# A REDEFINITION OF THE HADRONIC STRING

E.A. Remler

Department of Physics, The College of William and Mary
Williamsburg VA 23185, USA

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*Dedicated to Wiesław Czyż in honour of his 65th birthday*

The covariantly quantized momentum operator of the relativistic string in Minkowski space is redefined so that its domain is within the forward light cone. This leads, by definition, to a positive spectrum for the mass-squared operator.

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Hadronic strings arose from analyses of low momentum transfer ($|q|$) reactions [1]. Although this was circa 1970 prior to the advent of QCD, a satisfactory quantum theory of hadronic strings has not yet been created [2]. QCD itself independently leads to the idea of strings. In particular, lattice QCD in the string coupling low $|q|$ limit [3] is in fact a quantum hadronic string theory but it is not translation, rotation or boost invariant. The natural emergence of hadronic strings both from QCD and from the pre QCD analyses, strongly suggests that they provide the correct way to view low $|q|$ interactions.

Since there does not seem to be any principle prohibiting construction of a covariant Minkowski space string description of low $|q|$ QCD, difficulties encountered in its construction suggest that the model has not been properly defined. This paper discussed a possible redefinition.

In the usual formulation of string theory [2], (gauge) invariance under general coordinate transformations of the two surface parameters plays a central role. However, no theory can be expected to be invariant under this group unless it includes sufficient degrees of freedom. For example, classical electrodynamics is not by itself invariant under the full space-time

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coordinate transformation group; it is invariant only in conjunction with the metric when the latter is included as a dynamical variable.

Similarly, since the hadronic string does not describe high momentum components of QCD fields, it need not be invariant under the full gauge group. There is direct evidence that it is not in fact invariant. Strong coupling QCD implies the existence of baryons having junctions of three strings but boundary conditions at such junctions are inconsistent with gauge invariance. The problem originates in the model's idealization requiring momentum conservation and continuity across a point junction and thus over arbitrarily small distances and high momentum transfers.

Dropping gauge invariance, the basic open boson string action may be simply taken to be

$$S = \frac{\kappa}{2}\int d\xi^0 \int_{-1}^{1} d\xi^1 [\partial_0 x \cdot \partial_0 x - \partial_1 x \cdot \partial_1 x] . \tag{1}$$

The low $|q|$ restriction can be covariantly imposed either by truncating the string's normal mode expansion or by restricting it to a lattice in parameter space. In either case gauge invariance is lost and the surface parameters $\xi^0$, $\xi^1$ gain implicit physical meaning. For present purposes it is convenient to limit the mode expansion to $N$ terms.

Since gauge invariance usually determines the mass operator and allows nonphysical states to be decoupled from dynamics, these questions must be reconsidered. The mass-squared operator can be obtained heuristically as follows. The parameter space Hamiltonian is

$$\begin{aligned} H &= (2\kappa)^{-1}\int_{-1}^{1} d\xi^1 [p\cdot p + \kappa^2 \partial_1 x \cdot \partial_1 x] \\ &= (4\kappa)^{-1}[P^2 + \sum_{1}^{N} 2[p_n \cdot p_n + (\kappa\omega_n)^2 x_n \cdot x_n] , \\ p &= \kappa \partial_0 x , \end{aligned} \tag{2}$$

where the modes are given by

$$\begin{aligned} x^\mu(\xi) &= X^\mu(\xi^0) + \sum_{1}^{N} x_n^\mu(\xi^0) u_n(\xi^1) , \\ u_n(\xi^1) &= \cos(\omega_n(1+\xi^1)) , \\ \omega_n &= \frac{n\pi}{2} . \end{aligned} \tag{3}$$

This Hamiltonian generates displacements in $\xi^0$. The complete amplitude gets contributions from all $\xi^0$ and so must be an average of form

$$\Psi(x_1,\ldots,x_N) \sim \langle \exp(-iHT)\,\Psi_0(x_1,\ldots,x_N)\rangle_T\,. \tag{4}$$

But $T$ averaging projects out all functions except those for which

$$H\,\Psi = 0\,. \tag{5}$$

This defines the physical subspace of relativistic wave functions. The mass-squared operator therefore has the usual form

$$P^2 = -\sum_1^N 2[p_n \cdot p_n + (\kappa\omega_n)^2 x_n \cdot x_n]\,. \tag{6}$$

The main idea suggested here appears to be nothing but the usual one requiring that classical theory string histories be single valued surfaces and $\xi^0$ a timelike parameter. This can be expressed as the requirement

$$(\partial_0 x(\xi))^2 \geq 0\,; \quad \partial_0 x^0(\xi) > 0\,. \tag{7}$$

Although always assumed classically, it is not usually formulated kinematically upon covariant quantization. This is done here by simply defining the domain of the four-momentum operator $p(\xi)$ to be in the forward light cone. The result is a positive mass-squared spectrum by definition since

$$\left.\begin{array}{c} P = \int_{-\infty}^{+\infty} d\xi^1 p(\xi) \\ p(\xi)\cdot p(\xi) \geq 0 \\ p^0(\xi) \geq 0 \end{array}\right\} \Rightarrow P\cdot P \geq 0\,. \tag{8}$$

The theory truncated to one normal mode suffices to show the main effect of the four-momentum operator redefinition. The string becomes a version of the relativistic harmonic oscillator model and Eq. (6) for $N = 1$ becomes

$$\begin{aligned} &M^2\,\Psi(q) = -2\left[q^2 + (\kappa\omega)^2\left(\frac{i\partial}{\partial q}\right)^2\right]\Psi(q)\,,\\ &p(\xi)|_{\xi^1=\mp 1} = (M \pm q^0,\, \vec{q})\,,\\ &M \pm q^0 \geq |\vec{q}| \geq 0\,,\\ &q \equiv p_n|_{n=1}\,. \end{aligned} \tag{9}$$

More explicitly, for each angular momentum state we get

$$\tau = \frac{q^0}{M}; \quad \sigma = \frac{|\vec{q}|}{M}; \quad -1 \le \tau \le 1; \quad 0 \le \sigma \le 1 - |\tau|; \quad \lambda = \frac{M^2}{(\kappa\omega)};$$
$$\left[\left(\frac{\partial}{\partial\tau}\right)^2 - \sigma^{-2}\left(\frac{\partial}{\partial\sigma}\right)\sigma^2\left(\frac{\partial}{\partial\sigma}\right) + l(l+1)\sigma^{-2} + \lambda^2(\sigma^2 - \tau^2 - \tfrac{1}{2})\right]\Psi = 0. \tag{10}$$

Solutions to Eq. (10) which vanish on the boundaries determine allowed mass values and since it is linear, different mass solutions are linearly independent. However, it is not obvious that they are not orthogonal to one another relative to the expected norm

$$\int dq^0 \ldots dq^3\, \Psi^* \Psi \tag{11}$$

because the eigenvalue appears as part of the operator. On a formal level they should be since total momentum commutes with relative positions and momenta. This suggests that the boundary conditions applied to these eigenvalue equations need to be more carefully evaluated. For the present we simply assume that functions vanish on the boundaries.

The boundary conditions prevent the separation of variables in Eq. (10). To get a rough idea of the spectrum we can fix this by approximating the boundary as

$$-\tfrac{1}{2} \le \tau \le \tfrac{1}{2}; \qquad 0 \le \sigma \le \tfrac{1}{2}. \tag{12}$$

This covers most of the original domain of $q$. Furthermore, the neglected portion tends to be blocked by the harmonic oscillator potential. We take advantage of this and simplify further by ignoring the potential completely. The equation is now separable

$$\Psi = S(\sigma)T(\tau),$$
$$\left(\frac{\partial}{\partial\tau}\right)^2 T(\tau) = -\alpha^2 T(\tau),$$
$$\left[\sigma^{-2}\left(\frac{\partial}{\partial\sigma}\right)\sigma^2\left(\frac{\partial}{\partial\sigma}\right) - l(l+1)\sigma^{-2}\right]S(\sigma) = -\beta^2 S(\sigma),$$
$$\tfrac{1}{2}\lambda^2 = \beta^2 - \alpha^2. \tag{13}$$

Thus for Eq. (13) exactly and Eq. (10) approximately, the spectrum is just determined by the boundaries within which the string moves freely. This is asymptotic freedom at low $|q|$.

The spectral values are easy to obtain but the model is too rough for details to be relevant. The fact that a difference of squares enters directly reflects the hyperbolic nature of the differential equation. The derivation shows that negative values of this difference do not satisfy the eigenvalue condition. The redefinition of four-momenta means they are to be discarded. The set of eigenfunctions with positive mass-squared are, modulo the approximations made, complete within the Hilbert space of square integrable functions of timelike four-momenta.

Finally, we note that the four-momentum condition makes it possible to attach Dirac spinors to the ends of strings and is local so that when a string breaks, each part automatically satisfies the condition. These properties make realistic extensions of this model very promising.

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