CHIRAL QHD WITH VECTOR MESONS

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Dedicated to Wiesław Czyż — physicist and friend

Quantum hadrodynamics (QHD) is the formulation of the relativistic nuclear many-body problem in terms of renormalizable quantum field theory based on hadronic degrees of freedom. A model with neutral scalar and vector mesons (σ, ω) has had significant phenomenological success (QHD–I). An extension to include the isovector $\boldsymbol{\rho}$ through a Yang–Mills local gauge theory based on isospin, with the vector meson mass generated through the Higgs mechanism, also exists (QHD–II). Pions can be incorporated in a chiral-invariant fashion using the linear sigma model. The low-mass scalar of QHD–I is then produced dynamically through $\pi\pi$ interactions in this chiral-invariant theory. The question arises whether one can construct a chiral-invariant QHD lagrangian that incorporates the minimal set of hadrons $\{N, \omega, \boldsymbol{\pi}, \boldsymbol{\rho}\}$, where $N = \binom{p}{n}$ is the nucleon. These are the most important degrees of freedom for describing the low-energy nucleon–nucleon interaction and nuclear structure physics. In this paper we construct a chiral-invariant Yang–Mills theory based on the local gauge symmetry $\mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{SU}(2)_{\mathrm{L}}$. The baryon mass is generated through spontaneous symmetry breaking (as in the linear sigma model), and the vector meson masses are produced through the Higgs mechanism. The theory is parity conserving. Two baryon isodoublets with opposite hypercharge y are necessary to eliminate chiral anomalies. The minimal set of hadrons required consists of $\{N, \Xi; \sigma, \omega, \boldsymbol{\pi}, \boldsymbol{\rho}, \boldsymbol{a}; \eta, \xi\}$, where $\boldsymbol{a}$ is the chiral partner of the $\boldsymbol{\rho}$ (the $\boldsymbol{a}$ naturally obtains a higher mass in the model), and the η and ξ represent scalar and pseudoscalar Higgs particles. The parameters in this minimal theory consist of eight coupling constants and one mass (g_ω, $g_{0\pi} + y g_{1\pi}$, g_ρ, μ^2_{M}, λ_{M}, μ^2_{H}, λ_{H}; m_ω), where μ^2 and λ define the meson interaction potentials that lead to spontaneous symmetry breaking.

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1. Introduction and motivation

Two goals of modern nuclear physics are to study the properties of nuclear matter under extreme conditions of temperature and density, of interest for example in condensed stellar objects, supernovae, and relativistic heavy ion collisions, and to study the response of the nuclear system to large momentum transfers, of interest for example at CEBAF. In developing any theoretical extrapolation from existing empirical knowledge of nuclear behavior, it is essential to incorporate general principles of physics: quantum mechanics, special relativity, and microscopic causality. The only consistent theoretical framework we have for describing such a relativistic, interacting many-body system is relativistic quantum field theory based on a local lagrangian density. Such theories based on hadronic degrees of freedom (baryons and mesons) have had significant phenomenological success and have been the subject of numerous investigations in recent years. (For reviews, see Refs [1], [2], and [3].) Renormalizable theories of this type are known generically as *quantum hadrodynamics* (QHD).

A simple model (QHD–I) [4] based on baryons $N = \binom{\mathrm{p}}{\mathrm{n}}$, neutral scalar mesons σ coupled to the scalar baryon density $\overline{\psi}\psi$, and neutral vector mesons ω coupled to the conserved baryon current $\overline{\psi}\gamma_\mu\psi$ has been extensively studied and applied. It has been extended to include the ρ field through a Yang–Mills theory based on local isospin invariance (QHD–II) [5]; the vector meson mass is generated by the Higgs mechanism. Pions can be included in a chiral-invariant fashion through the linear sigma model. The low-mass scalar meson of QHD–I is then generated dynamically through the $\pi\pi$ interactions contained in the chiral-invariant lagrangian [6, 7]. Chiral invariance plays a central role in low-energy pion physics.

One may ask whether the model can be extended so that the vector mesons are also included in a chiral-invariant fashion. Our goal is to develop a QHD model with the following properties:

- It is based on hadronic degrees of freedom and contains at least N, ω, π, and ρ. These hadrons are the most important for nuclear phenomenology and form the basis for successful meson-exchange descriptions of the nucleon–nucleon interaction.
- It is invariant under isospin and chiral transformations.
- It is renormalizable.
- It conserves parity.

A model with these properties is constructed in this paper. We start from the linear σ model with global $\mathrm{SU(2)_R} \times \mathrm{SU(2)_L}$ symmetry, which requires both σ and π fields. This model is converted into a locally invariant Yang–Mills theory, necessitating the introduction of an axial-vector meson a, the chiral partner of the ρ. The baryon is given a mass through the spontaneous symmetry breaking of the σ model. Both vector mesons are given mass through the Higgs mechanism. Equal treatment of the left and right gauge fields guarantees parity conservation.

Chiral-invariant Yang–Mills theories are known to possess chiral anomalies [8–10]. The simplest way to understand the appearance of anomalies is by observing that while the classical action is invariant under local chiral transformations, the *fermion measure* in the quantum path integral is not [11, 12]. In the presence of anomalies, physical quantities such as the partition function or S matrix will be *gauge dependent.* This implies that a sensible quantum theory does not exist.

In the present model, chiral anomalies appear only in diagrams with a single isoscalar, vector vertex. Thus we can *eliminate* the anomalies through the following mechanism. The isoscalar, Lorentz vector ω, which couples to the baryon current in QHD–I, is assumed to couple more generally to the conserved strong *hypercharge* current, where the hypercharge operator is $\hat{Y} = \hat{B} + \hat{S}$. A second baryon isodoublet is introduced, with hypercharge $(y = -1)$ opposite to that of the nucleon $(y = +1)$. For example, this could be the $\Xi = \binom{\Xi^0}{\Xi^-}$. The chiral anomalies from these two baryon fields cancel. Physical quantities are now gauge invariant. The Ward identities are preserved. The theory appears to be renormalizable.

The final set of hadronic degrees of freedom, which is the minimal one required to achieve the desired goals, is $\{N, \Xi; \sigma, \omega, \pi, \rho, a; \eta, \xi\}$; the last two fields represent scalar and pseudoscalar Higgs particles. There are nine parameters in this model: eight coupling constants and one mass (g_ω, $g_{0\pi} + yg_{1\pi}$, g_ρ, $\mu_{\rm M}^2$, $\lambda_{\rm M}$, $\mu_{\rm H}^2$, $\lambda_{\rm H}$; m_ω). (Another parameter, the pion mass m_π, appears if the chiral symmetry is explicitly broken.) Here the parameters μ and λ enter in the *meson-meson potential* V responsible for the generation of the baryon masses in the sigma model ($V_{\rm M}$) and for the generation of the vector meson masses through the Higgs mechanism ($V_{\rm H}$); we write this potential generically as $V(x^2) = -\mu^2 x^2 + (\lambda/4)x^4$. The phenomenological consequences of the present model remain to be investigated.

A goal similar to that of the present paper has been pursued by Lovas and Sailer [13]. The present model differs in several respects, namely, in the minimization of the number of degrees of freedom and coupling constants, in the imposition of parity conservation, and in the necessary elimination of the chiral anomalies.

QHD must be viewed as a model of the underlying theory of *quantum chromodynamics* (QCD), which describes the strong interactions that bind colored quarks and gluons into the observed hadrons. To the extent that QHD can successfully describe nuclear matter under extreme conditions, it provides a powerful constraint on the low-momentum-transfer, large-distance, effective degrees of freedom of QCD. There is evidence from QCD sum rules that the strong isoscalar, Lorentz scalar and vector potentials predicted by QHD, and observed in nuclei, are a dynamical consequence of QCD [14].

Although lattice gauge-theory calculations provide an impressive means of directly exploring the consequences of the QCD lagrangian, the achievement of even a qualitative description of the relativistic, interacting, nuclear many-body system through these techniques appears to lie well in the future.

2. A chiral QHD model

2.1. The linear sigma model

To construct a chirally invariant model that contains the desired hadronic degrees of freedom (p, n, $\boldsymbol{\pi}$, ρ, and ω), we begin with the well-known linear sigma model [15–17]. This model contains a pseudoscalar (γ_5) coupling between pions and nucleons, and an auxiliary scalar field (denoted here by s) to implement the chiral symmetry. Since chiral symmetry is only approximate in nature, we will include a "small" symmetry-violating (SV) term to generate a mass for the pion[1]. We will also add a massive isoscalar vector field (representing the ω) to supply a repulsive nucleon–nucleon interaction, as in QHD–I. The isovector vector mesons will be omitted for now and added in the next subsection.

By demanding that the theory be local, Lorentz covariant, parity invariant, isospin and chiral invariant, and renormalizable, one is led to the form

$$\mathcal{L}_{\sigma\omega} = \mathcal{L}_{\text{chiral}} + \mathcal{L}_{\text{SV}}\,, \tag{2.1}$$

$$\begin{aligned}\mathcal{L}_{\text{chiral}} = {}& \overline{\psi}\left[\gamma_\mu(i\partial^\mu - g_{\text{v}}V^\mu) - g_\pi(s + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})\right]\psi + \tfrac{1}{2}(\partial_\mu s\partial^\mu s + \partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}) \\ & - \tfrac{1}{4}\lambda(s^2 + \boldsymbol{\pi}^2 - v^2)^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_{\text{v}}^2 V_\mu V^\mu + \delta\mathcal{L}\,,\end{aligned} \tag{2.2}$$

$$\mathcal{L}_{\text{SV}} = \epsilon s\,. \tag{2.3}$$

Here $\psi = \binom{\psi_p}{\psi_n}$, $\boldsymbol{\pi}$, and s are the nucleon, isovector pion, and neutral scalar meson fields, respectively, g_π is the pion–nucleon coupling constant, and $\boldsymbol{\tau}$ are the usual Pauli matrices. The parameters λ and v describe the strength of the meson self-interactions, and ϵ is a chiral-symmetry-violating parameter related to the pion mass; the exact chiral limit is obtained by setting $\epsilon = 0$. The form of the meson self-interactions allows for spontaneous symmetry breaking, which is used to give the nucleon a mass, as discussed

[1] Note that while the pion mass is small on the scale of hadronic masses, it is *not* small on the scale of nuclear physics observables!

below. The ω meson field is denoted by V^μ, its field strength tensor is $F^{\mu\nu} \equiv \partial^\mu V^\nu - \partial^\nu V^\mu$, and its coupling to the nucleons is given by g_v. Note that this $\sigma\omega$ model is renormalizable, as it contains no derivative couplings and is at most quartic in the meson fields; the counterterm contribution $\delta\mathcal{L}$ will henceforth be suppressed. The conventions used here are those of Refs [1–3].

The lagrangian $\mathcal{L}_{\rm chiral}$ is invariant under the infinitesimal global isospin transformations

$$\psi \longrightarrow \psi' = (1 + i\tfrac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{\alpha})\psi\,, \tag{2.4}$$

$$\boldsymbol{\pi} \longrightarrow \boldsymbol{\pi}' = \boldsymbol{\pi} - \boldsymbol{\alpha}\times\boldsymbol{\pi}\,, \tag{2.5}$$

where $\boldsymbol{\alpha}$ denotes three parameters that are independent of spacetime. (The other meson fields remain unchanged.) Using Noether's theorem, this invariance implies that the vector isovector current

$$\mathbf{T}^\mu = \tfrac{1}{2}\bar\psi\gamma^\mu\boldsymbol{\tau}\psi + \boldsymbol{\pi}\times\partial^\mu\boldsymbol{\pi} \tag{2.6}$$

is conserved ($\partial_\mu \mathbf{T}^\mu = 0$). This lagrangian is also invariant under the infinitesimal global chiral transformations ($\boldsymbol{\beta}$ = constant)

$$\psi \longrightarrow \psi' = (1 + i\tfrac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{\beta}\gamma_5)\psi\,, \tag{2.7}$$

$$\boldsymbol{\pi} \longrightarrow \boldsymbol{\pi}' = \boldsymbol{\pi} - s\boldsymbol{\beta}\,, \tag{2.8}$$

$$s \longrightarrow s' = s + \boldsymbol{\beta}\cdot\boldsymbol{\pi}\,, \tag{2.9}$$

which imply that there is a conserved *axial* isovector current

$$\mathbf{A}^\mu = \tfrac{1}{2}\bar\psi\gamma^\mu\gamma_5\boldsymbol{\tau}\psi - \boldsymbol{\pi}\partial^\mu s + s\partial^\mu\boldsymbol{\pi} \tag{2.10}$$

in the chiral limit $\epsilon = 0$. When $\epsilon \neq 0$, we obtain instead the PCAC relation

$$\partial_\mu \mathbf{A}^\mu = -\epsilon\boldsymbol{\pi}\,, \tag{2.11}$$

which follows from the field equations. (There is also a conserved baryon current $B^\mu = \bar\psi\gamma^\mu\psi$, as in QHD–I, since the lagrangian is invariant under global phase transformations of the baryon field.)

In the chiral limit, the conserved vector and axial-vector currents can be used to define the generators of isospin and chiral rotations, which can be combined to form generators for right- and left-handed isospin rotations. These latter generators satisfy a Lie algebra corresponding to the group $SU(2)_R \times SU(2)_L$ (see Ref. [17]). To illustrate this combined symmetry explicitly and to make the imposition of local gauge invariance more transparent, it is convenient to define right- and left-handed baryon fields:

$$\psi_R \equiv \tfrac{1}{2}(1+\gamma_5)\psi\,, \qquad \psi_L \equiv \tfrac{1}{2}(1-\gamma_5)\psi\,, \qquad \psi = \psi_R + \psi_L\,, \tag{2.12}$$

and to group the scalar and pion fields into a chiral four-vector:

$$\chi \equiv \frac{1}{\sqrt{2}}(s - i\boldsymbol{\tau}\cdot\boldsymbol{\pi}), \tag{2.13}$$

which is represented as a 2×2 matrix.

In terms of these new variables, $\mathcal{L}_{\text{chiral}}$ can be written (to within an additive constant) as

$$\mathcal{L}_{\text{chiral}} = \mathcal{L}_{\text{N}} + \mathcal{L}_{\pi} + \mathcal{L}_{\omega}, \tag{2.14}$$

where

$$\mathcal{L}_{\text{N}} = i(\overline{\psi}_{\text{R}}\gamma_{\mu}D^{\mu}\psi_{\text{R}} + \overline{\psi}_{\text{L}}\gamma_{\mu}D^{\mu}\psi_{\text{L}}) - \sqrt{2}g_{\pi}(\overline{\psi}_{\text{L}}\chi^{\dagger}\psi_{\text{R}} + \overline{\psi}_{\text{R}}\chi\psi_{\text{L}}), \tag{2.15}$$

$$\mathcal{L}_{\pi} = \tfrac{1}{2}\,\text{tr}(\partial_{\mu}\chi^{\dagger}\partial^{\mu}\chi) - V(\text{tr}\,\chi^{\dagger}\chi), \tag{2.16}$$

$$\mathcal{L}_{\omega} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_{\text{v}}^{2}V_{\mu}V^{\mu}. \tag{2.17}$$

Note the particularly simple form of the meson–nucleon interaction with these fields. We have written the baryon derivative as $D^{\mu} \equiv \partial^{\mu} + ig_{\text{v}}V^{\mu}$, which is a unit matrix in spin and isospin, and defined the meson self-interactions as

$$V(\text{tr}\,\chi^{\dagger}\chi) \equiv \frac{-v^{2}\lambda}{2}(\text{tr}\,\chi^{\dagger}\chi) + \frac{\lambda}{4}(\text{tr}\,\chi^{\dagger}\chi)^{2}. \tag{2.18}$$

The lower-case "tr" denotes a trace over isospin indices only. Here we have defined $\mu^{2} \equiv v^{2}\lambda/2$.

The transformation properties of the new fields are easily expressed by defining a unitary SU(2) rotation matrix

$$U(\boldsymbol{\omega}) \equiv \exp(\tfrac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{\omega}) = \cos(\omega/2) + i\hat{\boldsymbol{n}}\cdot\boldsymbol{\tau}\sin(\omega/2) \underset{\omega\to 0}{\longrightarrow} 1 + \tfrac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{\omega}, \tag{2.19}$$

where $\boldsymbol{\omega} \equiv \hat{\boldsymbol{n}}\omega$ denotes three real, constant parameters. There is one set of rotation matrices for SU(2)_{R} and another set for SU(2)_{L}. It is now obvious that $\mathcal{L}_{\text{chiral}}$ is separately invariant under the right-handed isospin transformations

$$\psi_{\text{R}} \underset{\text{R}}{\longrightarrow} U\psi_{\text{R}}, \quad \psi_{\text{L}} \underset{\text{R}}{\longrightarrow} \psi_{\text{L}}, \quad \chi \underset{\text{R}}{\longrightarrow} U\chi, \quad V^{\mu} \underset{\text{R}}{\longrightarrow} V^{\mu}, \tag{2.20}$$

and the left-handed isospin transformations

$$\psi_{\text{R}} \underset{\text{L}}{\longrightarrow} \psi_{\text{R}}, \quad \psi_{\text{L}} \underset{\text{L}}{\longrightarrow} U\psi_{\text{L}}, \quad \chi \underset{\text{L}}{\longrightarrow} \chi U^{\dagger}, \quad V^{\mu} \underset{\text{L}}{\longrightarrow} V^{\mu}. \tag{2.21}$$

One can verify that these transformations reproduce the infinitesimal isospin and chiral rotations given above, if one identifies $\boldsymbol{\alpha} = \frac{1}{2}(\boldsymbol{\omega}_{\rm R} + \boldsymbol{\omega}_{\rm L})$ and $\boldsymbol{\beta} = \frac{1}{2}(\boldsymbol{\omega}_{\rm R} - \boldsymbol{\omega}_{\rm L})$. Note that a mass term for the baryons is not allowed in $\mathcal{L}_{\rm chiral}$, since it is proportional to $\overline{\psi}\psi = \overline{\psi}_{\rm R}\psi_{\rm L} + \overline{\psi}_{\rm L}\psi_{\rm R}$, which is clearly not invariant. The baryon mass will be generated by spontaneous symmetry breaking, which we shall discuss below.

What about the properties of $\mathcal{L}_{\rm chiral}$ under parity transformations? If we denote the parity operator by $\hat{\mathcal{P}}$, the properties of scalar, pseudoscalar, vector, and spinor fields lead to the transformation laws [18]

$$\hat{\mathcal{P}}\chi(t,\boldsymbol{x})\hat{\mathcal{P}}^{-1} = \chi^{\dagger}(t,-\boldsymbol{x}) , \qquad \hat{\mathcal{P}}V^{\mu}(t,\boldsymbol{x})\hat{\mathcal{P}}^{-1} = V_{\mu}(t,-\boldsymbol{x}) ,$$
$$\hat{\mathcal{P}}\psi_{\rm L}(t,\boldsymbol{x})\hat{\mathcal{P}}^{-1} = \gamma^{0}\psi_{\rm R}(t,-\boldsymbol{x}) , \; \hat{\mathcal{P}}\psi_{\rm R}(t,\boldsymbol{x})\hat{\mathcal{P}}^{-1} = \gamma^{0}\psi_{\rm L}(t,-\boldsymbol{x}) . \quad (2.22)$$

Observe that the parity transformation interchanges the right- and left-handed spinor fields and flips the sign of the three-vector part of the vector field ($V^{\mu} \to V_{\mu}$). It is now easy to check that the action $S = \int {\rm d}^4x\mathcal{L}_{\sigma\omega}$ is invariant under these transformations; note that with our conventions, the change in dummy integration variables from $\boldsymbol{x}$ to $-\boldsymbol{x}$ produces $\partial^{\mu} \to \partial_{\mu}$.

2.2. Inclusion of vector mesons

Now that we have a lagrangian that is manifestly invariant under global $\mathrm{SU(2)_R \times SU(2)_L}$ transformations, we want to add isovector mesons by generalizing to a *local* gauge theory. First, we follow the well-known Yang–Mills procedures [19, 20] for the right- and left-handed isospin transformations, which will lead to a theory containing massless isovector-vector and isovector-axial vector mesons. We will then implement the Higgs mechanism to generate masses for these mesons.

Begin by defining right and left isovector gauge fields $\boldsymbol{r}^{\mu}$ and $\boldsymbol{\ell}^{\mu}$ that transform as follows under infinitesimal local transformations $\omega(x)$:

$$\boldsymbol{r}_{\mu} \underset{\rm R}{\longrightarrow} \boldsymbol{r}_{\mu} - \boldsymbol{\omega}_{\rm R} \times \boldsymbol{r}_{\mu} - G^{-1}\partial_{\mu}\boldsymbol{\omega}_{\rm R} , \qquad \boldsymbol{\ell}_{\mu} \underset{\rm R}{\longrightarrow} \boldsymbol{\ell}_{\mu} , \quad (2.23)$$
$$\boldsymbol{\ell}_{\mu} \underset{\rm L}{\longrightarrow} \boldsymbol{\ell}_{\mu} - \boldsymbol{\omega}_{\rm L} \times \boldsymbol{\ell}_{\mu} - G^{-1}\partial_{\mu}\boldsymbol{\omega}_{\rm L} , \qquad \boldsymbol{r}_{\mu} \underset{\rm L}{\longrightarrow} \boldsymbol{r}_{\mu} . \quad (2.24)$$

We will see shortly that parity invariance requires the same gauge coupling G for the left and right vector fields. The transformations of the other fields remain as in Eqs (2.20) and (2.21), except that $\omega_{\rm R}(x)$ and $\omega_{\rm L}(x)$ are now functions of spacetime.

The vector meson field tensors are defined by

$$\boldsymbol{R}_{\mu\nu} \equiv \partial_{\mu}\boldsymbol{r}_{\nu} - \partial_{\nu}\boldsymbol{r}_{\mu} - G(\boldsymbol{r}_{\mu}\times\boldsymbol{r}_{\nu}) , \quad \boldsymbol{L}_{\mu\nu} \equiv \partial_{\mu}\boldsymbol{\ell}_{\nu} - \partial_{\nu}\boldsymbol{\ell}_{\mu} - G(\boldsymbol{\ell}_{\mu}\times\boldsymbol{\ell}_{\nu}) , \quad (2.25)$$

and it is easy to verify that under infinitesimal transformations,

$$\boldsymbol{R}_{\mu\nu} \underset{\mathrm{R}}{\longrightarrow} \boldsymbol{R}_{\mu\nu} - \boldsymbol{\omega}_{\mathrm{R}} \times \boldsymbol{R}_{\mu\nu}\,, \quad \boldsymbol{L}_{\mu\nu} \underset{\mathrm{L}}{\longrightarrow} \boldsymbol{L}_{\mu\nu} - \boldsymbol{\omega}_{\mathrm{L}} \times \boldsymbol{L}_{\mu\nu}\,. \tag{2.26}$$

Thus kinetic-energy terms of the form $\boldsymbol{R}_{\mu\nu}\cdot\boldsymbol{R}^{\mu\nu}$ and $\boldsymbol{L}_{\mu\nu}\cdot\boldsymbol{L}^{\mu\nu}$ are locally gauge invariant. In contrast, mass terms of the form $\boldsymbol{r}_{\mu}\cdot\boldsymbol{r}^{\mu}$ or $\boldsymbol{\ell}_{\mu}\cdot\boldsymbol{\ell}^{\mu}$ are not invariant, so we cannot simply add mass terms for these mesons.

When the rotation parameters $\boldsymbol{\omega}_{\mathrm{R}}(x)$ and $\boldsymbol{\omega}_{\mathrm{L}}(x)$ depend on spacetime, the lagrangian of Eq. (2.14) is no longer invariant. The invariance can be restored, however, by defining *covariant derivatives* for the spinor and chiral meson fields. These covariant derivatives take the form

$$\begin{aligned} D_\mu\psi_{\mathrm{R}} &\equiv (\partial_\mu + ig_{\mathrm{v}}V_\mu + \tfrac{i}{2}G\,\boldsymbol{\tau}\cdot\boldsymbol{r}_\mu)\psi_{\mathrm{R}}\,, \\ D_\mu\psi_{\mathrm{L}} &\equiv (\partial_\mu + ig_{\mathrm{v}}V_\mu + \tfrac{i}{2}G\,\boldsymbol{\tau}\cdot\boldsymbol{\ell}_\mu)\psi_{\mathrm{L}}\,, \\ D_\mu\chi &\equiv \partial_\mu\chi + \tfrac{i}{2}G(\boldsymbol{\tau}\cdot\boldsymbol{r}_\mu)\chi - \tfrac{i}{2}G\chi(\boldsymbol{\tau}\cdot\boldsymbol{\ell}_\mu)\,. \end{aligned} \tag{2.27}$$

Note in particular the ordering of factors in the last line. The covariant derivatives transform exactly as the fields in Eqs (2.20) and (2.21). It is now straightforward to show that the lagrangian given by

$$\mathcal{L} = \mathcal{L}_{\mathrm{N}} + \mathcal{L}_\pi + \mathcal{L}_{\mathrm{G}}\,, \tag{2.28}$$

where

$$\mathcal{L}_{\mathrm{G}} \equiv \mathcal{L}_\omega - \tfrac{1}{4}\boldsymbol{R}_{\mu\nu}\cdot\boldsymbol{R}^{\mu\nu} - \tfrac{1}{4}\boldsymbol{L}_{\mu\nu}\cdot\boldsymbol{L}^{\mu\nu}\,, \tag{2.29}$$

and $\mathcal{L}_{\mathrm{N}}$, $\mathcal{L}_\pi$, and $\mathcal{L}_\omega$ are given by Eqs (2.15), (2.16), and (2.17), respectively, is locally $\mathrm{SU(2)_R} \times \mathrm{SU(2)_L}$ gauge invariant, provided that all derivatives are interpreted as the covariant derivatives from Eq. (2.27). For example, the scalar–pion lagrangian now reads

$$\mathcal{L}_\pi = \tfrac{1}{2}\,\mathrm{tr}\left[(D_\mu\chi)^\dagger D^\mu\chi\right] - V(\mathrm{tr}\,\chi^\dagger\chi)\,. \tag{2.30}$$

The parity invariance of the action $S = \int \mathrm{d}^4x\mathcal{L}$ can also be verified using the relations (2.22) together with

$$\hat{\mathcal{P}}\boldsymbol{r}^\mu(t,\boldsymbol{x})\hat{\mathcal{P}}^{-1} = \boldsymbol{\ell}_\mu(t,-\boldsymbol{x})\,, \quad \hat{\mathcal{P}}\boldsymbol{\ell}^\mu(t,\boldsymbol{x})\hat{\mathcal{P}}^{-1} = \boldsymbol{r}_\mu(t,-\boldsymbol{x})\,. \tag{2.31}$$

These last relations make it clear that the gauge coupling G must be the same for the left and right vector fields if parity invariance is to be maintained.

2.3. The Higgs sector

As noted above, local chiral gauge invariance precludes the addition of mass terms for the isovector mesons. To give these mesons masses, we shall use spontaneous symmetry breaking and the Higgs mechanism, as in the standard model of electroweak interactions [20–23]. We therefore introduce two complex doublets of spinless fields:

$$\phi_{\mathrm{R}} \equiv \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}_{\mathrm{R}} , \qquad \phi_{\mathrm{L}} \equiv \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}_{\mathrm{L}} , \tag{2.32}$$

which transform as the fundamental representation under global SU(2) transformations for each group:

$$\begin{aligned} \phi_{\mathrm{R}} &\underset{\mathrm{R}}{\longrightarrow} U\phi_{\mathrm{R}} , \qquad & \phi_{\mathrm{L}} &\underset{\mathrm{R}}{\longrightarrow} \phi_{\mathrm{L}} , \\ \phi_{\mathrm{L}} &\underset{\mathrm{L}}{\longrightarrow} U\phi_{\mathrm{L}} , \qquad & \phi_{\mathrm{R}} &\underset{\mathrm{L}}{\longrightarrow} \phi_{\mathrm{R}} . \end{aligned} \tag{2.33}$$

Thus any meson–meson potential that depends on $\phi_{\mathrm{R}}^{\dagger}\phi_{\mathrm{R}}$ or $\phi_{\mathrm{L}}^{\dagger}\phi_{\mathrm{L}}$ is invariant.

For the kinetic energies of these fields, we define the covariant derivatives

$$D_\mu\phi_{\mathrm{R}} \equiv (\partial_\mu + \frac{i}{2}G\,\boldsymbol{\tau}\cdot\boldsymbol{r}_\mu)\phi_{\mathrm{R}} , \qquad D_\mu\phi_{\mathrm{L}} \equiv (\partial_\mu + \frac{i}{2}G\,\boldsymbol{\tau}\cdot\boldsymbol{\ell}_\mu)\phi_{\mathrm{L}} . \tag{2.34}$$

Thus the combination $[(D_\mu\phi_{\mathrm{L}})^{\dagger}D^\mu\phi_{\mathrm{L}}]$ is locally gauge invariant, and similarly for ϕ_{R}.

Under parity transformations, we have

$$\hat{\mathcal{P}}\phi_{\mathrm{R}}(t,\boldsymbol{x})\hat{\mathcal{P}}^{-1} = \phi_{\mathrm{L}}(t,-\boldsymbol{x}) , \qquad \hat{\mathcal{P}}\phi_{\mathrm{L}}(t,\boldsymbol{x})\hat{\mathcal{P}}^{-1} = \phi_{\mathrm{R}}(t,-\boldsymbol{x}) , \tag{2.35}$$

which implies both that the gauge coupling G must be the same for the right and left fields and that the meson–meson potential must contain the same parameters for the right and left fields. Thus we are led to the gauge-invariant Higgs lagrangian

$$\begin{aligned} \mathcal{L}_{\mathrm{H}} \equiv {} & [(D_\mu\phi_{\mathrm{R}})^{\dagger}(D^\mu\phi_{\mathrm{R}}) + (D_\mu\phi_{\mathrm{L}})^{\dagger}(D^\mu\phi_{\mathrm{L}})] + \mu_{\mathrm{H}}^2(\phi_{\mathrm{R}}^{\dagger}\phi_{\mathrm{R}} + \phi_{\mathrm{L}}^{\dagger}\phi_{\mathrm{L}}) \\ & - \frac{\lambda_{\mathrm{H}}}{4}[(\phi_{\mathrm{R}}^{\dagger}\phi_{\mathrm{R}})^2 + (\phi_{\mathrm{L}}^{\dagger}\phi_{\mathrm{L}})^2] . \end{aligned} \tag{2.36}$$

By taking $\mu_{\mathrm{H}}^2 > 0$, the Higgs potential will allow for spontaneous symmetry breaking, and due to the couplings between the Higgs and gauge fields contained in the covariant derivatives, this will generate masses for the isovector mesons.

2.4. Summary

By combining the previous results, we can exhibit the lagrangian for our chiral QHD model. If we call this model QHD–III, the lagrangian is given by

$$\mathcal{L}_{\rm III} = \mathcal{L}_{\rm N} + \mathcal{L}_{\pi} + \mathcal{L}_{\rm G} + \mathcal{L}_{\rm H} + \mathcal{L}_{\rm SV} \,, \tag{2.37}$$

where $\mathcal{L}_{\rm N}$, $\mathcal{L}_{\pi}$, $\mathcal{L}_{\rm G}$, $\mathcal{L}_{\rm H}$, and $\mathcal{L}_{\rm SV}$ are given by Eqs (2.15), (2.30), (2.29) and (2.17), (2.36), and (2.3), respectively, the scalar–pion self-interactions are determined by Eq. (2.18), and the covariant derivatives are given by Eqs (2.27) and (2.34). The action resulting from this local, Lorentz-invariant lagrangian density is parity invariant, and in the absence of the symmetry-violating term $\mathcal{L}_{\rm SV}$, the lagrangian is invariant under *local* $\mathrm{SU(2)_R \times SU(2)_L}$ gauge transformations. It is a minimal lagrangian containing the desired degrees of freedom that obeys these constraints.

3. Representation in terms of physical fields

The model lagrangian $\mathcal{L}_{\rm III}$ has been constructed by imposing the required symmetry constraints on a system containing the desired hadron fields. The form in Eq. (2.37) is useful for demonstrating the chiral invariance of the theory, but for practical calculations, it is more efficient to rewrite the lagrangian in terms of physical fields and familiar parameters. This rewriting involves the following operations:

- Define vector meson fields with well-defined parity (representing the ρ and $\mathrm{a_1}$ mesons) to replace the chiral gauge fields $\boldsymbol{r}^{\mu}$ and $\boldsymbol{\ell}^{\mu}$.
- Spontaneously break the global chiral symmetry to give the nucleon a mass and rewrite the nucleon and pion sector in terms of fields with zero expectation value in the vacuum.
- Spontaneously break the local chiral symmetry to give the vector mesons mass and rewrite the field variables in the so-called "unitary gauge."
- Ensure that the resulting lagrangian contains no bilinear terms that mix fields. This is necessary to define the appropriate noninteracting parts of the lagrangian and the corresponding noninteracting propagators for use in the Feynman rules.

We now consider each of these procedures in turn.

Vector meson fields with well-defined parity can be constructed by taking linear combinations of the left and right gauge fields. We will denote the ρ meson field by $\boldsymbol{b}_{\mu}$ and the $\mathrm{a_1}$ field by $\boldsymbol{a}_{\mu}$, where

$$\boldsymbol{a}_{\mu} \equiv \frac{1}{\sqrt{2}}(\boldsymbol{r}_{\mu} - \boldsymbol{\ell}_{\mu}) \,, \qquad \boldsymbol{b}_{\mu} \equiv \frac{1}{\sqrt{2}}(\boldsymbol{r}_{\mu} + \boldsymbol{\ell}_{\mu}) \,. \tag{3.1}$$

The overall factors of $1/\sqrt{2}$ imply that the jacobian of this transformation is unity, so that the field-strength tensors become

$$\boldsymbol{R}_{\mu\nu}\cdot\boldsymbol{R}^{\mu\nu}+\boldsymbol{L}_{\mu\nu}\cdot\boldsymbol{L}^{\mu\nu}=\boldsymbol{A}_{\mu\nu}\cdot\boldsymbol{A}^{\mu\nu}+\boldsymbol{B}_{\mu\nu}\cdot\boldsymbol{B}^{\mu\nu}\,, \tag{3.2}$$

where

$$\begin{aligned}\boldsymbol{A}_{\mu\nu}&\equiv\partial_\mu\boldsymbol{a}_\nu-\partial_\nu\boldsymbol{a}_\mu-g_\rho(\boldsymbol{b}_\mu\times\boldsymbol{a}_\nu+\boldsymbol{a}_\mu\times\boldsymbol{b}_\nu)\,,\\ \boldsymbol{B}_{\mu\nu}&\equiv\partial_\mu\boldsymbol{b}_\nu-\partial_\nu\boldsymbol{b}_\mu-g_\rho(\boldsymbol{b}_\mu\times\boldsymbol{b}_\nu+\boldsymbol{a}_\mu\times\boldsymbol{a}_\nu)\,.\end{aligned} \tag{3.3}$$

Here we have defined $G\equiv\sqrt{2}g_\rho$ in terms of the physical ρ meson coupling constant. Because of parity conservation, this single coupling defines the interactions of both the ρ and a_1 mesons.

The properties of the $\boldsymbol{b}_\mu$ and $\boldsymbol{a}_\mu$ fields under parity transformations follow from Eq. (2.31) as

$$\hat{\mathcal{P}}\boldsymbol{b}_\mu(t,\boldsymbol{x})\hat{\mathcal{P}}^{-1}=\boldsymbol{b}^\mu(t,-\boldsymbol{x})\,,\qquad \hat{\mathcal{P}}\boldsymbol{a}_\mu(t,\boldsymbol{x})\hat{\mathcal{P}}^{-1}=-\boldsymbol{a}^\mu(t,-\boldsymbol{x})\,, \tag{3.4}$$

and thus the ρ is a polar vector meson and the a_1 is an axial vector meson[2]. These results also imply that the field strength tensors $\boldsymbol{B}^{\mu\nu}$ and $\boldsymbol{A}^{\mu\nu}$ have well-defined parity transformation properties. Moreover, the gauge transformations of the new fields can be deduced from Eqs (2.23) and (2.24):

$$\begin{aligned}\boldsymbol{a}^\mu&\longrightarrow\boldsymbol{a}^\mu-\boldsymbol{\alpha}\times\boldsymbol{a}^\mu-\boldsymbol{\beta}\times\boldsymbol{b}^\mu-g_\rho^{-1}\partial^\mu\boldsymbol{\beta}\,,\\ \boldsymbol{b}^\mu&\longrightarrow\boldsymbol{b}^\mu-\boldsymbol{\alpha}\times\boldsymbol{b}^\mu-\boldsymbol{\beta}\times\boldsymbol{a}^\mu-g_\rho^{-1}\partial^\mu\boldsymbol{\alpha}\,,\end{aligned} \tag{3.5}$$

where $\boldsymbol{\alpha}(x)\equiv\frac{1}{2}[\omega_{\rm R}(x)+\omega_{\rm L}(x)]$ and $\boldsymbol{\beta}(x)\equiv\frac{1}{2}[\omega_{\rm R}(x)-\omega_{\rm L}(x)]$. We will postpone rewriting the covariant derivatives in terms of the $\boldsymbol{b}^\mu$ and $\boldsymbol{a}^\mu$ fields.

To respect chiral symmetry, no baryon mass term is allowed in the lagrangian $\mathcal{L}_{\rm chiral}$. The baryon mass can be generated by spontaneous symmetry breaking, [17, 1], which arises from the form of the potential $V(\mathrm{tr}\,\chi^\dagger\chi)$. Symmetry breaking implies that the scalar field s has a nonzero vacuum expectation value $\langle s\rangle$ and that the pion is a massless Goldstone boson (in the limit $\epsilon=0$).

After defining a shifted scalar field $\sigma\equiv\langle s\rangle-s$ and the physical masses through

$$M=g_\pi\langle s\rangle\,,\quad \epsilon=\frac{M}{g_\pi}m_\pi^2\,,\quad \lambda=\frac{m_\sigma^2-m_\pi^2}{2M^2}g_\pi^2\,, \tag{3.6}$$

one can rewrite $\mathcal{L}_{\rm N}+\mathcal{L}_\pi+\mathcal{L}_{\rm SV}$ so that the particle masses are explicit. The algebra has been discussed in the indicated references and the final result

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[2] Note that the $a_1(1260)$, not the $b_1(1235)$, is the chiral partner of the $\rho(770)$, since the former has the correct G-parity.

will be given below. Note that the explicit violation of chiral symmetry is contained entirely in the parameter m_π, the pion mass.

A similar spontaneous symmetry breaking mechanism is applied to the Higgs potential to generate the vector meson masses. To maintain both parity and charge conservation, a nonzero vacuum expectation value can be given only to a combination of Higgs fields that is a neutral, positive-parity scalar. Thus we take the expectation values

$$\langle \phi_{\mathrm{R}} \rangle = \langle \phi_{\mathrm{L}} \rangle = \tfrac{1}{2} \begin{pmatrix} 0 \\ u \end{pmatrix} , \tag{3.7}$$

and redefine the eight spinless fields in ϕ_{R} and ϕ_{L} according to

$$\begin{aligned} \phi_{\mathrm{R}} &\equiv \tfrac{1}{2} \exp(-i\boldsymbol{\tau}\cdot\boldsymbol{\rho}/2u) \begin{pmatrix} 0 \\ u + \eta + \xi \end{pmatrix} , \\ \phi_{\mathrm{L}} &\equiv \tfrac{1}{2} \exp(-i\boldsymbol{\tau}\cdot\boldsymbol{\lambda}/2u) \begin{pmatrix} 0 \\ u + \eta - \xi \end{pmatrix} . \end{aligned} \tag{3.8}$$

Here $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$ are triplets of right- and left-handed spinless fields that will be absorbed by the isovector $\boldsymbol{b}^\mu$ and $\boldsymbol{a}^\mu$ fields. The remaining Higgs fields η and ξ are respectively scalar and pseudoscalar:

$$\hat{\mathcal{P}}\eta(t,\boldsymbol{x})\hat{\mathcal{P}}^{-1} = \eta(t,-\boldsymbol{x}) , \qquad \hat{\mathcal{P}}\xi(t,\boldsymbol{x})\hat{\mathcal{P}}^{-1} = -\xi(t,-\boldsymbol{x}) . \tag{3.9}$$

The algebra that follows after inserting the definitions (3.8) into $\mathcal{L}_{\mathrm{H}}$ is discussed in the literature [20] and will not be repeated here. The parameter u is chosen to cancel the linear term in the classical Higgs potential, and μ_{H}^2 and λ_{H} are replaced by the Higgs mass m_{H} and rho mass m_ρ according to

$$\mu_{\mathrm{H}}^2 = \tfrac{1}{2} m_{\mathrm{H}}^2 , \quad u^2 = \frac{8\mu_{\mathrm{H}}^2}{\lambda_{\mathrm{H}}} = \frac{4m_\rho^2}{g_\rho^2} , \quad \lambda_{\mathrm{H}} = \frac{m_{\mathrm{H}}^2 g_\rho^2}{m_\rho^2} . \tag{3.10}$$

(Both Higgs fields η and ξ have the same mass.) The remaining fields in $\mathcal{L}_{\mathrm{III}}$ are then redefined using a particular point transformation, which is often called "choosing the unitary gauge." (For the details of these manipulations, see Ref. [20].) The particle content of the theory now becomes transparent, as it is written in terms of fields with zero expectation value in the vacuum, and the isovector-vector mesons are massive.

After carrying out all of the above procedures, and after a slight reshuffling of mass terms between parts of the lagrangian, one arrives at the desired result:

$$\mathcal{L}_{\mathrm{III}} = \mathcal{L}_{\mathrm{N}} + \mathcal{L}_{\sigma\pi} + \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{H}} , \tag{3.11}$$

where the nucleon contribution is

$$\mathcal{L}_{\rm N} = \overline{\psi}\Big\{i\gamma^\mu\Big[\partial_\mu + ig_{\rm v}V_\mu + \tfrac{i}{2}\, g_\rho \boldsymbol{\tau}\cdot(\boldsymbol{b}_\mu + \gamma_5 \boldsymbol{a}_\mu)\Big] - (M - g_\pi\sigma) - ig_\pi\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\Big\}\psi\,. \tag{3.12}$$

The scalar and pion contribution is given by

$$\begin{aligned}\mathcal{L}_{\sigma\pi} =& \tfrac{1}{2}\Big[(\partial_\mu\sigma - g_\rho\boldsymbol{\pi}\cdot\boldsymbol{a}_\mu)^2 - m_\sigma^2\sigma^2\Big] \\ &+ \tfrac{1}{2}\Big[(\partial_\mu\boldsymbol{\pi} + g_\rho\sigma\boldsymbol{a}_\mu + g_\rho\boldsymbol{\pi}\times\boldsymbol{b}_\mu)^2 - m_\pi^2\boldsymbol{\pi}^2\Big] \\ &- \Big(\frac{g_\rho}{g_\pi}\Big)M\boldsymbol{a}_\mu\cdot(\partial^\mu\boldsymbol{\pi} + g_\rho\sigma\boldsymbol{a}^\mu + g_\rho\boldsymbol{\pi}\times\boldsymbol{b}^\mu) - V(\sigma,\boldsymbol{\pi})\,, \end{aligned}\tag{3.13}$$

$$V(\sigma,\boldsymbol{\pi}) = -\,g_\pi\frac{m_\sigma^2 - m_\pi^2}{2M}\,\sigma(\sigma^2 + \boldsymbol{\pi}^2) + g_\pi^2\frac{m_\sigma^2 - m_\pi^2}{8M^2}(\sigma^2 + \boldsymbol{\pi}^2)^2, \tag{3.14}$$

and the mass and kinetic terms for the vector fields are

$$\begin{aligned}\mathcal{L}_{\rm G} =& -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_{\rm v}^2V_\mu V^\mu - \tfrac{1}{4}\boldsymbol{B}_{\mu\nu}\cdot\boldsymbol{B}^{\mu\nu} + \tfrac{1}{2}m_\rho^2\boldsymbol{b}_\mu\cdot\boldsymbol{b}^\mu - \tfrac{1}{4}\boldsymbol{A}_{\mu\nu}\cdot\boldsymbol{A}^{\mu\nu} \\ &+\tfrac{1}{2}\Big[m_\rho^2 + \Big(\frac{g_\rho}{g_\pi}\Big)^2M^2\Big]\boldsymbol{a}_\mu\cdot\boldsymbol{a}^\mu\,, \end{aligned}\tag{3.15}$$

where the vector meson field tensors $\boldsymbol{A}_{\mu\nu}$ and $\boldsymbol{B}_{\mu\nu}$ are defined in Eq. (3.3). Finally, the lagrangian for the Higgs sector is

$$\begin{aligned}\mathcal{L}_{\rm H} =& \tfrac{1}{2}(\partial_\mu\eta\partial^\mu\eta - m_{\rm H}^2\eta^2) + \tfrac{1}{2}(\partial_\mu\xi\partial^\mu\xi - m_{\rm H}^2\xi^2) \\ &+ \tfrac{1}{2}\Big[g_\rho m_\rho\eta + \tfrac{1}{4}g_\rho^2(\eta^2+\xi^2)\Big](\boldsymbol{b}_\mu\cdot\boldsymbol{b}^\mu + \boldsymbol{a}_\mu\cdot\boldsymbol{a}^\mu) + \Big(g_\rho m_\rho\xi + \tfrac{1}{2}g_\rho^2\eta\xi\Big)\boldsymbol{b}_\mu\cdot\boldsymbol{a}^\mu \\ &- \Big(\frac{3m_{\rm H}^2g_\rho}{4m_\rho}\eta + \frac{3m_{\rm H}^2g_\rho^2}{16m_\rho^2}\eta^2\Big)\xi^2 - \frac{m_{\rm H}^2g_\rho}{4m_\rho}\eta^3 - \frac{m_{\rm H}^2g_\rho^2}{32m_\rho^2}(\eta^4+\xi^4)\,. \end{aligned}\tag{3.16}$$

As discussed in Ref. [1], the Higgs mesons are to be given a large mass so that they function as "regulators" that maintain renormalizability with minimal effects on the low-energy predictions of the theory.

An examination of the final line in Eq. (3.13) reveals that our manipulations are not yet complete, since there is still a bilinear term that mixes $\boldsymbol{a}_\mu$ and $\partial^\mu\boldsymbol{\pi}$. This coupling, which arises from the spontaneous symmetry breaking, can be removed by shifting the $\boldsymbol{a}_\mu$ field according to

$$\boldsymbol{a}_\mu \longrightarrow \boldsymbol{a}_\mu + \Big(\frac{g_\rho M}{g_\pi m_{\rm a}^2}\Big)\partial_\mu\boldsymbol{\pi} = \boldsymbol{a}_\mu + \frac{(m_{\rm a}^2 - m_\rho^2)^{1/2}}{m_{\rm a}^2}\,\partial_\mu\boldsymbol{\pi}\,, \tag{3.17}$$

where we have defined the $\mathbf{a}_1$ mass [see Eq. (3.15)] by

$$m_{\mathrm{a}}^2 \equiv m_\rho^2 + (g_\rho M/g_\pi)^2 \,. \tag{3.18}$$

The transformation (3.17) leaves $\partial_\mu \boldsymbol{a}_\nu - \partial_\nu \boldsymbol{a}_\mu$ unchanged, so the quadratic kinetic energy terms in $-\frac{1}{4}\boldsymbol{A}_{\mu\nu}\cdot\boldsymbol{A}^{\mu\nu}$ are unaffected. The canonical normalization for the pion field can now be achieved by rescaling it according to

$$\boldsymbol{\pi} \longrightarrow \left[1 - \left(\frac{g_\rho M}{g_\pi m_{\mathrm{a}}}\right)^2\right]^{-1/2} \boldsymbol{\pi} = \left(\frac{m_{\mathrm{a}}}{m_\rho}\right)\boldsymbol{\pi} \,. \tag{3.19}$$

It is easy to verify that with the indicated redefinitions, the noninteracting lagrangians for the $\boldsymbol{\pi}$ and $\boldsymbol{a}_\mu$ fields take their standard forms. Nevertheless, our previous identification of m_π as the pion mass (which is absent in the chiral-invariant theory) is incorrect due to the mixing of $\partial_\mu \boldsymbol{\pi}$ and $\boldsymbol{a}_\mu$; the correct physical pion mass can be restored by making the final replacement

$$m_\pi^2 \longrightarrow \left[1 - \left(\frac{g_\rho M}{g_\pi m_{\mathrm{a}}}\right)^2\right] m_\pi^2 = \left(\frac{m_\rho}{m_{\mathrm{a}}}\right)^2 m_\pi^2 \,. \tag{3.20}$$

Since the multiplicative factor on the right-hand side is essentially unity for reasonable masses and couplings, the explicit violation of chiral symmetry is still small. The $\mathbf{a}_1$ mass m_{a} remains as in Eq. (3.18). We will not exhibit the final expressions obtained by making the preceding replacements, since the results are not particularly illuminating.

The lagrangian given by Eqs (3.11)–(3.16) defines a minimal, parity conserving, locally chiral invariant model (when $m_\pi = 0$) containing the desired degrees of freedom (p, n, π, ρ, and ω). The additional fields σ, $\boldsymbol{a}_\mu$, η, and ξ necessarily appear to maintain the local chiral invariance. It is straightforward to verify that $\mathcal{L}_{\mathrm{III}}$ (with $m_\pi = 0$) is still invariant under a set of global isospin and chiral transformations, leading to the conserved vector and axial isovector currents

$$\begin{aligned}
\mathbf{T}^\mu =& \tfrac{1}{2}\overline{\psi}\gamma^\mu \boldsymbol{\tau}\psi + \boldsymbol{\pi}\times\left[\partial^\mu\boldsymbol{\pi} + g_\rho \boldsymbol{\pi}\times\boldsymbol{b}^\mu - \left(\frac{g_\rho}{g_\pi}\right)(M - g_\pi\sigma)\boldsymbol{a}^\mu\right] \\
&+ \boldsymbol{b}_\nu \times \boldsymbol{B}^{\nu\mu} + \boldsymbol{a}_\nu \times \boldsymbol{A}^{\nu\mu} \,,
\end{aligned} \tag{3.21}$$

$$\begin{aligned}
\mathbf{A}^\mu =& \tfrac{1}{2}\overline{\psi}\gamma^\mu\gamma_5\boldsymbol{\tau}\psi + \boldsymbol{\pi}\partial^\mu\sigma + \left(\frac{M}{g_\pi} - \sigma\right)\partial^\mu\boldsymbol{\pi} \\
&+ g_\rho\left(\frac{M}{g_\pi} - \sigma\right)\boldsymbol{\pi}\times\boldsymbol{b}^\mu - g_\rho\boldsymbol{\pi}(\boldsymbol{\pi}\cdot\boldsymbol{a}^\mu) \\
&- g_\rho\left(\frac{M}{g_\pi} - \sigma\right)^2 \boldsymbol{a}^\mu + \boldsymbol{b}_\nu\times\boldsymbol{A}^{\nu\mu} + \boldsymbol{a}_\nu\times\boldsymbol{B}^{\nu\mu} \,.
\end{aligned} \tag{3.22}$$

For a nonzero pion mass, we find the PCAC relation

$$\partial_\mu \mathbf{A}^\mu = -\epsilon\boldsymbol{\pi} = -\left(\frac{M m_\pi^2}{g_\pi}\right)\boldsymbol{\pi} \,, \tag{3.23}$$

which follows from the field equations. (To express the currents in terms of physical fields, one must redefine the a^μ and $\boldsymbol{\pi}$ fields as discussed above.)

Moreover, since the lagrangian $\mathcal{L}_{\rm III}$ still obeys the (now hidden) local $\mathrm{SU}(2)_\mathrm{R} \times \mathrm{SU}(2)_\mathrm{L}$ gauge invariance, and since all masses have been generated by spontaneous symmetry breaking and the Higgs mechanism, it is tempting to conclude that the field theory described by $\mathcal{L}_{\rm III}$ is *renormalizable*. There are, however, two problems with this conclusion. First, because of the explicit violation of the chiral symmetry when $m_\pi \neq 0$, the axial current is not conserved and instead obeys the PCAC relation (3.23). Since the proof of renormalizability in massive Yang–Mills theory relies on the conservation of the relevant current, it is possible that this symmetry violation destroys the renormalizability. However, an examination of Eqs (3.11)–(3.16) shows that the parameter m_π enters fairly innocuously; it will appear only in the pion propagator and in the $\sigma\boldsymbol{\pi}$ self-interactions, whose strength is arbitrary, since m_σ is a free parameter. It is possible that this "soft" violation of the symmetry will not destroy the required cancellations between baryon, gauge boson, and ghost loops (when the theory is quantized) that are necessary to maintain renormalizability. Nevertheless, we have no proof of this result. To ensure renormalizability, it may be necessary to compute quantum loops in this theory in the exact chiral limit, with $m_\pi = 0$. (One can certainly retain a finite m_π at the tree level.)

Second, and much more important, is the possibility of chiral anomalies. These are known to arise in chiral gauge theories, and one of the consequences is the loss of renormalizability [9]. It is therefore necessary to address this question in some detail, and we turn now to this point.

4. Cancellation of chiral anomalies

In the presence of both vector and axial-vector couplings to the mesons, it is possible that quantum loop corrections will modify the conservation of the axial current, change the axial Ward identities, and destroy the renormalizability of the theory [8, 9, 24]. More generally, as discussed below, the *fermion measure* in the quantum-mechanical path integral may not be invariant under chiral gauge transformations, and physical quantities then become gauge dependent [25, 11, 12, 26]. In other words, the symmetries of the classical lagrangian may not remain when the theory is quantized.

As a simple introduction, and to provide some insight into both the problem and the proposed solution, consider first the fermion-loop triangle diagrams in QED, as illustrated in Fig. 1.

Wick's theorem implies that these two diagrams provide separate contributions to the S matrix, and the combined contribution is proportional

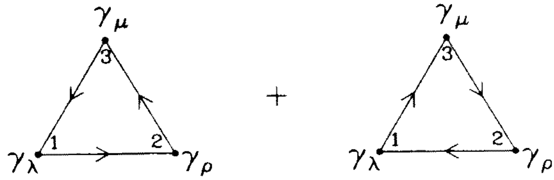

Fig. 1. Fermion-loop triangle diagrams in QED. Application of the Feynman rules to these two diagrams generates the analytic expression in Eq. (4.1).

to

$$L_1 + L_2 = \int d^4x_1 d^4x_2 d^4x_3 \; \mathrm{Tr}\{G_F(x_1 - x_3)\gamma_\mu G_F(x_3 - x_2)\gamma_\rho G_F(x_2 - x_1)\gamma_\lambda + G_F(x_1 - x_2)\gamma_\rho G_F(x_2 - x_3)\gamma_\mu G_F(x_3 - x_1)\gamma_\lambda\}, \quad (4.1)$$

where $G_F(x - y)$ is the noninteracting fermion propagator, and the uppercase "Tr" denotes a trace over Dirac indices. Now make use of the existence of the Dirac (charge conjugation) matrix C satisfying

$$C\gamma_\mu C^{-1} = -\gamma_\mu^T, \quad (4.2)$$

which implies

$$CG_F(x - y)C^{-1} = G_F(y - x)^T. \quad (4.3)$$

Insertion of $C^{-1}C$ between all the factors in Eq. (4.1) and use of the cyclic property of the trace then yields

$$L_1 + L_2 = \int d^4x_1 d^4x_2 d^4x_3 \; \mathrm{Tr}\{(-1)^3 G_F(x_3 - x_1)^T \gamma_\mu^T G_F(x_2 - x_3)^T \gamma_\rho^T G_F(x_1 - x_2)^T \gamma_\lambda^T + (-1)^3 G_F(x_2 - x_1)^T \gamma_\rho^T G_F(x_3 - x_2)^T \gamma_\mu^T G_F(x_1 - x_3)^T \gamma_\lambda^T\}. \quad (4.4)$$

Use of the cyclic property again, together with the relation

$$\mathrm{Tr}\{a^T b^T \ldots y^T z^T\} = \mathrm{Tr}\{zy \ldots ba\}, \quad (4.5)$$

leads to

$$L_1 + L_2 = \int d^4x_1 d^4x_2 d^4x_3 \; \mathrm{Tr}\{(-1)^3 G_F(x_1 - x_2)\gamma_\rho G_F(x_2 - x_3)\gamma_\mu G_F(x_3 - x_1)\gamma_\lambda + (-1)^3 G_F(x_1 - x_3)\gamma_\mu G_F(x_3 - x_2)\gamma_\rho G_F(x_2 - x_1)\gamma_\lambda\} = -L_2 - L_1. \quad (4.6)$$

Hence $L_1 + L_2 = 0$, and these two contributions cancel identically in QED. (This is Furry's theorem.)

Now include the possibility of inserting a γ_5 at any vertex in the loop, and consider triangle diagrams with an odd number of γ_5's, so that one is calculating a potential contribution to the axial-vector current. Since

$$C\gamma_5 C^{-1} = \gamma_5 , \qquad C\gamma_\mu\gamma_5 C^{-1} = (\gamma_\mu\gamma_5)^T , \tag{4.7}$$

one generates a factor of $(+1)$ at each vertex containing a $\gamma_\mu\gamma_5$, instead of the factor (-1) found above. Hence the contribution from the two triangle diagrams now *add*:

$$L_2 = +L_1 , \qquad \text{odd number of} \qquad \gamma_5{}'\text{s} . \tag{4.8}$$

Thus the sum of these diagrams can produce an anomalous contribution to the axial-vector current and its divergence.

Now suppose that the fermion is an isodoublet $\psi = \binom{\mathrm{p}}{\mathrm{n}}$, as in QHD, and that each of the vertices has an isovector coupling proportional to τ_i. Then, since the isospin trace factors out of each loop integral, the sum of those graphs with an odd number of γ_5's will again vanish:

$$\mathrm{tr}(\tau_i\tau_j\tau_k)L_1 + \mathrm{tr}(\tau_i\tau_k\tau_j)L_2 = \mathrm{tr}(\tau_i\{\tau_j, \tau_k\})L_1 = 0 . \tag{4.9}$$

Note that the sum of the loops vanishes here because the required trace of the τ matrices is zero. Loops with an even number of γ_5's can be shown not to produce anomalies [8]. Thus, in this $\mathrm{SU(2)_R} \times \mathrm{SU(2)_L}$ theory, there are no anomalies at the triangle level.

What happens if the loop contains an odd number of γ_5 matrices, but there is a coupling to an *isoscalar* vector meson at one vertex, so that the contributions of the two loops do not cancel, and the trace of the τ matrices does not vanish? This is the case in QHD with one axial vector vertex ($L_1 = L_2$) and two isovector vertices [$\mathrm{tr}(\tau_j\tau_k) \neq 0$]. Now, however, one can arrange for the triangle loops to cancel by the following device. Take the isoscalar vector meson to couple to a fermion charge, assumed for the ω to be the strong hypercharge $y = B + S$, with $y = 1$ for the nucleon; now add *a second fermion* isodoublet to the theory with identical vector and axial-vector couplings, but with opposite hypercharge, for example, the $\Xi = \binom{\Xi^0}{\Xi^-}$ with $y = -1$.

There are now *four* triangle diagrams, as illustrated in Fig. 2. Although the loops do not cancel exactly when the fermions have different masses, the anomalous contributions to the divergence of the axial current are independent of the fermion mass [9], and thus the anomaly from the second set of

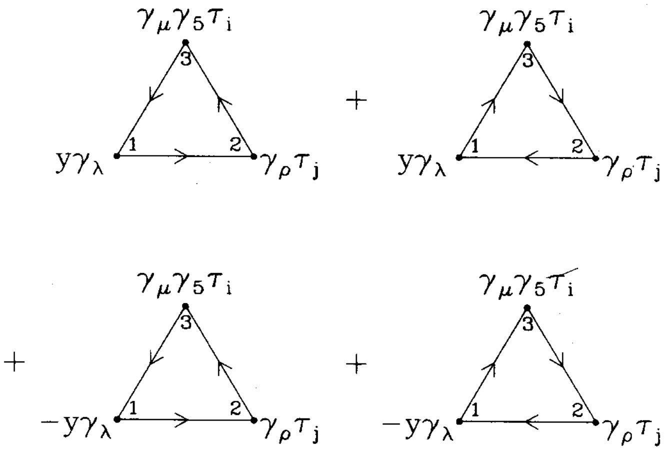

Fig. 2. Triangle-loop diagrams for $SU(2)_R \times SU(2)_L$ with an additional isoscalar vector meson coupled to the conserved hypercharge current and two fermion isodoublets with opposite hypercharge.

loops with $y = -1$ cancels that from the first set with $y = +1$. This model therefore eliminates chiral anomalies at the triangle level[3].

The preceding arguments apply only at the level of triangle loops. To investigate the entire problem of potential chiral anomalies, one needs more powerful methods. The chiral anomalies can be viewed as arising because the fermion measure in the quantum-mechanical path integral is, in general, not invariant under chiral transformations. This implies that in any quantum field theory with fermions coupled to vector or axial vector fields, it is impossible to satisfy *simultaneously* the vector and axial Ward identities derived from the lagrangian through Noether's theorem. When anomalies are present, physical quantities will be gauge dependent and the quantum gauge theory is ill-defined. However, if one can choose the particle content of the theory so that the complete measure is invariant under chiral transformations, then one will have succeeded in eliminating the anomalies.

Define the fermion integration measure as follows:

$$d\mu \equiv \mathcal{D}(\overline{\psi})\mathcal{D}(\psi), \tag{4.10}$$

and assume one has a gauge theory with a covariant derivative defined by

$$\begin{aligned} D_\mu &= \partial_\mu - V_\mu - A_\mu\gamma_5 , \\ V_\mu &\equiv \lambda_V^a V_\mu^a , \qquad A_\mu \equiv \lambda_A^a A_\mu^a . \end{aligned} \tag{4.11}$$

[3] Although we have not proved it, radiative corrections to the lowest-order loops do not change the value of the anomaly [8].

Here λ_A^a and λ_V^a are hermitian matrices in some intrinsic space, and repeated Latin indices are summed.

If we now define a local chiral gauge transformation by [see Eq. (2.7)]

$$\psi(x) \to \exp\{i\beta(x)\gamma_5\}\psi(x)\,,$$
$$\beta(x) \equiv \beta^a(x)\lambda_A^a\,, \tag{4.12}$$

the general transformation of the fermion measure under this chiral transformation follows from the analysis of Fujikawa [11, 12] and is given by Einhorn and Jones [26]:

$$\mathrm{d}\mu \to \mathrm{d}\mu \exp\left\{-2i\int \mathrm{d}^4x\,\beta^a(x)\mathcal{A}^a(x)\right\}, \tag{4.13}$$

where

$$\mathcal{A}^a(x) \equiv -\frac{1}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}\left\{\lambda_A^a\left[\tfrac{1}{2}G_{\mu\nu}^V G_{\alpha\beta}^V + \tfrac{1}{6}G_{\mu\nu}^A G_{\alpha\beta}^A + \tfrac{4}{3}\left(A_\mu A_\nu G_{\alpha\beta}^V + G_{\mu\nu}^V A_\alpha A_\beta + A_\mu G_{\nu\alpha}^V A_\beta\right) + \tfrac{16}{3}A_\mu A_\nu A_\alpha A_\beta\right]\right\} \tag{4.14}$$

The field tensors are defined as matrices in the intrinsic space:

$$G_{\mu\nu}^V \equiv \partial_\mu V_\nu - \partial_\nu V_\mu - [V_\mu, V_\nu] - [A_\mu, A_\nu]\,,$$
$$G_{\mu\nu}^A \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - [V_\mu, A_\nu] - [A_\mu, V_\nu]\,. \tag{4.15}$$

The result (4.14) agrees with that of Bardeen [8] and thus obeys the consistency conditions of Wess and Zumino [27]. It also generates the *minimal* anomalous contributions, in that any redefinition of the path integral (by adding counterterms to the lagrangian) will either violate the Ward identity for the *vector* current or add more terms to the right-hand side of (4.14) [8].

In the case at hand, the covariant derivative is given by [see Eq. (3.12)]

$$D_\mu \equiv \partial_\mu + ig_\mathrm{v}V_\mu + \tfrac{i}{2}g_\rho\boldsymbol{\tau}\cdot\mathbf{b}_\mu + \tfrac{i}{2}g_\rho\boldsymbol{\tau}\cdot\mathbf{a}_\mu\gamma_5\,, \tag{4.16}$$

and Eq. (4.14) becomes

$$\begin{aligned}\mathcal{A}^a(x) = \frac{1}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}\Bigg(\frac{\tau^a}{2}\Bigg\{&\tfrac{1}{2}(g_\mathrm{v}F_{\mu\nu} + \tfrac{1}{2}g_\rho\boldsymbol{\tau}\cdot\boldsymbol{B}_{\mu\nu})(g_\mathrm{v}F_{\alpha\beta} + \tfrac{1}{2}g_\rho\boldsymbol{\tau}\cdot\boldsymbol{B}_{\alpha\beta})\\ &+ \tfrac{1}{24}\,g_\rho^2\,\boldsymbol{\tau}\cdot\boldsymbol{A}_{\mu\nu}\boldsymbol{\tau}\cdot\boldsymbol{A}_{\alpha\beta} - \tfrac{i}{3}\,g_\rho^2\Big[\boldsymbol{\tau}\cdot\boldsymbol{a}_\mu\boldsymbol{\tau}\cdot\boldsymbol{a}_\nu(g_\mathrm{v}F_{\alpha\beta} + \tfrac{1}{2}g_\rho\boldsymbol{\tau}\cdot\boldsymbol{B}_{\alpha\beta})\\ &+ (g_\mathrm{v}F_{\mu\nu} + \tfrac{1}{2}g_\rho\boldsymbol{\tau}\cdot\boldsymbol{B}_{\mu\nu})\boldsymbol{\tau}\cdot\boldsymbol{a}_\alpha\boldsymbol{\tau}\cdot\boldsymbol{a}_\beta + \boldsymbol{\tau}\cdot\boldsymbol{a}_\mu(g_\mathrm{v}F_{\nu\alpha} + \tfrac{1}{2}g_\rho\boldsymbol{\tau}\cdot\boldsymbol{B}_{\nu\alpha})\boldsymbol{\tau}\cdot\boldsymbol{a}_\beta\Big]\\ &- \tfrac{1}{3}\,g_\rho^4\,\boldsymbol{\tau}\cdot\boldsymbol{a}_\mu\boldsymbol{\tau}\cdot\boldsymbol{a}_\nu\boldsymbol{\tau}\cdot\boldsymbol{a}_\alpha\boldsymbol{\tau}\cdot\boldsymbol{a}_\beta\Bigg\}\Bigg)\,.\end{aligned} \tag{4.17}$$

After some algebra, one can show that the change in measure under a local gauge transformation in the gauge theory of Eq. (3.11) reduces to

$$\mathrm{d}\mu \rightarrow \mathrm{d}\mu \, \exp\left\{-i\,\frac{g_{\mathrm{v}} g_{\rho}}{8\pi^2} \int \mathrm{d}^4x\, \widetilde{F}^{\alpha\beta}\, \boldsymbol{\beta}\cdot\boldsymbol{B}'_{\alpha\beta}\right\}, \tag{4.18}$$

where $\widetilde{F}^{\alpha\beta} \equiv \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu}F_{\mu\nu}$, with $F_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$, and

$$\boldsymbol{B}'_{\mu\nu} \equiv \partial_\mu \boldsymbol{b}_\nu - \partial_\nu \boldsymbol{b}_\mu - g_\rho(\boldsymbol{b}_\mu \times \boldsymbol{b}_\nu) + g_\rho(\boldsymbol{a}_\mu \times \boldsymbol{a}_\nu)\,. \tag{4.19}$$

The result in Eq. (4.18) is indeed *linear* in the field and coupling constant of the ω meson, and thus the anomalies can be cancelled by the mechanism discussed above. Note also that if $g_{\mathrm{v}} = 0$, the theory with local $\mathrm{SU(2)_R} \times \mathrm{SU(2)_L}$ invariance is explicitly anomaly free (as we saw above at the triangle level).

We therefore extend the theory so that the fermion carries an additional internal charge (the hypercharge $y = S + B$) and couple the isoscalar ω to the corresponding conserved hypercurrent $\overline{\psi}\gamma^\mu y\psi$. The baryon part of the lagrangian now takes a form analogous to Eq. (2.15), with one set of terms for the baryon doublet ψ and another for the cascade doublet Ξ. The covariant derivatives look exactly as in Eq. (2.27), except that we make the replacement $g_{\mathrm{v}} \rightarrow y g_{\mathrm{v}}$, and we also replace the coupling g_π in (2.15) with $g_{0\pi} + y g_{1\pi}$, which will allow us to generate different masses for the N and Ξ by spontaneous symmetry breaking.

The spontaneous chiral-symmetry breaking proceeds exactly as before, and the couplings $g_{0\pi}$ and $g_{1\pi}$ are adjusted to produce the observed nucleon and cascade masses. The algebra is straightforward, and one finds

$$g_{0\pi} = \tfrac{1}{2}g_\pi\left(1 + \frac{M_\Xi}{M}\right), \quad g_{1\pi} = \tfrac{1}{2}g_\pi\left(1 - \frac{M_\Xi}{M}\right), \tag{4.20}$$

where M is the nucleon mass and g_π is the pion–nucleon coupling. The remainder of $\mathcal{L}_{\mathrm{III}}$ follows exactly as before, and only the baryon part of the lagrangian changes. In the end, the new lagrangian can be written as

$$\mathcal{L}_{\mathrm{III}} = \mathcal{L}_{\mathrm{B}} + \mathcal{L}_{\sigma\pi} + \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{H}}\,, \tag{4.21}$$

where the baryon contribution is now

$$\begin{aligned}\mathcal{L}_{\mathrm{B}} =& \overline{\Psi}\{i\gamma^\mu[\partial_\mu + iyg_{\mathrm{v}}V_\mu + \tfrac{i}{2}\,g_\rho\boldsymbol{\tau}\cdot(\boldsymbol{b}_\mu + \gamma_5\boldsymbol{a}_\mu)] \\ &- (M - g_\pi\sigma) - ig_\pi\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\}\Psi\,.\end{aligned} \tag{4.22}$$

Here we have introduced the hypercharge doublet Ψ composed of the two isodoublets:

$$\Psi \equiv \begin{pmatrix}\psi \\ \Xi\end{pmatrix}, \tag{4.23}$$

and the following quantities in Eq. (4.22) are to be interpreted as matrices in the hypercharge space:

$$y = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad M = \begin{pmatrix} M & 0 \\ 0 & M_\Xi \end{pmatrix}, \quad g_\pi = \begin{pmatrix} g_\pi & 0 \\ 0 & g_\pi M_\Xi / M \end{pmatrix}. \tag{4.24}$$

All other terms involve unit matrices in the hypercharge space.

The addition of the second fermion with opposite hypercharge produces a combined fermion measure that is indeed invariant under chiral gauge transformations:

$$d\mu = \mathcal{D}(\overline{\psi})\mathcal{D}(\psi)\mathcal{D}(\overline{\Xi})\mathcal{D}(\Xi) \rightarrow d\mu, \tag{4.25}$$

and hence the theory will be free of chiral anomalies. This cancellation can be interpreted in the language of fermion loops by observing that the total hypercharge of all species in the baryon sector is zero. The phenomenology of this new model in the hypercharge $y = 1$ sector, for which it is specifically designed, should be little changed by the addition of the second fermion, which only contributes at the loop level in that sector.

Note also that since the (electromagnetic) charges of the baryons add to zero, this theory will still be anomaly free if the local gauge group is extended to include electromagnetic interactions, as discussed in chapter 7 of Ref. [1][4].

Several comments are in order about our anomaly cancellation mechanism and the mass generation for the new fermions. Note first that we cannot cancel the anomalies by changing only the sign of the coupling to the axial vector meson $\mathbf{a}_\mu$. This corresponds to switching the gauge fields $\boldsymbol{\ell}_\mu \leftrightarrow \mathbf{r}_\mu$ in the covariant derivatives (2.27), which clearly violates local gauge invariance. Moreover, we cannot generate a mass for the new fermion by coupling it to the Higgs field (as is done, for example, in the standard electroweak theory [22, 23]), because our fermions appear in both right- and left-handed doublets, which cannot be combined with a Higgs doublet to produce an isoscalar Yukawa coupling. We must generate the Ξ mass by coupling it to the scalar and pion fields, and we cannot introduce new scalar and pion fields, since the spontaneous chiral-symmetry breaking would then produce another isovector of massless "pions", which are not observed. Thus the coupling-to-mass ratio g_π/M must be the same for the nucleon and the Ξ [see Eq. (3.6)]. There is no advantage to making the Ξ extremely massive, since the $\boldsymbol{\pi}\Xi$ coupling must scale accordingly, and loop diagrams involving the Ξ will remain as large as those involving nucleons. In summary, we are

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[4] We remark that the usual argument for hadronic contributions to $\pi^0 \rightarrow \gamma\gamma$ decay no longer holds in this model.

essentially forced into the mechanism described above, which we implement with a new fermion that has a mass comparable to the other hadrons in the theory.

5. Discussion

The purpose of this paper is to construct a renormalizable quantum field theory based on hadrons (*quantum hadrodynamics*) that is isospin invariant, chirally invariant, parity conserving, and that contains p, n, π, ω, and ρ. These hadrons are the most important low-mass degrees of freedom for describing the nucleon–nucleon force and nuclear structure.

We begin with the linear sigma–omega model, which contains nucleons, pions, an auxiliary scalar field to implement the chiral symmetry, and isoscalar, Lorentz vector ω mesons. This model is invariant under global isospin and chiral transformations. These global symmetries are then elevated to local symmetries, which requires the addition of vector and axial-vector gauge fields representing the ρ meson and its chiral partner, the a_1. To maintain the local $SU(2)_R \times SU(2)_L$ symmetry, the masses of the baryons and vector mesons are generated by spontaneous symmetry breaking and the Higgs mechanism.

The model lagrangian is locally chiral invariant and parity conserving. The spontaneous symmetry breaking naturally produces an a_1 mass that is larger than the ρ mass, although the former is still substantially smaller than the empirical a_1 mass for reasonable values of the coupling constants. (Explicit chiral-symmetry violation may be responsible for the remainder of the mass shift.) Moreover, although the classical lagrangian respects the local chiral symmetry, quantum corrections produce anomalies that violate the symmetry and render the theory nonrenormalizable. Since the chiral anomalies occur only in fermion-loop diagrams that contain a single coupling to the ω meson, they are eliminated by assuming that the ω couples to the conserved strong hypercharge current and by introducing another isodoublet of baryons that couples to the ω with a hypercharge opposite to that of the nucleon. For definiteness, we take this additional baryon to be the cascade (Ξ). Note that this cancellation mechanism differs from that usually suggested in the literature, because we reverse the sign of the coupling to the *isoscalar* vector meson rather than the sign of the coupling to the *isovector* axial-vector meson [9]. The complete model, which we call QHD–III, is isospin and chirally invariant, parity conserving, and apparently renormalizable.

One of our goals is to minimize the number of degrees of freedom and coupling constants. This has been achieved, since the current model (QHD–III) contains only one more parameter than the simpler model (QHD–II) involving nucleons, pions, and isoscalar (ω) and isovector (ρ) vector mesons [1]. In fact, *no new parameters* are needed to implement the chiral symmetry, as long as parity conservation is enforced. The single extra parameter (M_Ξ) enters only to achieve the cancellation of the chiral anomalies. The minimal set of hadrons required is $\{N, \Xi; \sigma, \omega, \pi, \rho, a; \eta, \xi\}$. These results are in contrast to the model proposed by Lovas and Sailer [13], where the number of degrees of freedom is not minimal, parity conservation is not imposed, and the lagrangian still produces chiral anomalies when quantized.

The lagrangian for our minimal model is given in terms of the physical degrees of freedom by Eqs (4.21), (4.22), (3.13), (3.14), (3.15), and (3.16). The shifts in Eqs (3.17), (3.19), and (3.20) must also be made to achieve the correct forms for the noninteracting meson lagrangians. The resulting lagrangian can be quantized using path-integral methods and well-known gauge-theory procedures, such as the implementation of the Faddeev–Popov *Ansatz* to handle the local gauge invariance [20, 1]. Predictions can then be made (and tested) for various hadronic observables; for example, the decay width of the a_1 meson can be calculated in terms of parameters determined from other processes (such as ρ meson decay).

Moreover, if one enlarges the local gauge group to include the U(1) of hypercharge [1], electromagnetic interactions can be included, and some electromagnetic decays of these hadrons can be calculated. (The theory still remains anomaly free, since the sum of the charges of the baryons is zero.)

To obtain predictions for nuclear matter from QHD–III, one must utilize the procedures advocated for chiral models in Refs [2] and [3]. First, the scalar field (σ) in the QHD–III lagrangian should *not* be identified with the low-mass scalar field of QHD–I. The σ of QHD–III is instead to be assigned a large mass, and the mid-range scalar attraction between nucleons must be generated *dynamically* from the exchange of two correlated pions in the scalar-isoscalar channel [6, 7]. This correlated two-pion exchange can be simulated by introducing an "effective" low-mass scalar field, which can then be studied at the mean-field level. Moreover, the baryon, pion, and σ fields must be redefined using a chiral transformation, so that the lagrangian can be rewritten in terms of derivative couplings between the baryons and pions [1]. These procedures will produce a phenomenology resembling that of QHD–I (*i.e.*, large isoscalar scalar and vector interactions), while also including pions with derivative couplings to nucleons (which guarantees the soft-pion theorems) as well as chiral-invariant interactions with isovector vector and axial-vector mesons. Another degree of freedom central to low-energy nuclear dynamics, the $\Delta(1232)$ resonance, also arises *dynamically* in this model [28]. The effects of the additional (Ξ) baryons will appear only through loop corrections in the nonstrange sector, and hopefully these corrections will generate only modest changes to successful QHD predictions.

Finally, the Higgs mesons should also be assigned a large mass, so that they serve only to implement the renormalizability of the theory, with minimal impact on low-energy predictions.

To obtain a renormalizable model, we must introduce a single degree of freedom (denoted here as Ξ) from outside the "nuclear domain." Thus the present model is *not* intended to correctly describe the physics of the strange sector. For example, in a system with net hypercharge zero — equal numbers of nucleons and cascades — the source term for the ω meson will vanish. Without this short-range repulsion, the properties of the system will be sensitive to the details of the other short-range interactions. It is clearly necessary to augment the QHD–III lagrangian to include additional strange hadrons (K, Λ, Σ) with realistic interactions [for example, by using $SU(3)_R \times SU(3)_L$ symmetry] before meaningful results can be obtained in the strange sector.

In summary, the present model contains the important low-energy hadronic degrees of freedom for describing physics in the nuclear domain of up and down quarks. It manifests the isospin and chiral symmetry of the underlying QCD lagrangian. Moreover, it incorporates hadronic resonances dynamically while respecting these symmetries. The strong, mid-range, scalar attraction between nucleons, which is observed in nuclei and suggested by QCD sum rules [14], is a dynamical consequence of this chirally invariant model lagrangian. The investigation of relativistic nuclear many-body systems in a hadronic model that respects the symmetries of QCD is an important area for future research within the QHD framework.

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