DISCUSSION OF THE HIGH SPIN SPECTRA IN LIGHT NUCLEI IN TERMS OF THE ROTATING HARMONIC OSCILLATOR*

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Dedicated to Wiesław Czyż in honour of his 65th birthday

The potential of an anisotropic harmonic oscillator rotated externally with constant frequency is employed for the investigation of the basis for the high spin spectra in the sd-shell nuclei. The analysis emphasizes the search for proper nucleonic configurations. Various physical effects typical for the yrast region in these nuclei are discussed.

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1. Introduction

High spin spectra of light nuclei exhibit several remarkable features. Many interesting phenomena that have been predicted to occur in heavy nuclei at very high angular momentum show up in light nuclei already in the region which is directly accessible to experiments on the discrete nuclear states (*cf.* Ref. [1]). For example, the rotation of a light nucleus with $A \sim 20$ at angular momentum $I \sim 10$ requires rotational frequency ω which would correspond to $I > 300$ in a heavy nucleus (*cf.* Ref. [2]). Light nuclei may, therefore, be very useful for the investigation of some phenomena that are characteristic for very high spin behaviour such as for example: drastic shape changes including possible appearance of superdeformed states, sharp band crossings (often referred to as backbending phenomena), band

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terminations when the oblate nuclear shape is reached, rotations of the nucleus about the classically unfavoured axis, coexistence of different nuclear shapes *etc.*

Moreover, light nuclei, for example those with valence nucleons in the sd-shell (*i.e.* with $16 < A < 40$) may be rather attractive as objects of theoretical analysis. In fact, if the independent particle approach is accepted as a basis for the treatment there exist at least two favourable features that simplify considerably the analysis. One of them is the less important role played by the spin-orbit coupling and the other one — a possible unimportance of the two-body pairing force. Consequently, the solution for single particle orbitals is rather simple and may become an attractive test-ground for the validity of the model employed in the calculation.

An extensive analysis of the spin spectra in the sd-shell nuclei has been performed on the basis of the Nilsson potential rotating with a frequency ω about a fixed axis (procedure usually referred to as the cranking model) in a series of papers by the Lund group (Refs [1–8]). It has been possible to find an adequate understanding of many of the observed features of spectra in these nuclei. Apart from this cranked Nilsson approach some other more general approaches have been introduced such as for example those based on the Hartree–Fock method (Refs [9–13]).

In the present paper we propose to get a "step backwards" and to employ an even simpler model namely that of the rotating harmonic oscillator (h.o.). This model, although much less realistic, can provide a simple set of basic states that could then be used as a starting point for more refined calculations. It is well known that the cranking model for the rotating h.o. may be solved exactly (*i.e.* to all powers of rotational frequency ω) in a closed form (Refs [14–21]). An even simpler form of the solution is known in the case when the coupling between the major h.o. shell caused by rotation is neglected (Refs [18, 22, 23]).

It is worth mentioning that the rotating harmonic oscillator description formulated in this way may provide a basis that already incorporates all couplings following from the Coriolis term of the cranking model.

Our present aim is to discuss the above solutions and to provide a simple way of constructing the basis of a rotating harmonic oscillator. It is hoped that the h.o. potential may offer attractive possibilities of introducing appropriate quantum numbers for the yrast states, providing more insight in the fast rotating nuclei, of interpreting the rich structure of various rotational bands, of understanding the nature of deformation changes *etc.*

While the papers quoted above (Refs [14–21]) deal mostly with the analysis of a rotating system with a fixed nucleonic configuration, in the present paper we concentrate mostly on the method providing all the relevant h.o. configurations that occur in the yrast line in the range extending to high

angular momenta.

Sections 2 and 3 introduce the model and explain the calculation method. In Sections 4 and 5 applications to few selected nuclei are given. Finally, Section 6 summarizes the conclusions.

2. Diagonalization of the Hamiltonian

As it is well known the high angular momentum excitations in atomic nuclei are well described by the cranking Hamiltonian (Routhian)

$$H^{\omega} = H - \omega J_1 = \sum_{k=1}^{A} h^{\omega}(k) = \sum_{k=1}^{A} \{h(k) - \omega j_1(k)\} \tag{2.1}$$

corresponding to a rotation of a system of A nucleons about a fixed axis ("1" axis) with constant rotational frequency ω. Here

$$J_1 = \sum_{k=1}^{A} j_1(k) \tag{2.2}$$

denotes the projection of the total angular momentum I on the axis of rotation. For the original Hamiltonian H the triaxial harmonic oscillator has been adopted throughout this paper

$$H = \sum_{k=1}^{A} \sum_{\gamma=1}^{3} \omega_{\gamma} \left[b_{\gamma}^{+}(k) b_{\gamma}(k) + \tfrac{1}{2} \right] \tag{2.3}$$

corresponding to the nucleus with A nucleons. Quantities ω_1, ω_2, ω_3 are the three h.o. frequencies characterizing the range and shape of the potential. We assume that the volume enclosed by the equipotential surfaces remains constant independently of the nuclear shape. This leads to the usual (Ref. [24]) constraint for the frequencies

$$\omega_1 \omega_2 \omega_3 = \mathring{\omega}^3 \tag{2.4}$$

with $\mathring{\omega}$ being a deformation independent constant. Taking into account this constraint we can express the three h.o. frequencies ω_1, ω_2, ω_3 in terms of the constant $\mathring{\omega}$ and the two Nilsson quadrupole deformation parameters ϵ and γ defined through the relations

$$\omega_1 = \omega_0(\epsilon, \gamma) \left(1 + \frac{\epsilon \cos \gamma}{3} + \frac{\epsilon \sin \gamma}{\sqrt{3}} \right), \tag{2.5a}$$

$$\omega_2 = \omega_0(\epsilon \gamma) \left(1 + \frac{\epsilon \cos \gamma}{3} - \frac{\epsilon \sin \gamma}{\sqrt{3}} \right), \tag{2.5b}$$

$$\omega_3 = \omega_0(\epsilon, \gamma) \left(1 - \frac{2\epsilon \cos \gamma}{3} \right), \tag{2.5c}$$

together with

$$\omega_0(\epsilon,\gamma) = \omega^0\Big(1 - \frac{\epsilon^2}{3} + \frac{2\epsilon^3\cos\gamma}{9} - \frac{8\epsilon^3\cos^3\gamma}{27}\Big)^{-1/3}. \tag{2.6}$$

Quantities $b^+_\gamma(k)$ and $b_\gamma(k)$ appearing in Eq. (2.3) are the creation and annihilation operators of the h.o. quanta corresponding to the k-th nucleon and γ-th axis ($\gamma = 1,2,3$). Their phase have been chosen as in Ref. [18] (page 231, see also formulae (2.5) and (2.6) of Ref. [23]) in such a way that both H and J_1 have real matrix elements.

The description of nuclear rotation by means of the cranking Hamiltonian H^ω (Eq. (2.1)) may be also understood as a variational search for the lowest energy with a fixed value I of the angular momentum projection $\langle J_1\rangle$ on the rotation axis. Thus ω plays a role of a Lagrange multiplier. The constraint

$$\langle J_1\rangle \equiv I \tag{2.7}$$

for the angular momentum J_1 projection instead of the total angular momentum should be a good approximation in the range of large angular momenta ($I \gg 1$). The right-hand side of Eq. (2.7) depends on ω since the expectation value should be taken in an eigenstate of H^ω. The single nucleon angular momentum $j_1(k)$ may be expressed in terms of the h.o. operators $b^+_\gamma(k)$ and $b_\gamma(k)$ as

$$\begin{aligned} j_1(k) = j_I + j_{II} = {} & \frac{\omega_2+\omega_3}{2\sqrt{\omega_2\omega_3}}\big(b^+_2(k)b_3(k) + b^+_3(k)b_2(k)\big) \\ & - \frac{\omega_2-\omega_3}{2\sqrt{\omega_2\omega_3}}\big(b^+_2(k)b^+_3(k) + b_2(k)b_3(k)\big), \end{aligned} \tag{2.8}$$

where j_I describes the mixing of the h.o. quanta corresponding to $b^+_2(k)$ and $b^+_3(k)$ within each major h.o. shell and j_{II} — the coupling between different major h.o. shells due to rotation. Eqs (2.2) to (2.8) determine fully the cranking Hamiltonian (2.1) and its exact solution can be found in the literature (Refs [14–24]) in the explicit form providing the spin degrees of freedom are neglected. When the total Routhian E^ω is found for any given ω (for details see below) one may invert Eq. (2.7) *i.e.* find ω as a function of I. Finally, the total energy E may be calculated from the relation

$$E = E^\omega + \omega\langle J_1\rangle \tag{2.9}$$

as a function of angular momentum I. Now, let us come back to the procedure of determining E^ω.

The corresponding calculations are rather easy and have been discussed in Refs [14–21] and also in the papers quoted above in the case of fixed nucleonic configuration.

The solution of this problem follows from the fact that it is possible to transform the Hamiltonian (2.1) into the form of a "new" three-dimensional anisotropic harmonic oscillator which describes the normal modes of the system. The transformation when expressed in terms of the creation and annihilation operators for the h.o. quanta has the form

$$\alpha_\eta^+ = \sum_{\gamma=2,3} (\lambda_{\eta\gamma} b_\gamma^+ + \mu_{\eta\gamma} b_\gamma), \tag{2.10a}$$

$$\alpha_\eta = \sum_{\gamma=2,3} (\lambda_{\eta\gamma}^* b_\gamma + \mu_{\eta\gamma}^* b_\gamma^+), \tag{2.10b}$$

where $\eta = 2$ or 3. Here the particle index k has been omitted for the sake of simplicity. As can be seen this transformation describes a mixing between the creation and annihilation operators b_γ^+ and b_γ. When the transformation matrices $\lambda_{\eta\gamma}$ and $\mu_{\eta\gamma}$ are chosen in an appropriate way the transformed Hamiltonian h^ω entering Eq. (2.1) is of the form

$$h^\omega = \omega_1(\alpha_1^+\alpha_1 + \tfrac{1}{2}) + \Omega_2(\alpha_2^+\alpha_2 + \tfrac{1}{2}) + \Omega_3(\alpha_3^+\alpha_3 + \tfrac{1}{2}), \tag{2.11}$$

where $\alpha_1 = b_1$ and the two frequencies Ω_2 and Ω_3 corresponding to the new normal modes of the system depend on ω_2, ω_3 and ω:

$$\Omega_{2,3} = \left\{ \frac{\omega_2 + \omega_3}{2} + \omega^2 \pm [(\omega_2^2 - \omega_3^2)^2 + 8\omega^2(\omega_2^2 + \omega_3^2)]^1 1/2 \right\}^{1/2}. \tag{2.12}$$

The eigenvalues E^ω of the total cranking Hamiltonian H^ω are thus given by

$$E^\omega = \omega_1\Sigma_1 + \Omega_2\Sigma_2 + \Omega_3\Sigma_3, \tag{2.13}$$

where the three occupation sums Σ_1, Σ_2, Σ_3 are given by

$$\Sigma_\gamma = \sum_\nu \langle \nu(\alpha_\gamma^+\alpha_\gamma + \tfrac{1}{2}\nu\rangle = \sum_\nu (n_\nu + \tfrac{1}{2}) \tag{2.14}$$

with $\gamma = 1, 2$ or 3 and the sum Σ running over all occupied single nucleon states.

The three values of Σ_1, Σ_2 and Σ_3 are related to the particle occupation distribution over the h.o. shells. For a given configuration the set of Σ_1, Σ_2, Σ_3 is fully defined (however, the inverse is not true). The corresponding set of energy levels obtained with fixed Σ_1, Σ_2, Σ_3 and I increasing could be defined as a "rotational band" within the cranking model. The energy levels may then be obtained from Eq. (2.9) with $\omega = \omega_I$ determined for each I as a solution of Eq. (2.7).

The total energy E obtained in this way for any fixed value of I should then be further minimized with respect to the deformation. Results of the above procedure have been extensively discussed in previous papers (Refs [14–18]) as already mentioned.

However, this has been done only in the case of a fixed configuration $(\Sigma_1, \Sigma_2, \Sigma_3)$. On the other hand, it seems obvious that the rotating nucleus can pick up its angular momentum either by adjusting its orbitals to the increasing angular momentum at fixed configuration or else by changing its configuration (say, into another set of Σ_1, Σ_2 and Σ_3) as to accommodate more angular momentum. This is intimately related to the crossing of energy levels and will be discussed in the next section.

3. Selection of the lowest bands

3.1. The U(3) symmetry and the h.o. occupation sums

In the previous section we have derived a procedure for finding energies of the rotational bands following from the cranked h.o. potential for a fixed configuration that has been specified by the three numbers Σ_1, Σ_2 and Σ_3 (occupation sums). We shall now discuss how to select the interesting configurations *i.e.* those which come low in energy for a given angular momentum. For this purpose it is convenient to use the symmetry arguments provided by the theory of groups. Let us observe that the transformed cranking Hamiltonian (2.11) is entirely expressed by the bilinear forms of the boson operators α^+ and α. The Lie algebra formed by all the possible nine products

$$A_{\gamma\eta} = \sum_{k=1}^{A} \alpha_\gamma^+(k)\alpha_\gamma(k) \tag{3.1}$$

with $\gamma, \eta = 1,2,3$ is that of the unitary group U(3) in three dimensions as is well known for the case of a three-dimensional harmonic oscillator (*cf.* Ref. [25] see also Ref. [27]). Thus the U(3) group becomes a dynamical symmetry of the H^ω in the cranked h.o. model. It is well known (see Refs [25–27]) that the irreducible representations of U(3) may be labelled by the Young diagrams $[\lambda_1, \lambda_2, \lambda_3]$. For a given set of three integers λ_1, λ_2 and λ_3 the eigenstates of H^ω may be further labelled by additional (integer) quantum numbers p, q, r with

$$0 \leq p \leq \lambda_1 - \lambda_2\,, \tag{3.2a}$$

$$0 \leq q \leq \lambda_2 - \lambda_3\,, \tag{3.2b}$$

$$0 \leq r \leq \lambda_2 - \lambda_3 + p - q\,. \tag{3.2c}$$

For a given representation specified by the λ_1, λ_2 and λ_3 the additional quantum numbers p, q and r specify the basic vectors in the representation space. The notation used here has been taken from Ref. [27]. Numbers of oscillator quanta n_1, n_2 and n_3 along the three coordinate axes may be expressed by the quantum numbers $(\lambda_1, \lambda_2, \lambda_3, p, q, r)$ specifying the basic vectors in the representation space

$$n_1 = \lambda_2 + p - r\,, \tag{3.3a}$$

$$n_2 = \lambda_3 + q + r\,, \tag{3.3b}$$

$$n_3 = \lambda_1 - p - q\,. \tag{3.3c}$$

In this way we can determine all the values of Σ_1, Σ_2 and Σ_3 for each representation and each vector $\lambda_1\lambda_2\lambda_3 pqr\rangle$ within the representation. For a configuration of A nucleons distributed over N major h.o. shells filled and the remaining valence nucleons corresponding to the representation $[\lambda_1, \lambda_2, \lambda_3]$ we obtain

$$\Sigma_\gamma = \frac{a}{2} + \frac{2}{3}\sum_{m=0}^{N} m(m+1)(m+2) + n_\gamma\,, \tag{3.4}$$

with $\gamma = 1, 2, 3$. In this way one can find all possible sets of Σ_1, Σ_2, Σ_3 for a given distribution of nucleons over the nuclear shells and given U(3) representation $[\lambda_1\lambda_2\lambda_3]$. Our problem has thus been reduced to the search for all possible irreducible representations of the h.o. symmetry group U(3) for a given number of nucleons distributed over nuclear shells.

We have adopted the unitary group U(3) in three dimensions as an underlying symmetry of the rotating h.o. In fact, we could have used the special unitary group SU(3) instead. This follows from the existence of one U(3) generator $(A_{11} + A_{22} + A_{33})$ that commutes with all the other generators. The two numbers (λ, μ) labelling the irreducible representations of SU(3) could replace the three numbers λ_1, λ_2, λ_3 specifying the irreducible representations of U(3) with

$$\lambda = \lambda_1 - \lambda_3\,, \tag{3.5a}$$

$$\mu = \lambda_2 - \lambda_3\,, \tag{3.5b}$$

(see *e.g.* Ref. [26]). Then the sum $\lambda_1 + \lambda_2 + \lambda_3$ would not be necessary for the symmetry specification. We shall, however, remain within the U(3) formalism since it is sometimes convenient to remember that in our notation $\lambda_1 + \lambda_2 - \lambda_3$ is equal to the total number of oscillator quanta occupied by the valence particles.

3.2. Search for all possible U(3) representations corresponding to a given nucleonic symmetry by the method of plethysms

We shall now demonstrate how to find all possible representations $[\lambda_1\lambda_2\lambda_3]$ of U(3) corresponding to all possible configurations of particle occupying a set of h.o. shells in the nucleus. It is well known that the symmetry of the wave function of n nucleons in the shell model may be labelled by the Young diagram (see *e.g.* Refs [25–28]) with n boxes. In the case of neutrons and protons the spin–isospin part of the many nucleon wave function may be labelled by a Young diagram of no more than four rows. In order to obtain a fully antisymmetric total wave function the corresponding space part of the wave function has to be labelled by a transposed Young diagram $\{f\}$ with no more than four columns. We shall use curly brackets $\{\ \}$ for Young diagrams for the set of fermions (nucleons) while for Young diagrams for the boson U(3) representation we shall use square brackets $[\ ]$ as before. The problem is, therefore, to find all possible representations $[\lambda]$ for the set of bosons in an h.o. shell $[\mu]$ corresponding to nucleon symmetry given by the Young diagram $\{f\}$. This decomposition has been known in the theory of group representations as an operation of plethysms (Refs [29–34] see also Refs [25–28])

$$[\mu] * \{f\} = \Sigma G_{\mu f \lambda}[\lambda], \tag{3.6}$$

where the integer number $G_{\mu f \lambda}$ indicates the multiplicity of the appearance of representation $[\lambda]$ of U(3) in the plethysm $[\mu] * \{f\}$.

The procedure is best illustrated by the following example. Let us take n=3 nucleons in the sd-shell. The product state may be fully symmetric *i.e.* $\{f\} = \{3\}$, fully antisymmetric $\{f\} = \{111\}$ or else it may be of a mixed symmetry $\{f\} = \{21\}$ (see *e.g.* Ref. [27]). The number of one nucleon states in the sd-shell is six and the dimensions of $\{f\} = \{3\}$, $\{21\}$ and $\{111\}$ are 56, 70 and 20, respectively. Formulae for the dimensions of these representations are given in many textbooks (see *e.g.* Ref. [24] p. 118, Ref. [28] sections 4–6 or Ref. [27] chapter 10). Since the representation $\{f\} = \{21\}$ appears twice in the product function of three nucleons we obtain the dimensional check

$$56 + 2 * 70 + 20 = 216 = 6^3$$

as the total number of product states in the sd-shell must be 6^3. Now, we are interested in what would be the $[\lambda_1\lambda_2\lambda_3]$ content of each of these three possible particle states. We have thus to calculate the following plethysms

$$[2] * \{3\}, \quad [2] * \{21\} \text{ and } [2] * \{111\}.$$

Below we give only the results:

$$[2] * \{3\} = \overset{56}{[6]} + \overset{28}{[42]} + \overset{27}{[222]} + \overset{1}{\{\,\}},$$

$$[2] * \{21\} = \overset{70}{[51]} + \overset{35}{[42]} + \overset{27}{[321]} + \overset{8}{\{21\}},$$

$$[2] * \{111\} = \overset{20}{[33]} + \overset{10}{[411]} + \overset{10}{\{111\}}.$$

The numbers above each formula give dimensions of corresponding $[\lambda_1\lambda_2\lambda_3]$ and $\{f\}$ representations.

We shall not discuss here fully the methods for calculations of the plethysms (see Refs 34–35]). Some of the simplest cases have been given explicitly in Refs [25] and [26]. More extensive tables exist in the literature (see *e.g.* Ref. [34]). For the purpose of this paper all plethysms for $[\mu] = [2]$ (sd-shell) and some cases for $[\mu] = [3]$ (fp-shell) and [4] (sgdg-shell) are needed. They are given in Table A1. Moreover, few mixed cases for [2]+[3] are included in Table A2. Appendix A presents a very brief description of some methods and theorems used for the calculation of simple plethysms.

Using the results of Tables A1 and A2 we can immediately find all possible U(3) representations $[\lambda_1\lambda_2\lambda_3]$ corresponding to each nucleon symmetry $\{f\}$ and thus, determine all possible sets of the h.o. occupation factors Σ_1, Σ_2 and Σ_3.

4. Description of some typical configurations in the yrast region in the sd-shell nuclei

In this section we intend to discuss briefly the appearance of some most interesting (*i.e.* of the lowest energy) configurations in the sd-shell nuclei. As a representative example we shall use the ^{16}O nucleus. These configurations may then be eventually used as a quantal basis for the nuclear wave functions in the framework of some current nuclear models as *e.g.* the Nilsson model or the selfconsistent Hartree–Fock model. Such a quantal basis may be especially useful in the investigation of the high-spin rotational states as it incorporates already to all orders the existence of the rotational couplings characteristic for the cranking model. Moreover, treating the basic states as a zero-order approximation to the nuclear wave function one can hope to gain some preliminary orientation about the behaviour in the yrast region in nuclei from the sd-shell.

Let us now discuss briefly the behaviour of a nucleus with a fixed configuration $(\Sigma_1, \Sigma_2, \Sigma_3)$ that is rotated with increasing frequency ω. Let us first consider the case of rather small deformation, where the intershell rotational coupling given by the second term j_{II} in Eq. (2.8) may be neglected. The Coriolis and centrifugal forces manifest themselves tending to align gradually the nucleonic orbits with the rotation axis and by attempting to locate the nucleons as far as possible from this axis. These tendencies result in mixing the h.o. quanta corresponding to axes 2 and 3 within the h.o. major shells and in changing the nuclear shape towards the axial symmetry with respect to the rotation axis 1. Finally, in the limit of maximum possible angular momentum

$$I \longrightarrow I_m = \Sigma_3 - \Sigma_2 ,$$

the corresponding nuclear shape will reach the $\gamma = 60°$ line in the deformation plane (in some cases $\gamma = -120°$) and the rotational band terminates. Further increase of angular momentum is only possible by means of the rearrangement of the configuration. This often requires rearranging of the h.o. quanta over more shells. Simultaneously, it may be important to include the intershell coupling given by the second term j_{II} in Eq. (2.8).

All the configurations characterized by the three occupation sums $(\Sigma_1, \Sigma_2, \Sigma_3)$ may be found by the methods described in the preceding sections. Let us briefly summarize the procedure. First, for a given number of valence nucleons one should write down all possible Young diagrams $\{f\}$ that define all the possible symmetries in the corresponding fermion function. Then using Tables A1 and A2 one can find all possible irreducible representations $[\lambda_1\lambda_2\lambda_3]$ of the group U(3) contained in the relevant plethysm labelled by the fermion Young diagram $\{f\}$. Finally, methods described in Section 3.1 enable us to find all sets of the occupation sums $(\Sigma_1, \Sigma_2, \Sigma_3)$ existing within a given irreducible representation $[\lambda_1\lambda_2\lambda_3]$.

Following the above procedure we shall give few typical examples of the low-lying configurations in the nucleus ^{16}O. The spherical ground-state of this nucleus with the h.o. shells $N = 0$ and $N = 1$ filled completely would correspond to the h.o. basic state with $(\Sigma_1, \Sigma_2, \Sigma_3) = (12, 12, 12)$. Then the typical positive parity states are of the 2p–2h (*i.e.* 2 particle, 2 hole), 4p–4h ... *etc.* character. For example the lowest 2p–2h configurations may be found as $(\Sigma_1, \Sigma_2, \Sigma_3) = (10, 12, 16), (11, 11, 16), (10, 13, 15), (13, 10, 15), (11, 12, 15), (12, 11, 15)$ *etc.* Two configurations that differ from each other only by the order of Σ_1 and Σ_2 correspond to rotations about the lowest and intermediate axes (*i.e.* largest and intermediate moments of inertia). The lowest 4p–4h configurations in ^{16}O turn out to be $(\Sigma_1, \Sigma_2, \Sigma_3) = (8, 13, 20), (12, 8, 20), (8, 13, 19), (13, 8, 19), (9, 11, 20), (11, 9, 20)$ *etc.* The lowest 8p–8h configurations start with $(\Sigma_1, \Sigma_2, \Sigma_3) = (8, 8, 32)$. Fig. 1

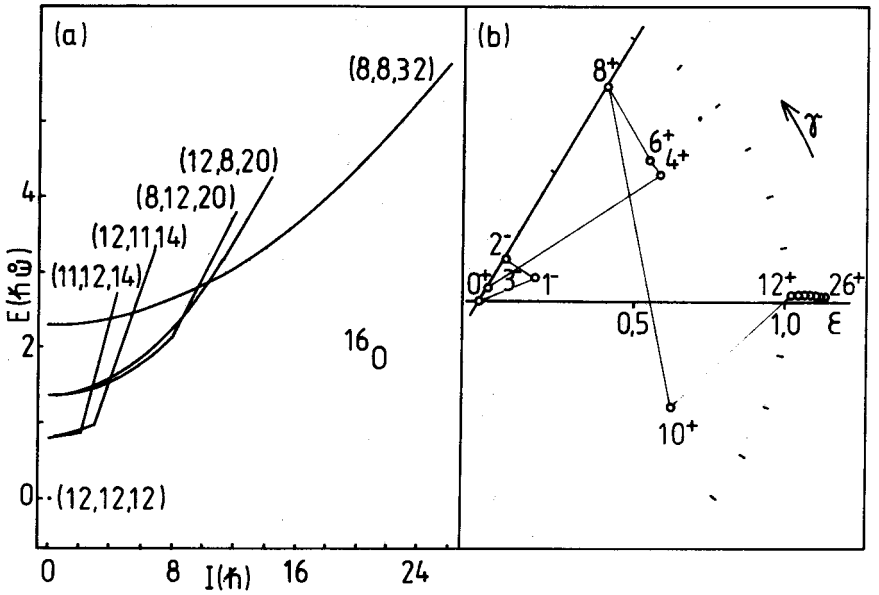

Fig. 1. Calculated energies (a) and deformations (b) of very high spin states of ^{16}O.

illustrates the energy curves corresponding to some configurations in ^{16}O that extend to high angular momenta (part (a) of the figure) and the spin trajectory in the deformation plane (ε, γ). Examples of the negative parity states could be first of all the 1p–1h states such as (11, 12, 14), (12, 11, 14) ... *etc.* while the 3p–3h states start with (9, 12, 18) and (12, 9, 18).

Similar procedure may be easily applied to all nuclei in the sd-shell.

5. Behaviour at very high angular momenta

In the preceding section we have illustrated methods for constructing the set of basic states that form a convenient representation for the states in the sd-shell nuclei and extend to the high-spin region. Alternatively these states could be considered as the zero-order approximation to the nuclear states arising from a description that is more realistic than that of the harmonic oscillator.

Now we would like to extend our considerations to the region of still higher angular momenta going beyond the termination of the few lowest energy rotational bands (formed out of the valence particles only). We expect that states characterized by very large angular momenta are built out of nucleons occupying higher oscillator shells. In the same time the

condition of selfconsistency between the deformation of the nuclear potential and density distribution will lead to the appearance of higher deformations.

Below we would like to present a procedure of finding the behaviour of the nucleus described by the model given above in the yrast region. For each fixed configuration (Σ_1, Σ_2, Σ_3) the nuclear energy E can be found from Eq. (2.9) including the minimization with respect to deformation parameters ε and γ. This procedure is repeated for several values of angular momentum I. In this way the trajectory in the (I, E) plane is determined for a given configuration. Now, various trajectories corresponding to given sets (Σ_1, Σ_2, Σ_3) form a family of curves in the (I, E) plane. The curves may cross at some points and one can select these parts of the curves that form the yrast line *i.e.* lie lowest in energy for any given I. The main problem is to find all the relevant configurations (out of many possible, *cf.* the preceding section) that contribute to some parts of the yrast line. Figs 1–3

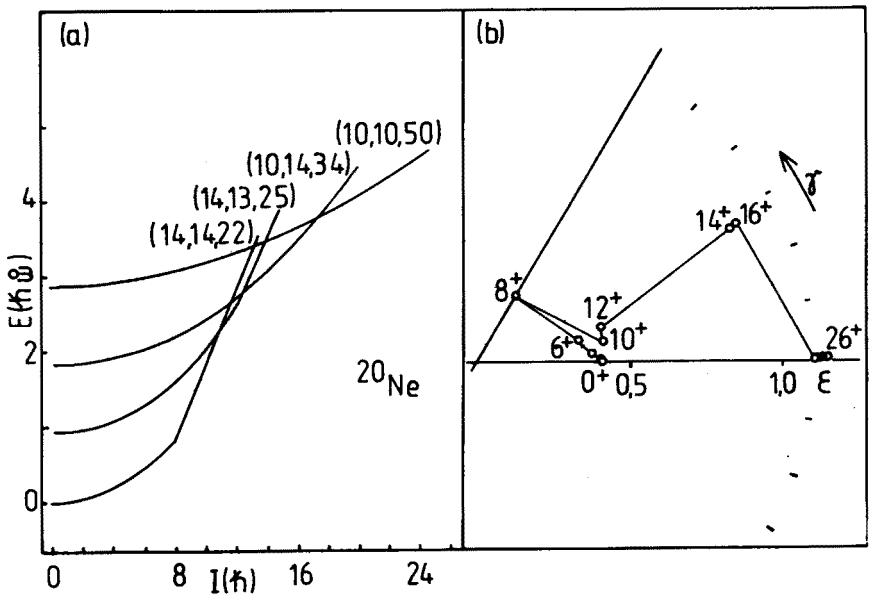

Fig. 2. Calculated energies (a) and deformations (b) of very high spin states of ^{20}Ne.

illustrate the results for three nuclei ^{16}O, ^{20}Ne and ^{28}Si (in the left portion of each figure). The right-hand side portions of Figs 1–3 illustrate the variations of the deformation parameters (ε, γ). Some of the curves in the (I, E) plane in the above mentioned figures exhibit very abrupt slope changes (*e.g.* curves labelled 11, 12, 14 and 12, 11, 14 for ^{16}O, or 14, 14, 22 for ^{20}Ne, or finally the 30, 18, 30 and 22, 22, 34 for ^{28}Si) at some singular

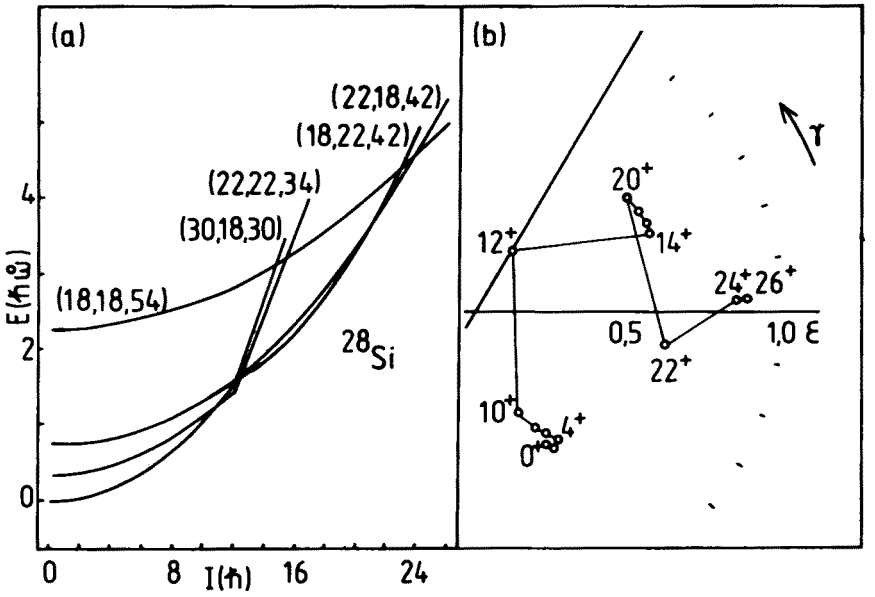

Fig. 3. Calculated energies and deformations of very high spin states of ^{28}Si.

points. These points correspond to the band terminations and the extensions beyond these points correspond to the new branch first found by Arvieu and Troudet [19–21]. One can see that the Arvieu–Troudet branches seldom approach the yrast line (the $I^\pi = 10^+$ state in ^{20}Ne may be an example of such an exception). Figure 3 shows the calculated deformations (ε, γ) of the yrast states up to very high angular momenta in ^{28}Si. In this case low spin states correspond to the rotation of a disc-type ellipsoid about an axis perpendicular to its symmetry axis, the 12^+ state is of a non-collective character while for still higher spins triaxial and then strongly elongated shapes are developed.

6. Conclusions

The rotating harmonic oscillator potential provides a very useful model for the description of nuclear rotation. The eigenstates of the rotating harmonic oscillator potential may be considered as the set of basic states that define a zero-order approximation that could then be used as a starting point for more realistic descriptions.

Alternatively, these states may offer a preliminary orientation in the properties of a rotating nucleus. Despite the simplicity of the model all qualitative features of the high spin states of light nuclei are well reproduced. The advantage of the rotating harmonic oscillator model consists in the fact

that it provides simple analytical solutions to the problem of many nucleons rotating in a single particle potential.

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Appendix A

Section 3.2 contained a short description of the idea of the method of plethysms. For the examples presented in that section it was sufficient to perform a simple analysis of dimensions of representations appearing in a decomposition of outer products of corresponding Young diagrams. However, in general this technique is not sufficient to find all plethysms needed to describe single particle configurations of nucleons occupying states of the rotating harmonic oscillator potential. The following theorems are necessary to fully solve this problem.

Theorem 1 (see Ref. [31])
For a given plethysm $\{\lambda\} * \{\pi\} = \sum_{\{\nu\}} G_{\lambda\pi\nu}\{\nu\}$ the following equality is fulfilled:

$$\sum_{\xi,\nu} \Gamma_{1\xi\nu}\{\xi\} = \Big(\sum_{\mu} \Gamma_{1\mu\pi}\{\lambda\} * \{\mu\}\Big)\Big(\sum_{\gamma} \Gamma_{1\gamma\lambda}\{\gamma\}\Big), \tag{A1}$$

where $\Gamma_{\alpha\beta\lambda}$ occurs as the coefficient in the decomposition of the outer product $\{\alpha\} - \{\beta\} = \sum \Gamma_{\alpha\beta\lambda}\{\lambda\}$.

Let the term $\{\alpha_1+\beta_1,\ \alpha_2+\beta_2,\ \ldots\}$ which is never zero in the product of $\{\alpha\}$ and $\{\beta\}$ be called the principal part of this product. Then the following theorems hold

Theorem 2 (see Ref. [33])
The principal parts of the products of terms in the expansion of $(\{\lambda\} * \{\omega\})(\{\mu\} * \{\eta\})$ appears as terms in the expansion of $\{\lambda_1+\mu_1,\ \lambda_2+\mu_2,\ \ldots\} * \{\nu\}$ whenever $\chi^{\omega}\chi^{\eta} = \chi^{\nu}$, where χ^{ω}, χ^{η} and χ^{ν} are the characters of the symmetric group of order $n!$ corresponding to the partitions (ω), (η), (ν) of n and (λ), (μ) may be other partitions.

Theorem 3 (see Ref. [32])
Principal parts of the products of terms in the expansion of $(\{\lambda\} * \{\omega\})(\{\lambda\} * \{\nu\})$ appear as terms in the expansion of the plethysm $\{\lambda\} * \{\omega_1 + \nu_1 +,\ \omega_2 + \nu_2, \ldots\}$.

Table A1 contains expansions of plethysms necessary to find all configurations of nucleons occupying states of $N = 2$ shell of the anisotropic harmonic oscillator in three dimensions.

For a more complicated case of particles occupying states belonging to some reducible representation of the type $\{\pi\} + \{\rho\}$ Littlewood (Ref. [29]) proved the following identity

$$(\{\pi\} + \{\rho\}) * \{\lambda\} = \sum \Gamma_{\alpha\beta\lambda}(\{\pi\} * \{\alpha\})(\{\rho\} * \{\beta\}). \qquad \text{(A2)}$$

This identity was used in calculations which led to the results contained in Table A2, which contains expansions of plethysms corresponding to up to 4 particles occupying $N = 2$ and $N = 3$ shells.

TABLE A1
$[\lambda] = [2]$

	{2}	{1,1}
[4]	1	
[3,1]		1
[2,2]	1	

	{3}	{2,1}	{1,1,1}
[6]	1		
[5,1]		1	
[4,2]	1	1	
[4,1,1]			1
[3,3]			1
[3,2,1]		1	
[2,2,2]	1		

	{4}	{3,1}	{2,2}	{2,1,1}	{1,1,1,1}
[8]	1				
[7,1]		1			
[6,2]	1	1	1		
[6,1,1]				1	
[5,3]		1		1	
[5,2,1]		1	1	1	
[4,4]	1		1		
[4,3,1]		1		1	1
[4,2,2]	1	1	1		
[3,3,2]				1	

Table A1 cont'd $[\lambda] = [2]$

	{4,1}	{3,2}	{3,1,1}	{2,2,1}	{2,1,1}	{1,1,1,1}
[9,1]	1					
[8,2]	1	1				
[8,1,1]			1			
[7,3]	1	1	1			
[7,2,1]	1	1	1	1		
[6,4]	1	1		1		
[6,3,1]	1	1	2	1	1	
[6,2,2]	1	2		1		
[5,5]			1			
[5,4,1]	1	1	1	1	1	
[5,3,2]	1	1	2	1	1	
[4,4,2]	1	1		1		1
[4,3,3]			1		1	

	{4,2}	{4,1,1}	{3,3}	{3,2,1}	{3,1,1,1}	{2,2,2}	{2,2,1,1}	{2,1,1,1,1}
[10,2]	1							
[10,1,1]		1						
[9,3]	1	1	1					
[9,2,1]	1	1		1				
[8,4]	2			1				
[8,3,1]	1	2	1	2	1			
[8,2,2]	2		1	1		1		
[7,5]		1	1	1				
[7,4,1]	2	2	1	2	1	1	1	
[7,3,2]	2	2	1	3	1		1	
[6,6]	1					1		
[6,5,1]	1	1		2	1		1	
[6,4,2]	3	1	1	3	1	2	1	1
[6,3,3]		2	1	1	2		1	
[5,5,2]		2	1	1	1		1	
[5,4,3]	1	1		2	1		1	1
[4,4,4]	1					1		

	{4,3}	{4,2,1}	{4,1,1,1}	{3,3,1}	{3,2,2}	{3,2,1,1}	{3,1,1,1,1}	{2,2,2,1}	{2,2,1,1,1}	{2,1,1,1,1,1}
[11,3]	1									
[11,2,1]		1								
[10,4]		1	1							
[10,3,1]		1	2	1	1					
[10,2,2]		1	1			1				
[9,5]		1	1	1						
[9,4,1]		2	3	1	1	1	1			
[9,3,2]		2	3	1	2	1	1			
[8,6]		1	1		1					
[8,5,1]		1	3	1	2	1	2			
[8,4,2]		3	5	1	2	3	2	1	1	
[8,3,3]		1	2	2	2		2			
[7,7]				1						
[7,6,1]		1	2	1	1	1	1	1		
[7,5,2]		2	4	2	3	2	3	1	1	
[7,4,3]		2	4	2	2	2	3	1	1	
[6,6,2]		1	2		2	1	1	1		
[6,5,3]		1	3	2	2	1	3	1	1	
[6,4,4]		1	2			2	1	1	1	1
[5,5,4]			1	1	1		1			

	{4,4}	{4,3,1}	{4,2,2}	{4,2,1,1}	{4,1,1,1}	{3,3,2}	{3,3,1,1}	{3,2,2,1}	{3,2,1,1,1}	{3,1,1,1,1,1}	{2,2,2,2}	{2,2,2,1,1}	{2,2,1,1,1,1}
[12,4]	1												
[12,3,1]		1											
[12,2,2]			1										
[11,5]		1											
[11,4,1]		1	2	1	1								
[11,3,2]		1	2	1	1	1							
[10,6]		1	1	1									
[10,5,1]		1	3	1	2	1	1						
[10,4,2]		2	4	2	4	1	1	1					
[10,3,3]			3		2	1	1						
[9,7]			1			1							
[9,6,1]		1	3	2	2	1	1	1					
[9,5,2]		1	6	3	4	3	2	2	1				
[9,4,3]		2	5	3	4	1	2	2	1				
[8,8]		1		1									
[8,7,1]			2	1	1	1	1	1					
[8,6,2]		2	4	5	3	1	2	2	3	1	1		
[8,5,3]		1	6	3	6	1	3	4	3	2		1	
[8,4,4]		2	2	4	2	1	1	1	2	1	1		
[7,7,2]			2		2		2	1	1			1	
[7,6,3]		1	4	3	4	1	2	2	3	2		1	1
[7,5,4]			4	2	4	1	2	2	3			1	1
[6,6,4]		1	1	3	1	1		1	2	1	1		
[6,5,5]			1		2		1	1		1		1	

Table A1 cont'd $[\lambda] = [2]$

	{4,4,1}	{4,3,2}	{4,3,1,1}	{4,2,2,1}	{4,2,1,1,1}	{4,1,1,1,1}	{3,3,3}	{3,3,2,1}	{3,3,1,1,1}	{3,2,2,2}	{3,2,2,1,1}	{3,2,1,1,1,1}	{2,2,2,2,1}	{2,2,1,1,1,1}
[13,4,1]	1													
[13,3,2]		1												
[12,6]	1													
[12,5,1]	1	1	1											
[12,4,2]	2	2	1	1										
[12,3,3]	1	1	1				1							
[11,7]		1												
[11,6,1]	2	2	2	1										
[11,5,2]	3	4	3	2	1		1	1						
[11,4,3]	3	4	3	2	1			1						
[10,8]	1	1												
[10,7,1]	2	2	2	1			1	1						
[10,6,2]	4	6	4	4	1			2	1	1				
[10,5,3]	4	6	7	4	2		2	3	1		1			
[10,4,4]	3	3	2	3	1	1		1	1	1				
[9,9]							1							
[9,8,1]	1	2	1	1				1						
[9,7,2]	2	5	4	3	1		2	3		1	1			
[9,6,3]	5	7	7	6	3		2	4	2	2	2			
[9,5,4]	3	6	6	5	3		1	4	1	1	2	1		
[8,8,2]	2	2	1	2				1	1	1				
[8,7,3]	2	5	5	4	2		1	4	1	1	2		1	
[8,6,4]	4	6	5	7	3	1		4	3	3	2	1	1	
[8,5,5]	1	2	4	2	2		2	2	1		2			1
[7,7,4]	1	2	3	2	2		2	2		1	2			1
[7,6,5]	1	3	3	3	2			2	1	1	2	1	1	
[6,6,6]	1			1		1			1	1				

Table A1 cont'd $[\lambda] = [2]$

	{4,4,2}	{4,4,1,1}	{4,3,3}	{4,3,2,1}	{4,3,1,1,1}	{4,2,2,2}	{4,2,2,1,1}	{4,2,1,1,1,1}	{3,3,3,1}	{3,3,2,2}	{3,3,2,1,1}	{3,3,1,1,1,1}	{3,2,2,2,1}	{3,2,2,1,1,1}	{2,2,2,2,2}	{2,2,2,1,1,1}
[14,4,2]	1															
[14,3,3]			1													
[13,6,1]	1	1														
[13,5,2]	1	1	1	1												
[13,4,3]	2	1	1	1												
[12,8]	1															
[12,7,1]	1	1	1	1												
[12,6,2]	4	2	1	3	1	1										
[12,5,3]	3	3	3	4	1		1		1							
[12,4,4]	3	1		2	1	1										
[11,9]			1													
[11,8,1]	2	1	1	2												
[11,7,2]	3	3	3	5	1	1	1		1	1						
[11,6,3]	6	5	4	8	3	2	2		1	1	1					
[11,5,4]	4	4	3	7	2	1	2	1	1	1	1					
[10,10]	1															
[10,9,1]	1	1	1	1					1							
[10,8,2]	5	2	2	5	1	2			1	1	1					
[10,7,3]	5	5	5	10	3	2	3		3	2	2		1			
[10,6,4]	8	5	3	11	5	5	3	1	2	2	3	1	1			
[10,5,5]	1	3	3	5	2		3		2	1	1			1		
[9,9,2]		1	2	2			1		1	1						
[9,8,3]	4	3	3	7	2	2	2		2	2	2		1			
[9,7,4]	4	5	5	11	4	3	5	1	3	4	3		2	1		
[9,6,5]	4	4	3	9	4	3	4	1	2	2	3	1	2	1		
[8,8,4]	4	1		5	2	3	1		1	1	2	1	1		1	
[8,7,5]	2	3	3	7	3	2	4	1	2	2	3		2	1		1
[8,6,6]	3	1		3	2	3	1	1		1	2	1	1		1	
[7,7,6]		1	1	2	1		2		1	1			1	1		

Table A1 cont'd $[\lambda] = [2]$

	{4,4,3}	{4,4,2,1}	{4,4,1,1,1}	{4,3,3,1}	{4,3,2,2}	{4,3,2,1,1}	{4,3,1,1,1,1}	{4,2,2,2,1}	{4,2,2,1,1,1}	{3,3,3,2}	{3,3,3,1,1}	{3,3,2,2,1}	{3,3,2,1,1,1}	{3,2,2,2,2}	{3,2,2,2,1,1}	{2,2,2,2,2,1}
[15,4,3]	1															
[14,6,2]	1	1														
[14,5,3]	1	1		1												
[14,4,4]	1	1														
[13,8,1]	1	1														
[13,7,2]	1	2	1	1	1											
[13,6,3]	3	4	1	2	1	1										
[13,5,4]	2	3	1	2	1	1										
[12,10]	1															
[12,9,1]	1	1		1												
[12,8,2]	3	4		2	2	1										
[12,7,3]	4	6	2	5	3	3		1		1						
[12,6,4]	5	8	2	4	4	4	1	1			1					
[12,5,5]	1	3	1	3	1	2			1	1						
[11,10,1]	1	1		1												
[11,9,2]	2	3	1	3	2	1				1						
[11,8,3]	5	8	2	6	5	4		1		1	1	1				
[11,7,4]	5	10	4	8	7	7	1	2	1	2	1	2				
[11,6,5]	4	8	3	6	5	6	1	2	1	1	1	1	1			
[10,10,2]	2	2		1	1						1					
[10,9,3]	3	5	1	5	3	3		1		2	1	1				
[10,8,4]	6	10	2	7	8	7	1	3		2	3	2	1	1		
[10,7,5]	4	9	4	9	7	9	1	4	2	3	2	3	1		1	
[10,6,6]	3	5	1	2	4	4	2	2			2	1	1	1		
[9,9,4]	1	3	2	4	3	3		1	1	2		2				
[9,8,5]	3	7	2	6	6	7	1	3	1	2	2	3	1	1	1	
[9,7,6]	2	5	3	5	5	6	1	3	2	2	1	3	1	1	1	
[8,8,6]	2	3		1	3	3	1	2			2	1	1	1		1
[8,7,7]		1	1	2	1	2		1	1	1		1			1	

Table A1 cont'd $[\lambda] = [2]$

	{4,4,4}	{4,4,3,1}	{4,4,2,2}	{4,4,2,1,1}	{4,4,1,1,1,1}	{4,3,3,2}	{4,3,3,1,1}	{4,3,2,2,1}	{4,3,2,1,1,1}	{4,2,2,2,2}	{4,2,2,2,1,1}	{3,3,3,3}	{3,3,3,2,1}	{3,3,3,1,1,1}	{3,3,2,2,2}	{3,3,2,2,1,1}	{3,2,2,2,2,1}	{2,2,2,2,2,2}
[16,4,4]	1																	
[15,6,3]	1	1																
[15,5,4]		1																
[14,8,2]	1	1	1															
[14,7,3]		2	1	1		1												
[14,6,4]	2	3	2	1			1											
[14,5,5]		1		1		1												
[13,10,1]	1	1																
[13,9,2]		2	1	1		1												
[13,8,3]	2	5	3	2		2	1	1										
[13,7,4]	2	6	3	4	1	3	2	2										
[13,6,5]	1	5	3	3		2	2	1	1									
[12,12,0]	1																	
[12,11,1]		1																
[12,10,2]	2	3	2			1	1											
[12,9,3]	2	6	3	3		4	2	2				1						
[12,8,4]	4	9	8	5		5	5	4	1	1			1					
[12,7,5]	1	9	5	7	1	6	4	5	2		1	1	1					
[12,6,6]	3	4	4	2	1	1	3	2	1	1				1				
[11,11,2]		1		1		1												
[11,10,3]	1	5	3	2		3	2	1					1					
[11,9,4]	1	9	5	6	1	7	4	5	1			1	2		1			
[11,8,5]	3	11	8	8	1	8	7	8	2	1	1	1	2	1	1	1		
[11,7,6]	1	7	5	7	1	6	5	6	3	1	1		2		1	1		
[10,10,4]	3	4	4	1		2	3	2		1		1	1	1				
[10,9,5]	1	7	5	6		7	5	6	2	1	1	1	3		1	1		
[10,8,6]	3	8	8	5	1	5	7	8	3	3	1	1	3	2	1	1	1	
[10,7,7]		3	1	4	1	4	2	4	1		2	1	1		1	1		
[9,9,6]		3	1	4	1	4	2	4	1		1	1	1		2	1		
[9,8,7]		3	3	3		3	3	4	2	1	1		2		1	1	1	
[8,8,8]	2		1				1	1		1				1				1

Table A1 cont'd $[\lambda] = [3]$

	{2}	{1,1}
[6]	1	
[5,1]		1
[4,2]	1	
[3,3]		1

	{3}	{2,1}	{1,1,1}
[9]	1		
[8,1]		1	
[7,2]	1	1	
[7,1,1]			1
[6,3]	1	1	1
[6,2,1]		1	
[5,4]		1	
[5,3,1]		1	1
[5,2,2]	1		
[4,4,1]	1		
[4,3,2]		1	
[3,3,3]			1

	{4}	{3,1}	{2,2}	{2,1,1}	{1,1,1,1}
[12]	1				
[11,1]		1			
[10,2]	1	1	1		
[10,1,1]				1	
[9,3]	1	2		1	
[9,2,1]		1	1	1	
[8,4]	1	1	2	1	
[8,3,1]		2	1	2	1
[8,2,2]	1	1	1		
[7,5]		2		1	
[7,4,1]	1	2	1	2	1
[7,3,2]	1	2	1	2	
[6,6]	1		1		1
[6,5,1]		1	1	2	
[6,4,2]	1	2	2	1	1
[6,3,3]		1		2	1
[5,5,2]		1		1	
[5,4,3]		1	1	1	
[4,4,4]	1				

Table A1 cont'd $[\lambda] = [3]$

	{4,1}	{3,2}	{3,1,1}	{2,2,1}	{2,1,1,1}	{1,1,1,1,1}
[14,1]	1					
[13,2]	1	1				
[13,1,1]			1			
[12,3]	2	1	1			
[12,2,1]	1	1	1	1		
[11,4]	2	2	1	1		
[11,3,1]	2	2	3	1	1	
[11,2,2]	1	2		1		
[10,5]	2	2	2	1		
[10,4,1]	3	3	3	3	2	
[10,3,2]	3	3	3	2	1	
[9,6]	2	2	1	1	1	
[9,5,1]	3	3	5	3	2	
[9,4,2]	4	5	4	4	2	1
[9,3,3]	2	1	4	1	2	
[8,7]	1	1	1	1		
[8,6,1]		3	3	3	2	1
[8,5,2]	4	5	5	4	3	
[8,4,3]	3	4	4	4	3	1
[7,7,1]	1	1	2		1	
[7,6,2]	3	3	3	3	2	1
[7,5,3]	3	3	6	3	3	
[7,4,4]	2	2	1	2	1	1
[6,6,3]	1	2	1	2	2	1
[6,5,4]	1	2	2	2	1	
[5,5,5]			1			

Table A1 cont'd $[\lambda] = [3]$

	{4,2}	{4,1,1}	{3,3}	{3,2,1}	{3,1,1,1}	{2,2,2}	{2,2,1,1}	{2,1,1,1,1}	{1,1,1,1,1,1}
[16,2]	1								
[16,1,1]		1							
[15,3]	1	1	1						
[15,2,1]	1	1		1					
[14,4]	3	1		1					
[14,3,1]	2	3	1	2	1				
[14,2,2]	2		1	1		1			
[13,5]	2	2	2	2					
[13,4,1]	4	4	2	4	2	1	1		
[13,3,2]	4	3	2	4	1		1		
[12,6]	4	2	1	2	1	1			
[12,5,1]	5	6	2	7	3	1	2		
[12,4,2]	8	5	3	8	3	3	2	1	
[12,3,3]	2	5	2	5	3		1		
[11,7]	2	2	2	3			1		
[11,6,1]	6	6	3	8	4	2	3	1	
[11,5,2]	9	9	6	12	5	3	5	1	
[11,4,3]	7	7	3	10	5	2	4	2	
[10,8]	3	1		2	1	1			
[10,7,1]	5	6	3	7	4	2	3	1	
[10,6,2]	11	8	4	14	7	5	5	3	
[10,5,3]	9	12	5	15	9	3	7	2	
[10,4,4]	6	3	2	6	3	4	3	2	1
[9,9]		1	2				1		
[9,8,1]	3	3	1	5	2	1	2	1	
[9,7,2]	7	8	5	11	5	2	6	2	1
[9,6,3]	10	11	6	15	10	5	8	4	1
[9,5,4]	7	8	4	13	6	3	6	3	
[8,8,2]	4	2	1	4	3	3	1	1	
[8,7,3]	6	7	3	11	6	2	5	3	
[8,6,4]	9	6	3	12	7	5	6	4	1
[8,5,5]	2	5	3	5	4	1	3		
[7,7,4]	2	5	3	4	3	1	3	1	1
[7,6,5]	3	3	1	6	3	1	1	2	
[6,6,6]	1		1		1	2		1	

Table A1 cont'd $[\lambda] = [4]$

	{2}	{1,1}
[8]	1	
[7,1]		1
[6,2]	1	
[5,3]		1
[4,4]	1	

	{3}	{2,1}	{1,1,1}
[12]	1		
[11,1]		1	
[10,2]	1	1	
[10,1,1]			1
[9,3]	1	1	1
[9,2,1]		1	
[8,4]	1	2	
[8,3,1]		1	1
[8,2,2]	1		
[7,5]		1	1
[7,4,1]	1	1	1
[7,3,2]		1	
[6,6]	1		
[6,5,1]		1	
[6,4,2]	1	1	
[6,3,3]			1
[5,5,2]			1
[5,4,3]		1	
[4,4,4]	1		

Table A1 cont'd $[\lambda] = [4]$

	{4}	{3,1}	{2,2}	{2,1,1}	{1,1,1,1}
[16]	1				
[15,1]		1			
[14,2]	1	1	1		
[14,1,1]				1	
[13,3]	1	2		1	
[13,2,1]		1	1	1	
[12,4]	2	2	2	1	
[12,3,1]		2	1	2	1
[12,2,2]	1	1	1		
[11,5]		3	1	2	
[11,4,1]	1	3	2	3	1
[11,3,2]	1	2	1	2	
[10,6]	2	2	2	1	1
[10,5,1]	1	3	2	4	1
[10,4,2]	2	4	3	2	1
[10,3,3]		1		2	1
[9,7]		2		2	
[9,6,1]	1	3	2	3	1
[9,5,2]	1	4	2	4	1
[9,4,3]	1	3	2	3	1
[8,8]	1		2		
[8,7,1]		2	1	2	1
[8,6,2]	2	3	3	2	1
[8,5,3]		3	2	4	1
[8,4,4]	2	2	2	1	
[7,7,2]		1		2	
[7,6,3]	1	2	1	2	1
[7,5,4]		2	1	2	1
[6,6,4]	1	1	1		
[6,5,5]				1	

TABLE A2

$([2]+[3])*\{1\} = [2]+[3]$

$([2]+[3])*\{2\} = [4]+[2,2]+[6]+[4,2]+[5]+[4,1]+[3,2]$

$([2]+[3])*\{1,1\} = [3,1]+[5,1]+[3,3]+[5]+[4,1]+[3,2]$

$([2]+[3])*\{3\} = [6]+[4,2]+[2,2,2]+[7]+[6,1]+2[5,2]+[4,3]+[4,2,1]+[3,2,2]+[8]+[7,1]+2[6,2]+[5,3]+[5,2,1]+[4,4]+[4,3,1]+[4,2,2]+[9]+[7,2]+[6,3]+[5,2,2]+[4,4,1]$

$([2]+[3])*\{2,1\} = [5,1]+[4,2]+[3,2,1]+[8,1]+[7,2]+[6,3]+[6,2,1]+[5,4]+[5,3,1]+[4,3,2]+[7]+2[6,1]+3[5,2]+[5,1,1]+2[4,3]+2[4,2,1]+[3,3,1]+[3,2,2]+[8]+2[7,1]+3[6,2]+[6,1,1]+3[5,3]+2[5,2,1]+[4,4]+2[4,3,1]+[4,2,2]+[3,3,2]$

$([2]+[3])*\{1,1,1\} = [3,3]+[4,1,1]+[7,1,1]+[6,3]+[5,3,1]+[3,3,3]+[6,1]+[5,2]+[5,1,1]+[4,3]+[4,2,1]+[3,3,1]+[7,1]+[6,2]+[6,1,1]+2[5,3]+[5,2,1]+[4,3,1]+[3,3,2]$

$([2]+[3])*\{4\} = [8]+[6,2]+[4,4]+[4,2,2]+[12]+[10,2]+[9,3]+[8,4]+[8,2,2]+[7,4,1]+[7,3,2]+[6,6]+[6,4,2]+[4,4,4]+[11]+[10,1]+2[9,2]+2[8,3]+[8,2,1]+2[7,4]+2[7,3,1]+2[7,2,2]+[6,5]+2[6,4,1]+2[6,3,2]+2[5,4,2]+[4,4,3]+[9]+[8,1]+2[7,2]+2[6,3]+[6,2,1]+[5,4]+[5,3,1]+2[5,2,2]+[4,4,1]+[4,3,2]+[10]+[9,1]+3[8,2]+2[7,3]+3[6,4]+2[7,2,1]+3[6,2,2]+2[6,3,1]+2[4,4,2]+2[5,4,1]+2[5,3,2]$

$([2]+[3])*\{3,1\} = [7,1]+[6,2]+[5,3]+[5,2,1]+[4,2,2]+[4,3,1]+[11,1]+[10,2]+2[9,3]+[9,2,1]+[8,4]+2[8,3,1]+[8,2,2]+2[7,5]+2[7,4,1]+2[7,3,2]+[6,5,1]+2[6,4,2]+[6,3,3]+[5,2,2],+[5,4,3]+[11]+2[10,1]+4[9,2]+5[8,3]+4[8,2,1]+5[7,4]+6[7,3,1]+4[7,2,2]+3[6,5]+6[6,4,1]+6[6,3,2]+5[5,4,2]+2[4,4,3]+[9]+2[8,1]+4[7,2]+4[6,3]+4[6,2,1]+3[5,4]+4[5,3,1]+4[5,2,2]+2[4,4,1]+3[4,3,2]+[10]+3[9,1]+5[8,2]+7[7,3]+5[6,4]+6[7,2,1]+3[6,2,2]+[6,3,1]+3[4,4,2]+6[5,4,1]+7[5,3,2]+[9,1,1]+2[5,5,1]+2[5,5,1]+[7,1,1]+2[8,1,1]+3[5,5]+2[6,2,2]+2[4,4,3]$

TABLE A2 (cont)

$$([2]+[3])*\{2,2\} = [6,2]+[4,4]+[4,2,2]+[5,2,1]+[10,2]+[9,2,1]+2[8,4]+[8,3,1]+[8,2,2]+[7,4,1]+[7,3,2]+[6,6]+[6,5,1]+2[6,4,2]+[5,4,3]+[10,1]+2[9,2]+[9,1,1]+3[8,3]+3[8,2,1]+3[7,4]+4[7,3,1]+2[7,2,2]+2[6,5]+4[6,4,1]+4[6,3,2]+2[5,5,1]+3[5,4,2]+2[5,3,3]+[4,4,3]+[8,1]+2[7,2]+[7,1,1]+2[6,3]+3[6,2,1]+2[5,4]+3[5,3,1]+2[5,2,2]+[4,4,1]+2[4,3,2]+4[8,2]+[8,1,1]+3[7,3]+4[7,2,1]+5[6,4]+5[6,3,1]+4[6,2,2]+4[5,3,2]+4[5,4,1]+3[4,4,2]+[4,3,3]+[10]+[9,1]$$

$$([2]+[3])*\{2,1,1\} = [5,2,1]+[5,3]+[3,3,2]+[4,3,1]+[6,1,1]+[10,1,1]+[9,3]+[9,2,1]+[8,4]+2[8,3,1]+[7,5]+2[7,4,1]+2[7,3,2]+2[6,5,1]+[6,4,2]+2[6,3,3]+[5,5,2]+[5,4,3]+[10,1]+2[9,2]+2[9,1,1]+4[8,3]+4[8,2,1]+4[7,4]+7[7,3,1]+2[7,2,2]+2[6,5,1]+6[6,4,1]+6[6,3,2]+3[5,5,1]+4[5,4,2]+4[5,3,3]+[4,4,3]+[8,1]+2[7,2]+2[7,1,1]+3[6,3]+4[6,2,1]+2[5,4]+5[5,3,1]+2[5,2,2]+2[4,4,1]+3[4,3,2]+[3,3,3]+3[8,2]+3[8,1,1]+6[7,3]+6[7,2,1]+4[6,4]+9[6,3,1]+3[6,2,2]+7[5,3,2]+6[5,4,1]+2[4,4,2]+3[4,3,3]+2[9,1]+3[5,5]$$

$$([2]+[3])*\{1,1,1,1\} = [4,3,1]+[8,3,1]+[7,4,1]+[6,6]+[6,4,2]+[6,3,3]+[7,1,1]+[6,2,1]+2[5,3,1]+[4,4,1]+[6,3]+[4,3,2]+[3,3,3]+[9,1,1]+[8,2,1]+3[7,3,1]+[8,3]+[7,4]+[6,5]+2[6,4,1]+2[6,3,2]+[5,5,1]+[5,4,2]+2[5,3,3]+[8,2]+[8,1,1]+[7,3]+2[7,2,1]+2[6,4]+3[6,3,1]+[6,2,2]+2[5,3,2]+2[5,4,1]+[4,4,2]+[4,3,3]$$

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