ON THE PASSAGE OF A FAST PARTICLE THROUGH A NUCLEUS*

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Dedicated to Wiesław Czyż in honour of his 65th birthday

With the aid of two exactly solvable models, interference effects are shown to be important for the passage of a fast particle through a nucleus when time dilation increases the lifetime of the excited states of the incident particle sufficiently.

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1. Introduction

In this paper we shall discuss, with the aid of two simple models, the passage of a fast nucleon through a nucleus. After the first collision with a nuclear nucleon, the incident nucleon will be excited. However, because of time dilation, the lifetime of the excited nucleon is greatly increased so that it will not have decayed appreciably before it collides with another nuclear nucleon. Its wave function will thus not have attained its free space asymptotic form at the time of the second collision. Quantum mechanically we can therefore no longer consider the two collisions to be independent and it is thus necessary to solve the three-body problem of the collision of the incident nucleon with two nuclear nucleons. In the case of the passage of a nucleon through a nucleus we must therefore consider its collision with

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N nucleons that it encountered and our problem becomes an $N+1$-body problem.

To assess how important this quantal effort is we shall solve exactly two model problems. In the first the incident nucleon has only one excited state, in the second two excited states. We find in the limit of strong coupling between the ground state and these excited states that the decay probability is substantially changed from what it would be if the collisions with each nuclear nucleon was independent. Because of this effect, nuclei will be more or less transparent than that predicted by a naive calculation.

The important parameters are functions of the dilated width

$$\Gamma = \frac{\Gamma_{\rm r}}{\gamma}, \qquad \gamma = \frac{E_{\rm L}}{mc^2},$$

where $\Gamma_{\rm r}$ is the width at rest. This is to be compared with the average time between collisions which is the interparticle distance r_0 divided by the velocity which is approximately c. The resultant parameter is

$$\frac{\Gamma_{\rm r}}{\gamma}\,\frac{r_0}{\hbar c}$$

which is expected to be small for large enough γ.

A second parameter which measures the strength of the coupling is valid when μ/Γ and λ/Γ are large where μ and λ are defined in Sections 2 and 3.

The models used in this paper are too schematic to permit quantitative comparison with experiment. However, the results show some of the issues which must be carefully considered in interpreting experimental data.

2. Model A

In both models it is assumed that the incident nucleon travels through the nucleus with a velocity v, close to light velocity. It suffers collisions at times $t_1, t_2 \ldots t_n$ where n is a small number of the order of the nuclear radius divided by the mean free path λ.

In Model A which has been discussed in Ref. [1], the nucleon is assumed to have only one excited state ψ_1 in addition to the ground state ψ_0. The excited state has a complex energy $(\epsilon - i\Gamma/2)$ while the ground state energy is taken to be zero. The constant Γ governs the decay of the excited state by the emission of pions, kaons, *etc.* The interesting regime is one in which the coupling of ψ_0 to ψ_1 is strong. In this limit the excited state will preferably decay to the ground state so that the system will be in the ground state ψ_0 one-half of the time the incident particle is in the nucleus. The emission governed by Γ will be reduced by at least one-half so that the total decay probability per unit time will be reduced by one-half and the mean free path correspondingly increased. However the total probability may be changed from that value as we shall see.

The wave function of the system is assumed to be a sum of terms, each of which describe the system presuming that a total of s collisions have occurred, $s = 1, \ldots, N$. Moreover, it is assumed that the collisions are impulsive so that the Schrödinger equation can be written

$$i\frac{\partial \Phi_s}{\partial t} = \mu \sum_{J=1}^{s} \delta\left(t - t_j\right) \chi_s\left(t^+\right) , \tag{1}$$

$$i\frac{\partial \chi_s}{\partial t} = E\chi_s + \mu \sum_{J=1}^{s} \delta\left(t - t_j\right) \Phi_s\left(t^+\right) . \tag{2}$$

In these equations for the s-th component total wave function, Φ_s is the wave function for the system in the ground state while χ_s refers to the excited state whose energy is complex, $E = \epsilon - i\Gamma/2$. The constants t_j are the times at which collisions occur. The coupling constant between the ground and excited state is given by μ. Integrating these equations one can obtain the recurrence relations:

$$\chi_s^{(\nu)} \equiv \chi_s\left(t_\nu^+\right) = \frac{1}{1+\mu^2}\left[e^{-ix_\nu}\chi_s^{(\nu-1)} - i\mu\Phi_s^{(\nu-1)}\right] , \tag{3}$$

$$\Phi_s^{(\nu)} \equiv \Phi_s\left(t_\nu^+\right) = \frac{1}{1+\mu^2}\left[\Phi_s^{(\nu-1)} - i\mu e^{-ix_\nu}\chi_s^{(\nu-1)}\right] . \tag{4}$$

In these equations

$$x_\nu = \left(t_\nu - t_{\nu-1}\right) E . \tag{5}$$

Defining the vector

$$\Psi_s^{(\nu)} = \frac{1}{\left(1+\mu^2\right)^{\nu/2}} \begin{pmatrix} \Phi_s^{(\nu)} \\ \chi_s^{(\nu)} \end{pmatrix} , \tag{6}$$

equations (3) and (4) may be written as follows

$$\Psi_s^{(\nu)} = M_{\nu,\nu-1}\Psi_s^{(\nu-1)} , \tag{7}$$

where

$$M_{\nu,\nu-1} \equiv \mathrm{e}^{-ix_\nu/2} \begin{pmatrix} \cos\frac{\vartheta}{2}\mathrm{e}^{ix_\nu/2} & -i\sin\frac{\vartheta}{2}e^{-ix_\nu/2} \\ -i\sin\frac{\vartheta}{2}\mathrm{e}^{ix_\nu/2} & \cos\frac{\vartheta}{2}e^{-ix_\nu/2} \end{pmatrix} \tag{8}$$

or

$$M_{\nu,\nu-1} = \mathrm{e}^{-ix_\nu/2} S(\vartheta) T(x_\nu) . \tag{9}$$

Here

$$S(\vartheta) = \begin{pmatrix} \cos\frac{\vartheta}{2} & -i\sin\frac{\vartheta}{2} \\ -i\sin\frac{\vartheta}{2} & \cos\frac{\vartheta}{2} \end{pmatrix} \tag{10a}$$

and

$$T(x_\nu) = \begin{pmatrix} \mathrm{e}^{ix_\nu/2} & 0 \\ 0 & \mathrm{e}^{-ix_\nu/2} \end{pmatrix} . \tag{10b}$$

The angle ϑ is defined by

$$\cos\frac{\vartheta}{2} = \frac{1}{\sqrt{1+\mu^2}} . \tag{11}$$

The solution of Eq. (7) in terms of an initial $\Psi^{(0)}$ is

$$\Psi_s^{(\nu)} = M_{\nu,\nu-1} M_{\nu-1,\nu-2} \ldots M_{1,0} \Psi^{(0)} . \tag{12}$$

This expression has been evaluated for the case of equal time intervals (x_ν independent of ν) in Ref. [1]. In this paper we consider only the strong coupling limit ($\vartheta \sim \pi/2$), but drop the assumption of equal time intervals. In the limit $\vartheta \sim \pi/2$

$$S(\vartheta) \longrightarrow S_\infty(\vartheta) \equiv \begin{pmatrix} 0 & -i\sin\frac{\vartheta}{2} \\ -i\sin\frac{\vartheta}{2} & 0 \end{pmatrix} = -i\sin\frac{\vartheta}{2}\sigma_x . \tag{13}$$

If we now take $\Psi^{(0)}$ to be α where

$$\alpha \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} , \qquad \beta \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{14}$$

then

$$\Psi_s^{(s)} = \left(-i\sin\frac{\vartheta}{2}\right)^s \mathrm{e}^{-i/2\Sigma x_j} \left[\sigma_x T(x_s) \sigma_x T(x_{s-1}) \ldots \sigma_x T(x_1)\right] \alpha . \tag{15}$$

Assume that s is even. Then after s collisions

$$\Psi_s^{(s)} = \left(-i\sin\frac{\vartheta}{2}\right)^s \mathrm{e}^{-i/2\Sigma x_j}\, \mathrm{e}^{-i/2\Sigma(-)^{j-1} x_j} \beta . \tag{16}$$

The exponent is therefore

$$-i\,(x_2 + x_4 + x_6 + \ldots + x_s)\,. \tag{17}$$

If the intervals x_2, x_4 *etc.* were all equal to $\bar{x}$ the exponent becomes $-i(s/2)\bar{x}$ so that

$$\Psi_s^{(s)} = \left(-i\sin\frac{\vartheta}{2}\right)^s \exp\left\{-\left(i\epsilon + \frac{\Gamma}{2}\right)\frac{s}{2}\tau\right\}\beta\,, \tag{18}$$

where

$$\bar{x} = E\tau\,.$$

Hence the attenuation is given by $\exp(-\Gamma/4\, s\tau)$, the rate equal to one-half of the rate of decay of the uncoupled χ.

Equation (18) gives the amplitude at the time given by $s\tau$. At the time given by $n\tau$, $n > s$

$$\Psi_s(n\tau) = \left(-i\sin\frac{\vartheta}{2}\right)^s \exp\left\{-\left(i\epsilon + \frac{\Gamma}{2}\right)\left[\frac{s}{2}\tau + (n-s)\tau\right]\right\}\beta\,. \tag{19}$$

Averaging classically over the number of collisions would lead to a decay rate which lies between that of the free decay of χ and the rate given by Eq. (18). However, such a calculation is incorrect since there is interference of the n-th component (n even) with all components which differ from n by an even number of steps. Thus for even n

$$\begin{aligned}\Psi(n\tau) = e^{-(iE\tau)\frac{n}{2}}\Bigg[&\left(-i\sin\frac{\vartheta}{2}\right)^n + \left(-i\sin\frac{\vartheta}{2}\right)^{n-2} e^{-iE\tau} \\ &+ \left(-i\sin\frac{\vartheta}{2}\right)^{n-4} e^{-iE(2\tau)} + \ldots + \left(-i\sin\frac{\vartheta}{2}\right)^{2} e^{-iE(\frac{n}{2}-1)\tau}\Bigg]\beta\,,\end{aligned}$$

$$\Psi(n\tau) = (-)^{n/2}\,\frac{e^{-iE\tau(n/2)}\left(\sin\frac{\vartheta}{2}\right)^n + e^{-iEn\tau}}{1 + e^{-iE\tau}\Big/\left(\sin^2\frac{\vartheta}{2}\right)}\,\beta\,. \tag{20}$$

We observe that when $E\tau$ is large, $\Psi(n\tau) \sim e^{-iE\tau n/2}$ yielding once again the decay rate of $\Gamma/4$. However, in the case of interest $\Gamma n\tau \ll 1$. Then

$$\Psi(n\tau) \longrightarrow \frac{1}{2}\left(1 + \left(\sin^2\frac{\vartheta}{2}\right)^n\right) - iE\tau\left[n\left(\frac{1}{2} + \frac{1}{4}\left(\sin^2\frac{1}{2}\vartheta\right)^n\right) - \frac{1}{2}\right] + \ldots\,. \tag{21}$$

As a consequence of interference, which in this case is destructive, the rate of decay is increased from $\Gamma/4$ to $3\Gamma/8$ for $\sin\frac{\vartheta}{2} = 1$ close to the free decay rate of the χ channel.

One can relax the assumption of equal time intervals, and the average over their distribution. This is readily carried out. The result no longer given by Eq. (21) but the conclusion remains that for small ΓT, where T is the total time interval the effect of interference is significant and cannot be neglected.

3. Model B

In Model A the system under strong coupling oscillated between the excited and ground states. To better conform to what we believe to be the experimental situation we would prefer the system to remain excited throughout its passage through the nucleus under strong couplings. Of course there is a probability that the system will not be excited on the first scattering, but may be excited at a later collision. These possibilities are realized when there are two excited states both coupled to the ground state. The equations replacing Eq. (1) are

$$\begin{aligned} i\frac{\partial\psi_0}{\partial t} &= \mu\Sigma\delta\left(t-t_i\right)\psi_1\left(t_i^+\right) + \lambda\Sigma\delta\left(t-t_i\right)\psi_2\left(t_i^+\right) , \\ i\frac{\partial\psi_1}{\partial t} &= \mu\Sigma\delta\left(t-t_i\right)\psi_0\left(t_i^+\right) + E_1\psi_1 , \\ i\frac{\partial\psi_2}{\partial t} &= \lambda\Sigma\delta\left(t-t_i\right)\psi_0\left(t_i^+\right) + E_2\psi_2 , \end{aligned} \tag{26}$$

where

$$E_\alpha = \epsilon_\alpha - \tfrac{1}{2}i\Gamma_\alpha , \qquad \alpha = 1,2 . \tag{27}$$

Integrating these equations permits one to establish recursion relations for $\psi_0^{(0)}$, $\psi_1^{(n)}$, $\psi_2^{(n)}$ in terms of $\psi_0^{(n-1)}$, $\psi_1^{(n-1)}$, $\psi_2^{(n-1)}$ where for example $\psi_0^{(n)} = \psi_0(t_n)$. These are

$$\psi_0^{(n)} = \frac{1}{1+\lambda^2+\mu^2}\left[\psi_0^{(n-1)} - i\mu\psi_1^{(n-1)}\mathrm{e}^{-ix_n} - i\lambda\psi_2^{(n-1)}\mathrm{e}^{-iy_n}\right] , \tag{28}$$

$$\psi_1^{(n)} = \frac{1}{1+\lambda^2+\mu^2}\left[\left(1+\lambda^2\right)\psi_1^{(n-1)}\mathrm{e}^{-ix_n} - \lambda\mu\psi_2^{(n-1)}e^{-iy_n} - i\mu\psi_0^{(n-1)}\right] \tag{29}$$

and

$$\psi_2^{(n)} = \frac{1}{1+\lambda^2+\mu^2}\left[\left(1+\mu^2\right)\psi_2^{(n-1)}\mathrm{e}^{-iy_n} - \lambda\mu\psi_1^{(n-1)}\mathrm{e}^{-ix_n} - i\lambda\psi_0^{(n-1)}\right] , \tag{30}$$

where

$$x_n = E_1 (t_n - t_{n-1}) \qquad \text{and} \qquad y_n = E_2 (t_n - t_{n-1}) .$$

Defining

$$\Psi^{(n)} \equiv \begin{pmatrix} \psi_0^{(n)} \\ \psi_1^{(n)} \\ \psi_2^{(n)} \end{pmatrix} \tag{31}$$

we obtain

$$\Psi^{(n)} = S(\mu, \lambda) T_n \Psi^{(n-1)} . \tag{32}$$

Here

$$S(\mu, \lambda) = \frac{1}{1 + \lambda^2 + \mu^2} \begin{pmatrix} 1 & -i\mu & -i\lambda \\ -i\mu & 1 + \lambda^2 & -\lambda\mu \\ -i\lambda & -\lambda\mu & 1 + \mu^2 \end{pmatrix} \tag{33}$$

and

$$T_n = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-ix_n} & 0 \\ 0 & 0 & e^{-iy_n} \end{pmatrix} . \tag{34}$$

Remarkably, for λ, $\mu \gg 1$, the coupling between the two excited states is much stronger than the coupling of each with the ground state satisfying the goal described at the beginning of this section.

One can immediately exhibit a general solution of Eq. (26) as follows:

$$\Psi^{(N)} = ST_N ST_{N-1} \dots ST_1 \Psi^{(0)} , \tag{35}$$

where

$$\Psi^{(0)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} .$$

We shall again discuss this solution in the limit of strong coupling, taking *first* the case when the system is excited on the first scattering. That is, for

$$S_\infty \longrightarrow \frac{1}{1 + \kappa^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -\kappa \\ 0 & -\kappa & \kappa^2 \end{pmatrix} , \qquad \kappa \equiv \frac{\mu}{\lambda} . \tag{36}$$

The first step using the exact S is

$$ST_1\Psi^{(0)} = \frac{1}{1+\lambda^2(1+\kappa^2)}\begin{pmatrix} 1 \\ -i\kappa\lambda \\ -i\lambda \end{pmatrix} \xrightarrow[\lambda\to\infty]{} -\frac{i}{\lambda(1+\kappa^2)}\begin{pmatrix} 0 \\ \kappa \\ 1 \end{pmatrix}. \tag{37}$$

Operating on this result with $S_\infty T_2$ yield

$$\Psi^{(2)} = -\frac{i\kappa}{\lambda(1+\kappa^2)^2}\left(e^{-ix_2} - e^{-iy_2}\right)\begin{pmatrix} 0 \\ 1 \\ -\kappa \end{pmatrix}. \tag{38}$$

We now establish the lemma that $\begin{pmatrix} 0 \\ 1 \\ -\kappa \end{pmatrix}$ is an eigenstate of $S_\infty T_n$:

$$\begin{aligned}
S_\infty T_n \begin{pmatrix} 0 \\ 1 \\ -\kappa \end{pmatrix} &= \frac{1}{1+\kappa^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -\kappa \\ 0 & -\kappa & \kappa^2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-ix_n} & 0 \\ 0 & 0 & e^{-iy_n} \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ -\kappa \end{pmatrix} \\
&= \frac{1}{1+\kappa^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -\kappa \\ 0 & -\kappa & \kappa^2 \end{pmatrix}\begin{pmatrix} 0 \\ e^{-ix_n} \\ -\kappa e^{-iy_n} \end{pmatrix} \\
&= \frac{1}{1+\kappa^2}\begin{pmatrix} 0 \\ e^{-ix_n} + \kappa^2 e^{-iy_n} \\ -\kappa e^{-ix_n} - \kappa^3 e^{-iy_n} \end{pmatrix} \\
&= \frac{1}{1+\kappa^2}\left(e^{-ix_n} + \kappa^2 e^{-iy_n}\right)\begin{pmatrix} 0 \\ 1 \\ -\kappa \end{pmatrix}.
\end{aligned}$$

In other words, after the second scattering the system remains in the state $\begin{pmatrix} 0 \\ 1 \\ -\kappa \end{pmatrix}$ for the remainder of the scatterings. Hence

$$\begin{aligned}
\Psi^{(N)} = -\frac{i\kappa}{\lambda}\left(\frac{1}{1+\kappa^2}\right)^N &\left(e^{-ix_N} + \kappa^2 e^{-iy_N}\right)\left(e^{-ix_{N-1}} + \kappa^2 e^{-iy_{N-1}}\right) \\
&\ldots\left(e^{-ix_2} - e^{-iy_2}\right)\begin{pmatrix} 0 \\ 1 \\ -\kappa \end{pmatrix}.
\end{aligned} \tag{39}$$

We turn now to the amplitude which is generated when the first excitation occurs at the third scattering. (The amplitude for excitation at the second scattering is of the order of $(1/\lambda)$ compared to this term). Toward this end we calculate $ST_2ST_1\Psi^{(0)}$ exactly. This yields in the limit $\lambda \gg 1$

$$ST_2ST_1\Psi^{(0)} \to -\frac{1}{\lambda\left(1+\kappa^2\right)^2} \times \left[\frac{1}{\lambda}\left(\kappa^2 e^{-ix_2} + e^{-iy_2}\right)\begin{pmatrix}1\\0\\0\end{pmatrix} + i\kappa\left(e^{-ix_2} - e^{-iy_2}\right)\begin{pmatrix}0\\1\\-\kappa\end{pmatrix}\right] . \quad (40)$$

The second term agrees with Eq. (38). The first term can now be developed using additional steps $S_\infty T_N S_\infty T_{N-1} \dots S_\infty T_3$ to obtain

$$\Psi^{(N)} = -\frac{i\kappa}{\lambda\left(1+\kappa^2\right)^N}\left[\left(e^{-ix_N} + \kappa^2 e^{-iy_N}\right)\dots\left(e^{ix_5} + \kappa^2 e^{-iy_5}\right)\right] \times \left[\left(e^{-ix_4} + \kappa^2 e^{-iy_4}\right)\left(e^{-ix_3} + \kappa^2 e^{-iy_3}\right)\left(e^{-ix_2} - e^{-iy_2}\right) - \left(e^{-ix_4} - e^{-iy_4}\right)\left(\kappa^2 e^{-ix_2} + e^{-iy_2}\right)\right]\begin{pmatrix}0\\1\\-\kappa\end{pmatrix} . \quad (41)$$

Additional corrections would come from the sixth, eighth, *etc.* collisions. Interference is now explicitly exhibited. To illustrate the effect of the interference, we assume that the x_n are equal to $\bar{x}$ and the y_n to $\bar{y}$, and that they are small. Then

$$\Psi^{(N)} = \frac{\kappa(\bar{y}-\bar{x})}{\lambda\left(1+\kappa^2\right)^2}\left[1 - i(N-4)\frac{\left(\bar{x}+\kappa^2\bar{y}\right)}{1+\kappa^2}\right]\left(\frac{\kappa^2}{1+\kappa^2}\right) \times \left[1 - i\left(2\bar{y} + \frac{\bar{x}}{1+\kappa^2} + \frac{2\bar{x}-\bar{y}}{\kappa^2\left(1+\kappa^2\right)^2}\right)\right]\begin{pmatrix}0\\1\\-\kappa\end{pmatrix} . \quad (42)$$

This is to be compared with the result which would be obtained using Eq. (39). In the same limit

$$\Psi^{(N)} = \frac{\kappa\left(\bar{x}-\bar{y}\right)}{\lambda\left(1+\kappa^2\right)^2}\left[1 - i(N-4)\frac{\left(\bar{x}+\kappa^2\bar{y}\right)}{1+\kappa^2}\right]\left[1 - \frac{2i\left(\bar{x}+\kappa^2\bar{y}\right)}{1+\kappa^2}\right]\begin{pmatrix}0\\1\\-\kappa\end{pmatrix} . \quad (43)$$

For $\kappa = 1$, the last square bracket in Eq. (42) becomes

$$\left[1 - \frac{3}{2}(\bar{x} + \bar{y})\right]$$

while the corresponding term in Eq. (43) is $1 - i(\bar{x} - \bar{y})$. Once more we see the importance of interference.

Finally, note that the solutions for model A and model B depend on the times between collisions, $t_N - t_{N-1}$, $t_{N-1} - t_{N-2}$ *etc.* One should average over these time intervals. The assumed distributions could be tailored to take into account phenomena like shadowing by reducing the probability that a collision occurs when the time interval is small. The calculation is not difficult. The qualitative conclusion that interference effects are important acquires if anything more validity. Clearly, classical kinetic theory calculations are suspect.

REFERENCES

[1] H. Feshbach, in *Asymptotic Realms of Physics*, Eds A.H. Guth, K. Huang, R.L. Jaffe, The MIT Press, Cambridge, MA, 1983.