SQUEEZING AND SU(3)-INVARIANCE IN MULTIPARTICLE PRODUCTION

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Dedicated to Wiesław Czyż in honour of his 65th birthday

This note is written as a tribute to professor Wiesław Czyż on the occasion of his 65-th birthday. It is a variation on a paper (*Acta Phys. Pol.* **B9**, 433 (1978)) entitled "A quasi-classical description of isospin-conservation in multiparticle production", written when he and his wife spent some time in Utrecht. In that paper the correlations between charged and neutral pions were calculated and expressed in terms of the average charge multiplicity and the charge dispersion. Similar calculations will be reported here, but now also K- and η-mesons will be included. Again the conservation law has a large effect on the multiplicity distribution of the particles produced in a high energy reaction.

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Suppose that in a high energy collision only mesons are produced belonging to the pseudo scalar octet

$$\begin{array}{ccccc} & \mathrm{K}_0 & & \mathrm{K}_+ & \\ \pi_- & & \pi_0, \eta & & \pi_+ \\ & \mathrm{K}_- & & \mathrm{K}_{\bar{0}} & \end{array} \tag{1}$$

and that the isospin and hypercharge are conserved. The many meson final state must then be an SU(3) singlet. Such a state is not uniquely defined, but the concept of independent particle production will suggest a special form for this singlet. For scalar particles it is known to lead to a coherent state, while in the present case the result is a so called squeezed state, which also plays an important role in quantum optics [2]. In the following it will be shown how such a many particle singlet state can be constructed with the octet bosons (1). For that purpose eight boson creation operators $a_1^*, \ldots, a_8^*$ are defined, which are connected to the octet creation operators as follows

$$\pi_\pm^* = \frac{1}{\sqrt{2}}(a_1^* \pm i a_2^*); \quad \pi_0^* = a_3^*; \quad \eta^* = a_8^*;$$
$$K_\pm^* = \frac{1}{\sqrt{2}}(a_4^* \pm a_5^*); \quad K_0^* = \frac{1}{\sqrt{2}}(a_6^* + i a_7^*); \quad K_{\bar{0}}^* = \frac{1}{\sqrt{2}}(a_6^* - i a_7^*). \quad (2)$$

The generators of SU(3) can be written in terms of these a_k^* and their hermitean conjugates:

$$\lambda_k = -2i f_{k\ell m} a_\ell^* a_m \qquad (k = 1, \ldots, 8), \qquad \left(\sum_{\ell,m=1}^{8} \text{implied} \right), \quad (3)$$

where the $f_{k\ell m}$ are the structure constants of SU(3), which satisfy the relation [3]

$$f_{kp\ell} f_{jm\ell} - f_{jp\ell} f_{km\ell} = f_{kj\ell} f_{pm\ell}. \quad (4)$$

From (3) and (4) follows immediately that

$$[\lambda_j, \lambda_k] = 2i f_{jk\ell} \lambda_\ell, \quad (5)$$

which are indeed the commutation relations for SU(3). The λ_k's, it should be stressed, are not Gell–Mann's 3×3 matrices, but rather an infinite dimensional representation of them. The other octet operators

$$t_k = d_{k\ell m} \lambda_\ell \lambda_m, \quad (6)$$

constructed with the fully symmetric coefficients $d_{k\ell m}$, automatically obey the correct rules under SU(3) transformations, *i.e.*

$$[\lambda_j, t_k] = 2i f_{jk\ell} t_\ell. \quad (7)$$

The Casimir operators

$$C_2 = \lambda_k \lambda_k \qquad \text{and} \qquad C_3 = \lambda_k t_k = d_{k\ell m} \lambda_k \lambda_\ell \lambda_m \quad (8)$$

are scalars, so they commute with all λ_i. Table I and Table II give examples of other octet and scalar operators.

TABLE I

Octet operators

$f_{k\ell m}\lambda_\ell\lambda_m$	$=$	$3i\lambda_k$
$f_{k\ell m}\lambda_\ell t_m$	$=$	$f_{k\ell m}t_\ell\lambda_m = 3it_k$
$d_{k\ell m}\lambda_\ell t_m$	$=$	$d_{k\ell m}t_\ell\lambda_m = (\frac{1}{3}C_2+1)\lambda_k$
$f_{k\ell m}t_\ell t_m$	$=$	$i(C_2+3)\lambda_k$
$d_{k\ell m}t_\ell t_m$	$=$	$\frac{2}{3}C_3\lambda_k+(1-\frac{1}{3}C_2)t_k$

TABLE II

Scalar operators

$t_k t_k$	$=$	$\frac{1}{3}C_2^2+C_2$
$f_{k\ell m}\lambda_k\lambda_\ell\lambda_m$	$=$	$3iC_2$
$f_{k\ell m}\lambda_k\lambda_\ell t_m$	$=$	$3iC_3$
$f_{k\ell m}\lambda_k t_\ell t_m$	$=$	$iC_2^2+3iC_2$
$f_{k\ell m}t_k t_\ell t_m$	$=$	$iC_2C_3+3iC_3$
$d_{k\ell m}\lambda_k\lambda_\ell\lambda_m$	$=$	C_3
$d_{k\ell m}\lambda_k\lambda_\ell t_m$	$=$	$\frac{1}{3}C_2^2+C_2$
$d_{k\ell m}\lambda_k t_\ell t_m$	$=$	$\frac{1}{3}C_2C_3+C_3$
$d_{k\ell m}t_k t_\ell t_m$	$=$	$\frac{2}{3}C_3^2-\frac{1}{9}C_2^3+C_2$

The expressions can be proved using a number of identities between $f_{jk\ell}$ and $d_{jk\ell}$ [4]. They are, however, not independent of λ_i, t_i, C_2 and C_3. The simplest scalar operator C_2, moreover, is of 4-th order in a_k and a_k^*, and is therefore too complicated to serve as Hamiltonian, of which the ground state is a good candidate for a many particle singlet state.

Since, however,

$$[\lambda_j, a_k] = 2if_{jk\ell}a_\ell \qquad \text{and} \qquad [\lambda_j, a_k^*] = 2if_{jk\ell}a_\ell^*, \tag{9}$$

the operators a_k and a_k^* themselves are octet operators. They can therefore be used to construct other scalar operators, like

$$N = a_k^*a_k; \quad G = a_ka_k; \quad F_kF_k; \quad H_k^*H_k; \quad H_kH_k; \quad H_k^*H_k^*, \quad \textit{etc.}, \tag{10}$$

where

$$F_k = d_{k\ell m}a_\ell^*a_m \qquad \text{and} \qquad H_k = d_{k\ell m}a_\ell a_m. \tag{11}$$

Only N, G and G^* are of second order, while the other scalars are of higher order in a_k and a_k^*. For reason of simplicity, therefore, the Hamiltonian is chosen as

$$H = N + 4 + \tfrac{1}{2}(tG + t^* G^*), \tag{12}$$

where 4 is added for later convenience and t is a complex number of magnitude less than one. For $|t| > 1$ the Hamiltonian is of the same form as $H = p^2 - q^2$ and is then not bounded, neither from above nor from below [5]. When the Hamiltonian (12) is written in the form

$$H = \sum_{k=1}^{8} \left[\left(a_k^* a_k + \tfrac{1}{2} \right) + \tfrac{1}{2} t a_k a_k + \tfrac{1}{2} t^* a_k^* a_k^* \right], \tag{13}$$

a physical interpretation suggests itself immediately. In addition to a mass term for each fundamental particle and its zero point energy, the Hamiltonian contains an interaction term, which shows that pairs of particles of the same kind are created or destroyed independently of other particles.

It is now assumed that the mesons produced in a high energy collision will quickly approach an equilibrium state in which their interaction is described by the Hamiltonian (13). If the temperature is low enough this will be the ground state, which is an SU(3) singlet. For higher temperatures also other singlet eigenstates will be present, but this possibility will not be considered now.

It is not difficult to diagonalise the Hamiltonian (13) by a Bogoliubov transformation

$$a_k = u^* c_k - v c_k^*, \qquad \text{with inverse} \qquad c_k = u a_k + v a_k^*, \tag{14}$$

where $c_1, \ldots, c_8$ are new Bose operators. If

$$t = \mathrm{e}^{2i\alpha} \tanh\gamma \quad \text{then} \quad u = \mathrm{e}^{i\alpha} \cosh\frac{\gamma}{2} \quad \text{and} \quad v = \mathrm{e}^{-i\alpha} \sinh\frac{\gamma}{2}. \tag{15}$$

The Hamiltonian then takes the form

$$H = \Lambda \sum_{k=1}^{8} (c_k^* c_k + \tfrac{1}{2}), \tag{16}$$

with

$$\Lambda = \frac{1}{\cosh\gamma}. \tag{17}$$

Let $|\gamma\rangle$ be the ground state, defined by

$$c_k|\gamma\rangle = 0 \qquad \text{for} \qquad k = 1, \ldots, 8. \tag{18}$$

If this state is written as $|\gamma\rangle = f(a_1^*, \ldots, a_8^*)|0\rangle$, where $a_k|0\rangle = 0$, the function $f(x_1, \ldots, x_8)$ must be a solution of

$$u\frac{\partial f}{\partial x_k} + v x_k f = 0 \qquad (k = 1, \ldots, 8). \tag{19}$$

This equation is easily solved and the normalized state $|\gamma\rangle$ is found to be

$$|\gamma\rangle = (1 - |w|^2)^2 \exp\left(-\frac{1}{2} w \sum_k a_k^* a_k^*\right) |0\rangle \tag{20}$$

with

$$w = \mathrm{e}^{-2i\alpha} \tanh\frac{\gamma}{2}. \tag{21}$$

For special values of α the state $|\gamma\rangle$ is the squeezed state referred to before. Its name can be understood from the following consideration. Take one of the eight modes and define a coordinate and momentum operator by

$$x = \frac{1}{\sqrt{2}}(a^* + a) \qquad \text{and} \qquad p = \frac{i}{\sqrt{2}}(a^* - a). \tag{22}$$

The product of the uncertainties $\Delta x \cdot \Delta p$, where $(\Delta x)^2 = \overline{(x - \overline{x})^2}$ and $(\Delta p)^2 = \overline{(p - \overline{p})^2}$, is equal to

$$\Delta x \cdot \Delta p = \tfrac{1}{2}\sqrt{1 + \sin^2 2\alpha \cdot \sinh^2 \gamma}. \tag{23}$$

For increasing γ it will be shown that the average multiplicity is also growing. From (23) it is therefore seen that the wavefunction $\psi(x) = \langle x|\gamma\rangle$ will produce a large value of $\Delta x \cdot \Delta p$ in the high energy limit $\gamma \longrightarrow \infty$. Two exceptional cases arise when either $\alpha = 0$ or $\alpha = \pi/2$, because then $\Delta x \cdot \Delta p = 1/2$, no matter how large the value of γ. One therefore obtains a Gaussian minimum width packet. In the first case the x-distribution is very narrow

$$\Delta x = \frac{1}{\sqrt{2}}\mathrm{e}^{-\frac{1}{2}\gamma} \qquad \text{and} \qquad \Delta p = \frac{1}{\sqrt{2}}\mathrm{e}^{\frac{1}{2}\gamma} \qquad \text{for} \qquad \alpha = 0, \tag{24}$$

while the p-distribution is narrow in the second case:

$$\Delta x = \frac{1}{\sqrt{2}}\mathrm{e}^{\frac{1}{2}\gamma} \qquad \text{and} \qquad \Delta p = \frac{1}{\sqrt{2}}\mathrm{e}^{-\frac{1}{2}\gamma} \qquad \text{for} \qquad \alpha = \frac{\pi}{2}. \tag{25}$$

This sudden narrowing of the wavefunction for certain values of its phase is a typical quantum mechanical effect and is called "squeezing" [2].

Returning to Eq. (20) for the multiparticle state $|\gamma\rangle$ it is now possible to calculate the probability $P(\{n_\ell\})$ to find n_1 particles of type $1,\ldots,n_8$ particles of type 8. Since

$$|\{n_\ell\}\rangle = \prod_{\ell=1}^{8} \frac{a_\ell^{*n_\ell}}{\sqrt{n_\ell!}}|0\rangle, \tag{26}$$

this probability is $P(\{n_\ell\}) = |\langle\{n_\ell\}|\gamma\rangle|^2$. It is easily calculated and found to be equal to

$$P(\{n_\ell\}) = \prod_{\ell=1}^{8} p(n_\ell), \tag{27}$$

where $p(n) = 0$ for odd n and

$$p(2n) = \frac{|\frac{w}{2}|^{2n}}{\cosh\frac{\gamma}{2}} \binom{2n}{n}. \tag{28}$$

Apart from the replacement of SU(2) by SU(3), the probability (27) is the same as obtained in Ref. [1] from a very different point of view.

In order to calculate averages and correlations between the number of particles of different types, the number operators are first expressed in terms of a_k and a_k^*. The result is given in Table III.

TABLE III

Number operators

$n(\pi_\pm)$	$=$	$\frac{1}{2}(a_1^*a_1 + a_2^*a_2) \mp \frac{1}{2}(a_1^*a_2 - a_2^*a_1)$
$n(\pi_0)$	$=$	$a_3^*a_3$
$n(K_\pm)$	$=$	$(a_4^*a_4 + a_5^*a_5) \mp \frac{i}{2}(a_4^*a_5 - a_5^*a_4)$
$n(K_{\frac{0}{0}})$	$=$	$\frac{1}{2}(a_6^*a_6 + a_7^*a_7) \mp \frac{i}{2}(a_6^*a_7 - a_7^*a_6)$
$n(\eta)$	$=$	$a_8^*a_8$

Averages are now easily calculated using Eqs (14) and (18). With $n_i = a_i^*a_i$ (no summation) it is seen that the number of charged- and of neutral particles are

$$N_c = n_1 + n_2 + n_4 + n_5 \qquad \text{and} \qquad N_0 = n_3 + n_6 + n_7 + n_8. \tag{29}$$

Their averages are

$$\overline{N}_c = \overline{N}_0 = 4\overline{n}_i = 4|v|^2 = 4\sinh^2\frac{\gamma}{2}. \tag{30}$$

With

$$\overline{n_i n_j} = |v|^4 + 2|u|^2 \cdot |v|^2 \delta_{ij} \tag{31}$$

the correlations become

$$D_c^2 = \overline{N_c^2} - \overline{N}_c^2 = 8|u|^2 \cdot |v|^2 = \tfrac{1}{2}\overline{N}_c^2 + 2\overline{N}_c, \tag{32}$$

$$\overline{N_c N_0} - \overline{N}_c \overline{N}_0 = 0, \tag{33}$$

and

$$D^2 = \overline{N^2} - \overline{N}^2 = 16|u|^2 \cdot |v|^2 = 2D_c^2. \tag{34}$$

Other correlations between charged and neutral particles can be calculated similarly, but will not add much to the conclusion that in independent pair production the SU(3) invariance gives an important contribution to these correlations.

No dependence on the phase of the many particle state is seen. This would be different if it were possible to measure the average value of the neutral current

$$J = \frac{i}{2}(\pi_+^* \pi_-^* - \pi_+ \pi_-). \tag{35}$$

The average value of this operator is

$$\overline{J} = \langle \gamma | J | \gamma \rangle = \sin 2\alpha \cdot |u| \cdot |v| = \tfrac{1}{2} \sin 2\alpha \cdot \sinh \gamma. \tag{36}$$

For high energies this would be a large number, proportional to $\overline{N}_c$. If, however, the phase α would, for some reason, be energy dependent and pass through 0 or $\pi/2$ for some energy, then there would be a dramatic change in J, caused by the squeezing of the final state.

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