THE MEASUREMENT OF RED-SHIFT AND FLAT TIME

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The measurement of the red-shift (R-S) effect of the general theory of relativity (GTR) resorts to the existence of two essentially different kinds of metrical objects: a) the macro-corpuscules of rigid bodies and the clocks which constitute reference bodies $\bar{S}$ and b) the light signals never at rest with regard to any reference body, which interconnect different regions of the metrical spacetime found by $\bar{S}$'s. Our aim is to show that the comparison of frequencies of the photons emitted by atomic clocks at different spacetime regions must call for flat time continuum. Thus quantum physics which integrates the wave and the corpuscular aspects of microobjects would distinguish flat spacetime. It must be remembered that owing to its nonlocality, quantum theory is capable of reproducing stable, extended objects from the hypothetical structureless constituents. Thus it provides us with a complete theory, whereas any classical theory ($\hbar = 0$), in particular the GTR, remains incomplete.

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1. "Intercommunicated" character of flat time

We shall say that flat spacetime of Galileo (G_4) and of Minkowski (L_4) are "intercommunicated", because the synchronization of clocks of a given reference body (frame) $\bar{S}(S)$ is performed by signals which — potentially — cross the whole spacetime and interact with macro-corpuscules of measuring rods and clocks. This coexistence of corpuscules with signals becomes strongly enhanced by the special theory of relativity (STR) in which light signals never at rest with regard to the corpuscules make the synchronization of clocks relativized to the given reference frame (body) $S(\bar{S})$, as the light signals (in vacuum) have the velocity equal to the universal constant c independent of S.

As a result the concept of metrical continuum of physics must consistently account for two basically different physical entities: a) the *corpuscular* clocks and measuring rods and b) the *light-waves* never at rest with regard to the macro-corpuscules of rods and clocks which constitute metrical reference bodies $\bar{S}$ and their abstraction — reference frames S. Both kinds of reality carry their metrical patterns which must be consistent with one another. The wavelength λ and the wave period $T = 1/\nu$ account for the space and time intervals, respectively, carried by light signals. This wandering spacetime pattern of light signals makes possible the confrontation of the space and time extensions of corpuscular realities at different spacetime regions, which is of fundamental importance for any measuring process. Consequently, we can expect a consistent description of the metrical properties of both corpuscules and waves to impose some constraints onto the "practical geometry" of physics.

In Einstein's philosophy of his GTR one starts with the fourdimensional spacetime continuum with scalar four-interval ds^2 which should determine the metrical spacetime parametrized by arbitrary (continuous) gaussian coordinates [1]. This obviously implies the general covariance of GTR equations. However, to use Bridgman's terminology [2], any experimentally testable inference from a theory requires a clear-cut "*physical text*" of its symbols. Here GTR encounters difficulties of which Einstein himself was fully aware, *cf.* Section 5. Fock was of the opinion that the purely mathematical purport of the general covariance is too poor to pretend to a physical principle [3]. Let us emphasize that the change of parametrization of spacetime, in which space and time variables are mixed, does not mean a change of "observation point" that requires a well defined behaviour of measuring devices. For example, the relativistic distortions of measuring rods and clocks implied by STR show that accelerated motion of a laboratory affects its intrinsic metrical properties thus making the data of such a laboratory ("point of observation") quite vague.

In Fock's opinion, GTR is a theory of gravitation [3] of "insular problems" where the asymptotic zone free of gravitation exists and where space and time are flat, *i.e.* with well-defined physical text. As a matter of fact, all textbooks discussing GTR effects make use of Fock's interpretation of GTR. We intend to show that the measurement of the R-S effect must resort to flat time and hence the question arises of whether atomic clocks move "good" or "bad".

The Schwarzschild solution of the GTR equations [4] for a given center M with mass M ("infinitely" large) and parametrized by the r, t variables in which the gravitational potential $\phi = -GM/r$ takes the form:

$$ds^2 = dr^2\left(1 + \frac{2\phi(P)}{c^2}\right)^{-1} - c^2dt^2\left(1 + \frac{2\phi(P)}{c^2}\right) = dl^2 - c^2d\tau^2 . \quad (1.1)$$

We restrict our consideration to the radial and time dimensions as the only ones relevant for further analysis. Here P denotes a point in the space of the rest-frames S of M and time t, called the "world-time", is identified with the flat time of the asymptotic zone ($r \to \infty$) of the inertial reference frame S, which is consistent with the analytic form of ds^2. This physical text of t works in favour of Fock's rather then Einstein's interpretation of GTR.

We shall be dealing with space regions where $|\phi/c^2| \ll 1$ and thus it is justified to restrict the analysis to the first-order approximation of ϕ/c^2, where according to (1.1)

$$d\tau(P) = dt\left(1 + \frac{\phi(P)}{c^2}\right) = dt\left(1 - \frac{GM}{rc^2}\right). \tag{1.2}$$

According to GTR, the $d\tau(P)$ determines the local-time interval of any good clock localized at macro-point P, *i.e.*, apart from its physical nature, because τ reflects the curvature of space which precedes any reality embedded in that space. Our point is that the very physical text of τ must call for the global (flat) world-time t which, owing to the light-signals, accounts for the intercommunicated nature of physical spacetime. Indeed, we assume that the hyperplanes

$$t = \text{const}. \tag{1.3}$$

remain space like for each value of the "const." and according to this text of flat time t the detection of the R-S effect becomes possible. Thus the variable t in ds^2 cannot be regarded as an arbitrary gaussian coordinate. Let us emphasize that the measurement of the R-S effect consists in comparing the motions of two clocks localized in two spacetime regions with different gravitational potential ϕ.

The objection to the local time τ as the time indicated by good clocks results simply from the following argument. Let us assume that t is indicated by good clocks at rest in the asymptotic zone of $S(r \to \infty)$ free of gravitation. With the help of light signals these clocks determine the hyperplanes (1.3) in the whole spacetime, which justifies the assumption that all clocks in S are synchronized at some given "zero-hour", *i.e.*:

$$t = \tau(P) = 0. \tag{1.4}$$

Thus,

$$\tau(P) = \int_0^t dt\left(1 + \frac{\phi(P)}{c^2}\right) = \left(1 + \frac{\phi(P)}{c^2}\right)t = K \geq 0 \tag{1.5}$$

represents the hypersurface of events of the same "τ-hour" indicated by the local clocks which, according to GTR, move "good". Since the factor

$(1 + \phi/c^2)$ is less than one, any two events on the $0r$-semi axis on the hypersurface (1.5) become, with increasing t, separated by time-like four--intervals when, from (1.1),

$$dl^2 < c^2 d\tau^2 \quad \text{or} \quad dr^2 < c^2 \left(1 - \frac{GM}{rc^2}\right)^4 dt^2 . \tag{1.6}$$

Indeed, the intervals dt, dr on the hypersurface (1.5) satisfy the equality

$$\left(1 - \frac{GM}{rc^2}\right) dt + \frac{GM}{r^2 c^2} t dr = 0 \tag{1.7}$$

consistent with the inequality (1.6) if:

$$r < r_K = \left(\frac{GM}{c}\right)^{1/2} K^{1/2} \underset{t\to\infty}{\longrightarrow} \infty . \tag{1.8}$$

This means that if $r < r_K \to \infty$, all (good) clocks on the $0r$-semi axis become disynchronized in a way which can be checked by real signal, *i.e.*, in the usual sense of the word "disynchronization".

2. Red-shift effect in GTR

Let $dn(P_1)$ denote the number of vibrations of a stationary wave passing a macro-point P_1 (in the space of a fixed reference frame S) during the flat-time interval dt, and $dn(P_2)$ the number of vibrations of the same wave passing another point P_2 at rest in S during the same interval dt. The spacelikeness and the parallelness of hyperplanes (1.3) imply that

$$\frac{dn(P_1)}{dt} dt \equiv \nu_t(P_1; P) dt = \nu_t(P_2; P) dt \equiv \frac{dn(P_2)}{dt} dt , \tag{2.1}$$

where $\nu_t(Q; P)$ denotes the frequency of the wave emitted at any point P, detected at any point Q and measured in the flat-time units, and so

$$\nu_t(P_2; P_1) = \nu_t(P_1; P_1) . \tag{2.2}$$

In other words, the wave-frequency measured in flat time must be an integral of the wave motion which, in the language of frequencies "complementary" to that of time, expresses the intercommunicated character of flat time t. The equality (2.2) enables one to confront the time rates of two clocks being at rest at two different (macro-) points P_1 and P_2 in a single region P_2 of a corpuscular detector.

According to the general principle of relativity, the number of vibrations $dn(P)$ measured in the local time $\tau(P)$ of any good GTR-clock (in particular, an atomic clock) is independent of P. Consequently,

$$\nu_0 = \frac{dn(P)}{d\tau(P)} \equiv \nu_{\tau(P)}(P;P) \tag{2.3}$$

determines the proper frequency ν_0 measured in the local time of all good GTR-clocks regardless of their structure. Since dt in (1.2) denotes the flat time interval which results in (2.2), the frequencies of the same clocks measured in t amount to:

$$\begin{aligned}\nu_t(P_1;P_1) &= \frac{dn(P_1)}{dt} = \frac{dn(P_1)}{d\tau(P_1)}\frac{d\tau(P_1)}{dt} \\ &= \nu_0\Big(1+\frac{\phi(P_1)}{c^2}\Big) = \nu_t(P_2;P_1)\,.\end{aligned} \tag{2.4}$$

Thus we get the ratio of frequencies, measurable at the macro-point P_2, which expresses the R-S effect of GTR,

$$\frac{\nu_t(P_2;P_1)}{\nu_t(P_2;P_2)} = \frac{1+\frac{\phi(P_1)}{c^2}}{1+\frac{\phi(P_2)}{c^2}} \simeq 1-\frac{\Delta\phi}{c^2}\,; \quad \Delta\phi = \phi(P_2)-\phi(P_1)\,, \tag{2.5}$$

proved on a laboratory scale by famous experiments of Pound and Rebka [5].

Without light-signals which make the notion of flat (global) time t realistic, the R-S effect could never be proved, as, according to GTR, any corpuscular clock shifted from P_1 to P_2 would behave in the same manner as the one that has already been at P_2. In order to understand the R-S effect better we put forward a "flat model" of this effect, which enables us to recognize the role of curved space of GTR. Moreover, within the flat model we can analyze the energy–momentum aspect of an atomic clock complementary to its space-time localization, which from the viewpoint of quantum theory seems to be of essential importance.

3. Gravitational interaction in flat spacetime

Owing to the Moesbauer effect we have got macroscopic atomic-clocks which practically do not suffer from recoil when emitting-absorbing photons thus having perfectly defined frequencies in a fixed lab-system. In consequence, the R-S effect has become testable on a laboratory scale. Note that heavy atomic clocks m, although much like any particle subject to the uncertainty relations [6], become macroscopically well-localized at "macro--points" as required by any classical theory, in particular, by GTR.

The very fact that the inertial mass of an object is at the same time the "coupling constant" of gravitational interaction implies that in the n-body system ($n \geq 2$) the Lorentz–Poincaré symmetry L of L_4 can only be the asymptotic, kinematic symmetry of one-body systems. Thus the flat model proposed below will still resort to the absolute-relation nature of interaction [7]. In our case the "relational" nature of gravitational interaction will be concealed, because we assume that M is an "infinitely" heavy center and hence, relations will coincide with the space coordinates of flat space represented in the rest-frame S of M.

For our present purpose it is enough to know that the hypothesis of the "relationism" [7] imposes a hierarchy in establishing the Hamiltonian $\mathcal{H}$ of an atomic clock m interacting with a very ("infinitely") heavy center M, such as the earth for the earth-gravitational phenomena. Then we start with an absolute-internal Hamiltonian $\hat{h}$ (operator) of the system m entirely separated from its external degrees of freedom. Consequently, from the Schroedinger equation:

$$\hat{h}|i,f\rangle = w_{i,f}|i,f\rangle; \qquad w_{i,f} = m_{i,f}c^2 \tag{3.1}$$

one obtains the initial and final states $|i,f\rangle$ of m, where the transitions between them result in the emission–absorption processes of photons thus creating atomic-clocks. In the absence of gravitational force, *i.e.* for a free object m, after determining the states $|i,f\rangle$ and their absolute eigen-masses $m_{i,f}$, we attach to m the standard one-body free Hamiltonians

$$\mathcal{H}_0 = c(m^2c^2 + p^2)^{1/2} \tag{3.2}$$

which accounts for relativistic kinematics in the so far arbitrary, inertial reference frame and $m = m_i$ or $m = m_f$.

Strong inequality $M \gg m$ justifies to recognize M as an external center of the gravitational potential $\phi = -GM/r$, and therefore, even that M interacts with the clock m, its rest-frame S remains inertial. Consequently, $(m^2 + p^2/c^2)^{1/2}$ denotes the absolute-relational inertia of m with regard to M, if p is the momentum of m in S. Therefore, the equality of heavy and inertial masses of m modifies $\mathcal{H}_0$ according to:

$$\begin{aligned}\mathcal{H} &= c(m^2c^2 + p^2)^{1/2} + \left(m^2 + \frac{p^2}{c^2}\right)^{1/2} \phi(P) \\ &= c(m^2c^2 + p^2)^{1/2}\left(1 + \frac{\phi(P)}{c^2}\right) = \left(1 + \frac{\phi(P)}{c^2}\right)\mathcal{H}_0 \,.\end{aligned} \tag{3.3}$$

In particular, for mass-less particles like photons, their Hamiltonian $\mathcal{H}^{\mathrm{ph}}$ takes (in the same S) the form:

$$\mathcal{H}^{\mathrm{ph}} = cp\left(1 + \frac{\phi(P)}{c^2}\right); \qquad p = |\boldsymbol{p}|\,. \tag{3.4}$$

Gravitation would manifest its universal nature by the fact that $\mathcal{H}$ is the product of $\mathcal{H}_0$ and the factor $(1 + \phi/c^2)$ responsible for the gravitational interaction. This implies the light-velocity dependence on ϕ, which clearly shows that the equality of inertial and heavy masses breaks the kinematic symmetry L.

Indeed, the Hamiltonian (3.3) determines the Lagrangean

$$\mathcal{L} = -mc^2 \left(\left(1 + \frac{\phi(P)}{c^2}\right)^2 - \frac{\dot{x}^2}{c^2} \right)^{1/2}, \tag{3.5}$$

where $\dot{x} = v$ is the velocity of m in S. Hence

$$C(P) = c\left(1 + \frac{\phi(P)}{c^2}\right) \tag{3.6}$$

denotes the upper value of the velocity of any mass-particle m (in S) which, of course, coincides with the velocity of a mass-less object, as $C(P) = \partial\mathcal{H}^{\mathrm{ph}}/\partial p$. We also see that our flat model results, similarly as GTR, in a Schwarzshild-like singularity on the sphere of the radius

$$r = r_0 = \frac{GM}{c^2} \tag{3.7}$$

(twice smaller than the Schwarzshild one r_S from (1.1)) where $C(P)$ vanishes; $C(r_0) = 0$.

In the NR limit $c \to \infty$ the energy–mass relation disappears. Hence $r_0 \to 0$ and after subtracting the term mc^2 from $\mathcal{H}$ we obtain the standard NR Hamiltonian $\mathcal{H}_{\mathrm{NR}}$ of m in the external gravitational field equal to:

$$\mathcal{H}_{\mathrm{NR}} = \lim_{c\to\infty}(\mathcal{H} - mc^2) = \frac{p^2}{2m} + m\phi(P). \tag{3.8}$$

The independence of the inertia of m of the velocity of m makes the gravitational interaction consistent with symmetry G of the Galilean spacetime G_4. Of course in this limit there is no room for "purely relativistic" objects like photons and for the R-S effect itself strictly connected with finite universal constant c.

4. Red-shift effect within flat model

An essential difference between the flat model and the GTR approaches to the R-S effect consists in that the former accounts for physical attributes of a real atomic-clock, in particular, for the time–energy complementarity of the clocks m revealed by photons, while the latter (GTR) ignores the reality of a clock regarding the background of curved spacetime as preceding any object embedded in this continuum hence subject to its symmetry.

Let us assume that all clocks m are at rest in the rest-frame S of the center M when, according to (3.3), the initial and final rest-energies $W_{i,f} = (E_{i,f})|_{\underline{v}=0}$ of m localized at P are equal to:

$$W_{i,f} = w_{i,f}\left(1 + \frac{\phi(P)}{c^2}\right) . \tag{4.1}$$

Taking into account the fact that macro-clocks m practically do not suffer from recoil when emitting–absorbing photons (Moesbauer effect), the energy conservation results, by virtue of (3.4), in the energy of photon (in S) emitted at macro-point P_1 which is its integral of motion, and hence

$$\begin{aligned} h\nu(P_1;P_1) = h\nu(P_2;P_1) &= c\,p(P_2;P_1)\left(1 + \frac{\phi(P_2)}{c^2}\right) \\ &= \Delta w\left(1 + \frac{\phi(P_1)}{c^2}\right) ; \quad \Delta w \equiv w_i - w_f . \end{aligned} \tag{4.2}$$

Flat model leaves no ambiguity as to the physical text of the space and time intervals and therefore we have omitted the subscripts indicating the measures in which the photon characteristics are determined. In particular, flat time makes the frequency of photon its integral of motion as stated in (4.2). Thus

$$\nu(P_2;P_1) = \nu(P_1;P_1) = \left(\frac{\Delta w}{h}\right)\left(1 + \frac{\phi(P_1)}{c^2}\right)$$

coinciding with (2.4) if

$$\nu_0 = \frac{\Delta w}{h} . \tag{4.3}$$

The P-independence of ν_0 results here from the P-independence of the internal energy w of the clock m. Whereas the P-dependence of $\nu(P;P)$ from (4.2) due to the dependence of W from (4.1) on $\phi(P)$ indicates that atomic--clocks move "bad" and their indications should be corrected by the factor $(1 + \phi(P)/c^2)$ which can be determined independently.

From (4.2) one immediately obtains the R-S effect measurable in a single space region P_2, as:

$$\frac{\nu(P_2;P_1)}{\nu(P_2;P_2)} = \frac{1 + \frac{\phi(P_1)}{c^2}}{1 + \frac{\phi(P_2)}{c^2}} \simeq 1 - \frac{\Delta\phi}{c^2} \tag{4.4}$$

which formally coincides with (2.5). Moreover, the connection of ν with the energy levels of m shows that the wavelength and momentum of photons are related by the de Broglie relation which proves that the flat model of the R-S effect remains consistent with quantum theory.

As a matter of fact, from the "wave" equality

$$\lambda(P_2; P_1) = \frac{C(P_2)}{\nu(P_2; P_1)} \tag{4.5}$$

and taking into account (3.6) and (4.2) we get

$$\lambda(P_2; P_1) = \left(\frac{hc}{\Delta w}\right) \frac{1 + \frac{\phi(P_2)}{c^2}}{1 + \frac{\phi(P_1)}{c^2}} . \tag{4.6}$$

However, from (4.2) we also obtain that

$$p(P_2; P_1) = \left(\frac{\Delta w}{c}\right) \frac{1 + \frac{\phi(P_1)}{c^2}}{1 + \frac{\phi(P_2)}{c^2}} , \tag{4.7}$$

thus (4.6) and (4.7) result in the de Broglie equation

$$\lambda(P_2; P_1) p(P_2; P_1) = h \tag{4.8}$$

which proves the universality of the Planck constant h in the world with gravitation.

Eq. (4.6) also shows that $\lambda(P; P)$, *i.e.* the wavelength of photon measured in the vicinity of its source m is independent of $\phi(P)$, as

$$\lambda(P; P) = \left(\frac{hc}{\Delta w}\right) \equiv \lambda_0 . \tag{4.9}$$

This stands in agreement with the absolute extension of the source-structure embedded in the flat space of our model, which together with the de Broglie relation (4.8) shows that the hypothesis of flat space and time remains consistent with quantum nonlocality [8]. The consistency of quantum mechanics with flat model of gravitation seems to be of fundamental importance, because nonlocal quantum mechanics is capable of reproducing extended, stable structures as its particular solutions which is required from a complete theory. Note that no classical theory ($h = 0$) — not the GTR in particular — can reproduce measuring rods and clocks from point constituents, which settles its incompleteness, *cf.* Section 5.

Let us emphasize that the arguments in favour of the flat model of gravity does not imply that the hypothesis of curved spacetime is "wrong".

What is more, two other "classical" GTR effects, the deflection of light and the advance of the perihelion of Mercury, distinguish between the GTR and the flat model predictions working in favour of the GTR. The flat model predicts the same effects (*cf.* Appendix) though twice smaller than the corresponding GTR ones. It is remarkable, as shown by Moeller [9], that in the r, t parametrization of ds^2 from (1.1), one half of these GTR effects is due to the curved space and the other half to the light–velocity dependence of ϕ. In this situation it is not the R-S effect, as originally maintained by Einstein, but the other two that would favour the GTR hypothesis.

5. Main assumptions of GTR

By a complete theory we mean here a theory whose laws result in solutions describing the structure of measuring rods and clocks which constitute the metrical space and time of measurement. Well aware that neither the STR nor GTR are complete, Einstein always pointed to the fundamental importance of the completeness of the theory. At the time when he put forward the STR he wrote: "... that at the present stage of the theory, measuring rods and clocks must be taken from experiment, as the theory is unable to reproduce them as its solutions." [10]. It is therefore interesting to confront Einstein's later opinion on this problem in his GTR. Einstein maintains that the following three assumptions constitute GTR [11]:

1° "Physical entities must be described by continuous functions spanned on four coordinates parametrizing spacetime recognized as the first (unanalyzable) physical continuum. As far as this parametrization preserves the compactness of the continuum its specification is irrelevant (gaussian coordinates).

2° Field quantities are represented by tensor components and among the tensors a symmetric one, g_{jk}, exists which determines ds^2 accounting for gravitational field.

3° Physical objects exist which in macrophysics measure the absolute four-interval ds^2."

As a matter of fact, the mathematical structure of the GTR is based on assumptions 1° and 2°, as maintained by Einstein. However, no theory exists consistent with these assumptions that would be complete. In a complete theory — Einstein continues — there would be no room for assumption 3°, as the metrical properties of rods and clocks would follow from such a theory. In our opinion, this imperfection accompanies any classical theory because of the locality of the relativistic spacetime continuum. On the other hand, since quantum theory based on flat spacetime enables one to reconstruct extended objects from (hypothetical) point-constituents, the metrical spacetime based on these objects makes the hypothesis of flat spacetime consistent.

Appendix

Perihelion Motion and Light Deflection

Let us start with the Lagrangean (3.5) ($c = 1$)

$$\mathcal{L} = -m\Big((1+\phi)^2 - \dot{\boldsymbol{x}}^2\Big)^{1/2}$$

which determines the corresponding Hamiltonian $\mathcal{H}$ of m equal to

$$\mathcal{H} = (1+\phi)\big(m^2 + p^2\big)^{1/2} = m(1+\phi)^2\big((1+\phi)^2 - \dot{\boldsymbol{x}}^2\big)^{-1/2} = W\,,$$

where W is the total energy of m in the gravitational field ϕ.

Since $\phi = -GM/r$, the angular-momentum conservation implies the (classical) motion of m in the plane perpendicular to the angular-momentum and hence, as usually, we introduce the polar coordinates r, φ on this plane. The two integrals of motion, the total energy W and the length l of the angular momentum where:

$$W = m(1+\phi)^2\big((1+\phi)^2 - \dot{r}^2 - r^2\dot{\varphi}^2\big)^{-1/2} \qquad (1.i)$$

$$l = \frac{\partial L}{\partial \dot{\varphi}} = mr^2\dot{\varphi}\big((1+\phi)^2 - \dot{r}^2 - r^2\dot{\varphi}^2\big)^{-1/2} \qquad (1.ii)$$

result in:

$$\dot{\varphi} = \frac{l}{W}\frac{(1+\phi)^2}{r^2}\,. \qquad (2)$$

This enables us to put the equation for $u(\varphi) \equiv 1/r(\varphi)$ and from equations (1.i) (or/and (1.ii)) one easily finds that:

$$u^2(\varphi) + u'^2(\varphi) = \left(\frac{W}{l}\right)^2 \big(1+\phi(u)\big)^{-2} - \left(\frac{m}{l}\right)^2 \qquad (3)$$

hence

$$u''(\varphi) + u(\varphi) = -\left(\frac{W}{l}\right)^2 \big(1+\phi(u)\big)^{-3}\frac{d\phi}{du}\,. \qquad (4)$$

By putting $\phi = -au$ ($a = GM$), Eqs (3) and (4) result in the perihelion motion ($m \neq 0$) and in the deflection of light-ray ($m = 0$) in the gravitational field of the center M.

a) *Perihelion motion*

Taking into account the fact that $d\phi/du = -a$ and restricting oneself to the first-order relativistic correction ($\sim 1/c^2$), we get $(1-au)^{-3} \simeq 1 + 3au$ and Eq. (4) then takes the form:

$$u''(\varphi) + \left(1 - 3a^2 \left(\frac{W}{l}\right)^2\right) u(\varphi) = a \left(\frac{W}{l}\right)^2 . \tag{5}$$

Without explicitly solving this equation we see that the bound motion of m when $W < m$ is non-periodic and the perihelion (aphelion) motion of the trajectory of m rotates in the same direction as the object m with the successive position of perihelion shifted by the angle:

$$\Delta\varphi = 3\pi a^2 \left(\frac{m}{l}\right)^2 . \tag{6}$$

If one assumes an almost circular motion with the radius R, then, within our approximation, $l = mRv$, $v = (GM/R)^{1/2}$ and finally

$$\Delta\varphi = 3\pi \frac{GM}{Rc^2} . \tag{7}$$

As already mentioned in the text, this effect is twice smaller than the corresponding one which follows from the GTR.

b) *Deflection of light-ray*

From the same Eqs (3) and (4) with $m = 0$ we obtain a trajectory of the light-ray which always represents an unbound curve. Assuming that b is the impact parameter of the light-ray with respect to the center M, which satisfies the strong inequality

$$b \gg a = \frac{GM}{c^2} \quad \text{where} \quad l = bW ,$$

in the same $1/c^2$-approximation the light-ray trajectory is given by one branch of the hyperbola, namely:

$$r = \frac{\frac{b^2}{a}}{1 + \left(\frac{b}{a}\right) \cos\varphi} . \tag{8}$$

Thus, $\cos\varphi = -(a/b)$ determines the direction of the asymptotes and, within the same $1/c^2$-approximation, we end up with the total deflection angle

$$\delta\varphi = 2\frac{GM}{bc^2} , \tag{9}$$

again twice smaller than that of the GTR.

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