence of this centrifugal barrier that makes the symmetry breaking effects so significant for the S-waves, which are the pathological case. Our conclusions should not have been applied to the P-wave $3/2^+$ meson–baryon resonances, but to the S-wave 0^+ meson–meson resonances; which unfortunately, had not yet been discovered.

With the discovery of quarks and quantum chromodynamics it is now clear why the SU(3) extension of isospin was such a successful approximate symmetry. The u, d, and s quarks have small current quark masses and neglecting their mass differences is a good approximation in many cases. The light quark flavor SU(3) is then realized in nature by the appearance of SU(3) multiplets of hadrons and approximate SU(3) symmetry of matrix elements. Moreover, the u and d quark masses are quite small and if they are neglected altogether we have, in addition, an approximate chiral SU(2)×SU(2) symmetry, realized in nature by the existence of a Nambu–Goldstone boson, the pion [12].

More recently it has been realized that there is also an approximate symmetry among the heavy quarks c, b, and t [13]. To the extent these quarks are heavy enough to neglect their recoil it does not matter how heavy they are; *i.e.* one can ignore the differences in the inverses of their masses. Calculating systematically the $1/m$ corrections to this naive approximation is currently being explored. Of course, there will be no additional chiral symmetry as for the light quarks. Nevertheless, if the quark symmetry breaking in the Lagrangian is only due to the quark masses one can readily compute the divergences of the axial vector currents:

$$\partial A^{(\mathrm{q})}(x) = m_{\mathrm{q}}\bar{\mathrm{q}}(x)\mathrm{q}(x)\,. \tag{4}$$

Taking matrix elements been the $\mathrm{q}\bar{\mathrm{q}}$ pseudoscalar meson states and the vacuum leads to a number of intersecting relations between quark masses and matrix elements and pseudoscalar meson masses and their decay constants. For the π^+ and K^+ one finds

$$\frac{M_\pi^2 F_\pi}{M_{\mathrm{K}}^2 F_{\mathrm{K}}} = \frac{(m_{\mathrm{u}} + m_{\mathrm{d}})}{(m_{\mathrm{u}} + m_{\mathrm{s}})}\frac{\langle 0|\mathrm{u}\bar{\mathrm{d}}|\pi^+\rangle}{\langle 0|\mathrm{u}\bar{\mathrm{s}}|\mathrm{K}^+\rangle}\,. \tag{5}$$

Using the light quark SU(3) flavor symmetry of the matrix elements, one finds the estimate

$$\frac{1}{2}\frac{m_{\mathrm{u}} + m_{\mathrm{d}}}{m_{\mathrm{s}}} \simeq \frac{1}{30}\,, \tag{6}$$

which provides some insight into the origin of the chiral SU(2)×SU(2) symmetry which results when m_{u} and m_{d} are neglected [12].

Looking next at the pseudoscalar mesons composed of one heavy and one light quark we find similar relations. For example, consider the D^+, D^0 and D_s^+ mesons, which form a light quark flavor SU(3) triplet. One finds, analogous to Eq. (5) that [14]

$$\frac{F_{\mathrm{D}^+}}{F_{\mathrm{D}_s^+}} = \left(\frac{M_{\mathrm{D}_s}}{M_{\mathrm{D}}}\right)^2\left(\frac{m_{\mathrm{c}} + m_{\mathrm{d}}}{m_{\mathrm{c}} + m_{\mathrm{s}}}\right)\frac{\langle 0|\mathrm{c}\bar{\mathrm{d}}|\mathrm{D}^+\rangle}{\langle 0|\mathrm{c}\bar{\mathrm{s}}|\mathrm{D}_s^+\rangle}\,, \tag{7}$$

which implies

$$\frac{F_{\mathrm{D}^+}}{F_{\mathrm{D}_s^+}} \simeq 1.2\,.$$

For the pseudoscalar bottom and top mesons we have the relations

$$\frac{F_{\mathrm{B}^0}}{F_{\mathrm{B}_s^0}} = \left(\frac{M_{\mathrm{B}_s}}{M_{\mathrm{B}}}\right)^2\left(\frac{m_{\mathrm{b}} + m_{\mathrm{d}}}{m_{\mathrm{b}} + m_{\mathrm{s}}}\right)\frac{\langle 0|\mathrm{b}\bar{\mathrm{d}}|\mathrm{B}^0\rangle}{\langle 0|\mathrm{b}\bar{\mathrm{s}}|\mathrm{B}_s^0\rangle} \simeq 1 \tag{8}$$

and

$$\frac{F_{\mathrm{T}^+}}{F_{\mathrm{T}_s^+}} = \left(\frac{m_{\mathrm{t}} + m_{\mathrm{d}}}{m_{\mathrm{t}} + m_{\mathrm{s}}}\right)\frac{\langle 0|\mathrm{t}\bar{\mathrm{d}}|\mathrm{T}^+\rangle}{\langle 0|\mathrm{t}\bar{\mathrm{s}}|\mathrm{T}_s^+\rangle} \simeq 1\,. \tag{9}$$

Similarly, based on the heavy quark symmetry one expects

$$\frac{F_{\rm B}}{F_{\rm D}} \simeq \frac{M_{\rm D}}{M_{\rm B}} \simeq 0.3 \tag{10}$$

and

$$\frac{F_{\rm T}}{F_{\rm B}} \simeq \frac{M_{\rm B}}{M_{\rm T}} \ll 1 \tag{11}$$

to lowest order in $1/m$ if the matrix elements exhibit the heavy quark symmetry. However, the $1/m$ corrections could be quite substantial, especially for the c quark case, Eq. (7).

The study of symmetries over the years, which began for me when Wiesław and I were at Stanford has been very rewarding, although the path was sometimes confusing, particularly in those early stages of our understanding. I was indeed fortunate to have begun this path sharing an office with Wiesław who was always so kind and encouraging.

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