A MEASUREMENT OF Λ_c^+ SPIN USING THE $\Lambda_c^+ \rightarrow pK^-\pi^+$ DECAY CHANNEL*

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(Received May 12, 1992)

We attempt to determine the spin of the charmed baryon Λ_c^+ investigating the angular distribution of the direction of the normal to the decay plane in the Jackson frame for the three-body weak decay $\Lambda_c^+ \rightarrow pK^-\pi^+$. The method is effective even for a small number of events. This is demonstrated for a sample of 121 $\Lambda_c^+ \rightarrow pK^-\pi^+$ decays from NA32 experiment. The results are entirely consistent with $J = 1/2$ assignment for the Λ_c^+. The spin formalism for a three-body weak decay of a baryon is extensively described.

PACS numbers: 13.30.–a

1. Introduction

Although the mass and lifetime of the Λ_c^+ [1] have been measured with high accuracy, its $J = 1/2$ spin assignment is still based only on the quark model. The experimental difficulty is connected with a lack of a clean and numerous sample of decays. All the previous investigations of the Λ_c^+ decay distributions were assuming spin $J = 1/2$. We mean here the CLEO [1] and ARGUS [2] studies of $\Lambda_c^+ \rightarrow \Lambda^0\pi^+$ decay and our earlier paper [3] which was using the $\Lambda_c^+ \rightarrow pK^-\pi^+$ channel. The advantage of the latter is the clean experimental signature and larger branching fraction. In this paper we continue the analysis of the same, practically background-free, sample of 121 $\Lambda_c^+ \rightarrow pK^-\pi^+$ events collected in the ACCMOR [4–7] experiment. This time we investigate various spin hypotheses for such a three-body weak

* Work partly supported by the KBN Grant No. 20-38-09101

** Alexander von Humboldt Foundation Fellow

[1] Throughout the paper a particle symbol stands for particle and antiparticle *i.e.* any reference to a specific state implies the charge-conjugate state as well.

decay of a fermion. Since the experiment and data handling were already described in our earlier paper [3], special attention is given here to the spin formalism for the rarely discussed three-body weak decay of a fermion.

The paper is organized as follows: In Section 2 we recall the main formulae dealing with the angular distribution of decay products. In Section 3 we apply the formulae to the above-mentioned experimental sample, while the conclusions are given in Section 4. Appendix A gives explicit expressions for the moments of the angular distribution in terms of the elements of the spin density matrices. Appendix B contains a detailed description of the full (*i.e.* including the azimuthal-angle dependence) angular distribution.

2. Angular distribution of decay products

For a two-body decay it is the direction of one of the secondary particles which determines the angular distribution. For three-body decays this role is played by the normal $\vec{n}$ to the decay plane. According to Berman and Jacob [8] the angular distribution of $\vec{n}$ in the rest frame of the decaying particle is given by the following formula[2]

$$I(\theta,\phi)=\frac{2J+1}{4\pi}\sum_{M,M'}\varrho^J_{MM'}\sum_{\mu}f^J_{\mu}\,D^{J*}_{M\mu}(\phi,\theta,0)D^J_{M'\mu}(\phi,\theta,0)\,, \qquad (1)$$

where $\varrho^J_{MM'}$ are the elements of the spin density matrix for the production of the particle of spin J and f^J_μ represent the phenomenological parameters related to its decay. It is convenient to decompose $I(\theta,\phi)$ in the basis of spherical harmonics

$$I(\Omega)=\sum_{l,m}\langle Y^{l*}_m\rangle\,Y^l_m(\Omega)\,, \qquad (2)$$

where Ω stands for θ, ϕ. Using the well-known identity

$$Y^l_m(\theta,\phi)=\sqrt{\frac{2l+1}{4\pi}}\,D^{l*}_{m0}(\phi,\theta,0)\,, \qquad (3)$$

as well as (see Chung [10])

$$\int dR\,D^{j_1}_{m_1M_1}\,D^{j_2}_{m_2M_2}\,D^{j_3*}_{m_3M_3}=\frac{8\pi}{2j_3+1} \times\langle j_1m_1;j_2m_2|j_3m_3\rangle\langle j_1M_1;j_2M_2|j_3M_3\rangle\,. \qquad (4)$$

[2] The angles θ and ϕ specify the direction of the analyser [9]. This may be either the normal to the decay plane or the direction of the decay product. We prefer to study here the direction of the normal because the acceptance depends only moderately on $\cos\theta_{GJ}$ and ϕ_{GJ}, while the acceptance dependence of other angular variables characterizing the final state is much stronger.

one can calculate the multipole moments $\langle Y^{lm} \rangle$. After integration we obtain

$$\langle Y^l_m \rangle = \sqrt{\frac{2l+1}{4\pi}} R_{lm} g_l \,, \tag{5}$$

where[3]

$$R_{lm} = \sum_{M,M'} \varrho^J_{MM'} \langle J, M; l, m | J, M' \rangle \tag{6}$$

describe the production, while

$$g_l = \sum_{\mu} f^J_{\mu} \langle J, \mu; l, 0 | J, \mu \rangle \tag{7}$$

are responsible for the decay and $\langle j_1, m_1; j_2, m_2 | j, m \rangle$ are Clebsch–Gordan coefficients.

Finally the distribution (1) can be written as:

$$I(\theta, \phi) = I_0 + I_1(\theta) + I_2(\theta, \phi) \,, \tag{8}$$

where $I_0 = 1/4\pi$ represents an isotropic term, while $I_1(\theta)$ is given by Legendre polynomials ($x \equiv \cos\theta$)

$$I_1(\theta) = \frac{1}{4\pi} \sum_{l=1}^{2J} d_l \; P_l(x) \,, \tag{9}$$

and

$$d_l = (2l+1) g_l R_{l0} \,. \tag{10}$$

The g_l and R_{l0} coefficients for spin $J = {}^1\!/_2$, ${}^3\!/_2$ and ${}^5\!/_2$ are given in Appendix A. The last term $I_2(\theta, \phi)$ is the sum of terms proportional to $\exp(\pm im\phi)$ ($m \geq 1$) and thus vanishes when the distribution (8) is integrated over the azimuthal angle ϕ. On the other hand integration over θ leads to the following ϕ-dependent distribution

$$I(\phi) = \frac{1}{2\pi} \left(1 + \sum_{m=1}^{2J} (S^+_m \cos m\phi + S^-_m \sin m\phi) \right) . \tag{11}$$

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[3] According to Kotański and Zalewski [11] as well as to Dąbkowski [9] the coefficients R_{lm} depend linearly on the statistical tensors of rank l in the decomposition of spin density matrix ϱ^J. When the reference frame is rotated the R_{lm} does not mix with $R_{l'm}$ ($l \neq l'$).

where $S_m^{\pm}$ coefficients are somehow analogous to d_l moments (Appendix B).

3. Experimental results

We use the Gottfried–Jackson reference system in which decaying Λ_c^+ is at rest. The OZ axis of the right-handed reference frame is along the beam direction. The OX axis points towards the Λ_c^+ momentum in the laboratory frame. The normal to the decay plane is given by

$$\vec{n} = \frac{\vec{Q} \times \vec{k}}{|\vec{Q} \times \vec{k}|}, \tag{12}$$

where $\vec{Q}$ and $\vec{k}$ are the momenta (in the Λ_c^+ rest frame) of the decay proton and kaon, respectively. Angles θ_{GJ} and ϕ_{GJ} are the polar and the azimuthal angle of $\vec{n}$, respectively.

In general there are $(2J+1)$ d_l moments for spin J. Fortunately, their number may be reduced by parity conservation in the production process. This yields the following identity (see Chung [10]) for for the density matrix in the Jackson frame:

$$\varrho_{mm'} = (-1)^{m-m'} \varrho_{-m,-m'} \,. \tag{13}$$

which leads to identical vanishing[4] of d_l moments for odd l. Thus $J > 1/2$ will result in a symmetric but anisotropic distribution.

We have tried to describe the experimental distribution of $\cos\theta_{\mathrm{GJ}}$ in our sample of 121 $\Lambda_c^+ \to \mathrm{pK}^-\pi^+$ events by $J = 1/2$, $J = 3/2$ and $J = 5/2$ hypotheses. The experimental acceptance, which depends on $\cos\theta_{\mathrm{GJ}}$, was folded in our fits. The results obtained with the help of the maximum-likelihood method are listed in Table I both for the whole sample and for the subsample of $p_T(\Lambda_c^+) > 0.7$ GeV/c. It should be recalled here (see Eqs (26), (27) in Appendix A) that the possible range of d_2 is (-1,1) for $J = 3/2$ and even more for both d_2 and d_4 in case of $J = 5/2$. The measured d_2 and d_4 moments are much smaller and essentially consistent with zero. Fig. 1 shows a comparison of the $J = 1/2$ hypothesis with the experimental distribution.

The distribution in the azimuthal angle ϕ_{GJ}, which depends on the off-diagonal elements of the spin density matrix $\varrho^J_{MM'}$ in the production and on g_l coefficients, is essentially flat both for the total sample (χ^2/DoF=7.3/9)

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[4] In our previous paper [3] we used the transversity frame for which one has to deal with all d_l moments since there is no relation between the diagonal elements of the spin density matrix.

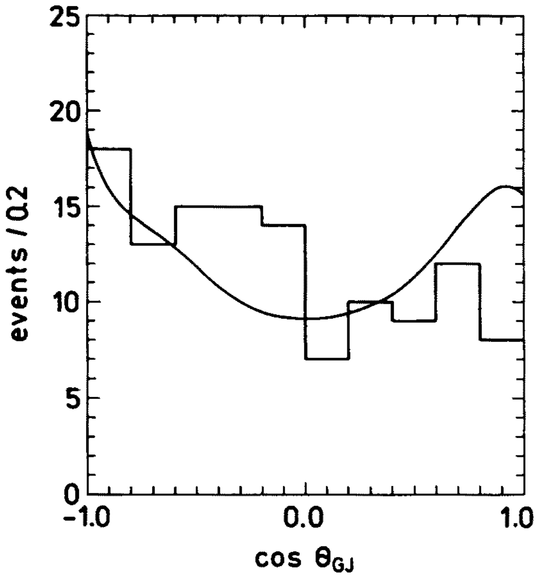

Fig. 1. The distribution of $\cos\theta_{GJ}$ for 121 decays $\Lambda_c^+ \to pK^-\pi^+$. The solid line represents the result of the fit to acceptance corrected distribution assuming the spin $^1/_2$ of Λ_c^+.

TABLE I

The results of maximum-likelihood fits for the d_l moments.

Spin	121 events	$\frac{\chi^2}{\text{DoF}}$	$p_T > 0.7$ GeV/c	$\frac{\chi^2}{\text{DoF}}$
$J = \frac{1}{2}$	$d_1 \equiv 0$	$\frac{11.2}{9}$	$d_1 \equiv 0$	$\frac{10.1}{9}$
$J = \frac{3}{2}$	$d_2 = -0.26 \pm 0.18$	$\frac{8.7}{8}$	$d_2 = 0.20 \pm 0.28$	$\frac{9.4}{8}$
$J = \frac{5}{2}$	$d_2 = -0.31 \pm 0.20$ $d_4 = 0.20 \pm 0.24$	$\frac{2.9}{7}$	$d_2 = 0.20 \pm 0.28$ $d_4 = -0.01 \pm 0.35$	$\frac{3.0}{7}$

and the subsample $p_T > 0.7$ GeV/c events (χ^2/DoF=6.6/9). These $S_m^\pm$ coefficients, which do not vanish identically[5] for $J = {}^1/_2$ or $J = {}^3/_2$, are shown to be consistent with zero in Table II.

The results for high p_T are shown here for the following reason. The

[5] Eq. (13) implies that S_m^+ vanish for odd m and S_m^- for even m (see Appendix B for the definition of $S_m^\pm$ coefficients).

integrated distribution $I(\theta)$ for spin $J \geq {}^3/_2$ and for vanishing higher ($l \geq 2$) d_l moments will be a linear function of $x = \cos\theta_{GJ}$. This may result from vanishing higher g_l or R_{l0} coefficients. Such a particular alignment of high spin ($J \geq {}^3/_2$) would imitate the $J = {}^1/_2$ case. In order to investigate this possibility we have also studied the subsample of high p_T events. As seen in Tables I, II the corresponding moments are also consistent with zero. This strengthens our conclusion on spin $J = {}^1/_2$ assignment for Λ_c^+.

TABLE II

The results of maximum-likelihood fits for the $S_m^\pm$ moments.

Spin Λ_c^+	121 events	$\frac{\chi^2}{\text{DoF}}$	$p_T > 0.7\,GeV/c$	$\frac{\chi^2}{\text{DoF}}$
$J = \frac{1}{2}$	$S_1^- = -0.06 \pm 0.13$	$\frac{7.3}{8}$	$S_1^- = -0.19 \pm 0.19$	$\frac{6.2}{6}$
$J = \frac{3}{2}$	$S_2^+ = 0.13 \pm 0.13$	$\frac{4.8}{5}$	$S_2^+ = 0.25 \pm 0.19$	$\frac{1.3}{5}$
	$S_1^- = -0.06 \pm 0.13$		$S_1^- = -0.19 \pm 0.19$	
	$S_3^- = -0.19 \pm 0.13$		$S_3^- = -0.06 \pm 0.19$	

4. Conclusions

The angular distribution of the normal to the decay plane in our sample of 121 $\Lambda_c^+ \to pK^-\pi^+$ decays is fully consistent with the spin $J = {}^1/_2$. All the higher moments are consistent with zero as shown in Tables I, II. This supports the quark model assignment for Λ_c^+. We demonstrate simultaneously that the decay channel in question is suitable for spin studies.

The authors are very grateful to the ACCMOR collaboration for a kind permission to use their data. Acceptance calculations done by Mr A. Bożek are highly appreciated. One of us (M. J.) thanks the University of Karlsruhe for hospitality and the other (K. R.) is grateful to the Werner Heisenberg Institute for support.

Appendix A

In this appendix we give expresions for the moments d_l of the angular distribution (8) in terms of the elements of spin density matrices ϱ^J_{MM} and f^J_μ for the weak decay of a baryon.

For J=1/2 we have $g_l = a_-/\sqrt{3}$ and $R_{l0} = p_-/\sqrt{3}$, where

$$a_- = \varrho^{\frac{1}{2}}_{\frac{1}{2},\frac{1}{2}} - \varrho^{\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2}} \tag{14}$$

$$p_- = f^{\frac{1}{2}}_{\frac{1}{2},\frac{1}{2}} - f^{\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2}} \,. \tag{15}$$

In the following we omit the J index in $\varrho^J_{MM'}$ and f^J_μ. Instead, we explicitly specify the value of J in the text. In addition we simplify the formulae introducing short notations

$$\left.\begin{aligned} a_\pm &= \varrho_{\frac{1}{2},\frac{1}{2}} \pm \varrho_{-\frac{1}{2},-\frac{1}{2}} \\ b_\pm &= \varrho_{\frac{3}{2},\frac{3}{2}} \pm \varrho_{-\frac{3}{2},-\frac{3}{2}} \end{aligned}\right\} \quad \text{for} \quad J = {}^3\!/_2,\ {}^3\!/_2 \tag{16}$$

and

$$c_\pm = \varrho_{\frac{5}{2},\frac{5}{2}} \pm \varrho_{-\frac{5}{2},-\frac{5}{2}} \qquad \text{for} \quad J = {}^5\!/_2 \tag{17}$$

Similarly for f^J_μ we denote

$$\left.\begin{aligned} p_\pm &= f_{\frac{1}{2}} \pm f_{-\frac{1}{2}} \\ q_\pm &= f_{\frac{3}{2}} \pm f_{-\frac{3}{2}} \end{aligned}\right\} \quad \text{for} \quad J = {}^3\!/_2,\ {}^3\!/_2 \tag{18}$$

and

$$r_\pm = f_{\frac{5}{2}} \pm f_{-\frac{5}{2}} \qquad \text{for} \quad J = {}^5\!/_2 \,. \tag{19}$$

The Clebsch–Gordan coefficients as well as the coefficients g_l, R_{l0} and $R^\pm_{lm}$ (see Appendix B) for $J = {}^3\!/_2$ and ${}^5\!/_2$ have been calculated algebraically. This has been done using the most effective algorithm based on the formula given in Ref. [12]

$$\begin{aligned} \langle j_1, m_1; j_2, m_2 | j, m\rangle &= \frac{\Delta(j_1, j_2, j)\, \delta^m_{m_1+m_2}}{(j + j_1 - j_2)!(j + j_2 - j_1)!} \\ &\left(\frac{(j_1 - m_1)!(j_2 - m_2)!(j - m)!(j + m)!(2j + 1)}{(j_1 + m_1)!(j_2 + m_2)!}\right)^{\frac{1}{2}} \\ &\sum_z (-1)^{j_1 - m_1 + z} \frac{(j + j_2 - m_1 - z)!(j_1 + m_1 + z)!}{z!(j_1 - m_1 - z)!(j - m - z)!(j_2 - j + m_1 + z)!} \,. \end{aligned} \tag{20}$$

The summation goes over all z values for which the above formula makes sense. δ^i_j is the Kronecker delta function. The function

$$\Delta(a, b, c) = \left(\frac{(a + b - c)!(a - b + c)!(-a + b + c)!}{(a + b + c + 1)!}\right)^{\frac{1}{2}} \tag{21}$$

is nonvanishing only for $a, b, c \geq 0$ and $|a-b| \leq c \leq a+b$.

For $J = {}^3/_2$ the coefficients R_l describing the production are as follows

$$\begin{aligned} R_1 &= \frac{1}{\sqrt{15}}(a_- + 3b_-)\,, \\ R_2 &= \frac{1}{\sqrt{5}}(1 - 2a_+)\,, \\ R_3 &= \frac{1}{\sqrt{35}}(b_- - 3a_-)\,. \end{aligned} \tag{22}$$

Also for $J = {}^3/_2$ the g_l factors can be written as

$$\begin{aligned} g_1 &= \frac{1}{\sqrt{15}}(p_- + 3q_-)\,, \\ g_2 &= \frac{1}{\sqrt{5}}(1 - 2p_+)\,, \\ g_3 &= \frac{1}{\sqrt{35}}(q_- - 3p_-)\,. \end{aligned} \tag{23}$$

For $J = {}^5/_2$ there are five R_l:

$$\begin{aligned} R_1 &= \frac{1}{\sqrt{35}}(a_- + 3b_- + 5c_-)\,, \\ R_2 &= -\frac{1}{\sqrt{70}}(9a_+ + 6b_+ - 5)\,, \\ R_3 &= -\frac{1}{\sqrt{210}}(2a_- + 7b_- - 5c_-)\,, \\ R_4 &= \frac{1}{\sqrt{42}}(a_+ - 4b_+ + 1)\,, \\ R_5 &= \frac{1}{\sqrt{462}}(10a_- - 5b_- + c_-)\,, \end{aligned} \tag{24}$$

and five g_l:

$$\begin{aligned} g_1 &= \frac{1}{\sqrt{35}}(p_- + 3q_- + 5r_-)\,, \\ g_2 &= -\frac{1}{\sqrt{70}}(9p_+ + 6q_+ - 5)\,, \\ g_3 &= -\frac{1}{\sqrt{210}}(2p_- + 7q_- - 5r_-)\,, \\ g_4 &= \frac{1}{\sqrt{42}}(p_+ - 4q_+ + 1)\,, \\ g_5 &= \frac{1}{\sqrt{462}}(10p_- - 5q_- + r_-)\,. \end{aligned} \tag{25}$$

Varying $\varrho^J_{MM'}$ and f^J_μ we determine the range of possible d_l values. For $J = {}^3/_2$ this gives

$$\begin{aligned} |d_1| &\leq \tfrac{9}{5}\,, \\ |d_2| &\leq 1\,, \\ |d_3| &\leq \tfrac{9}{5}\,, \end{aligned} \tag{26}$$

while for $J = {}^5/_2$:

$$\begin{aligned} |d_1| &\leq \tfrac{15}{7}\,, \\ d_2 &\in \left[-\tfrac{10}{7}, \tfrac{25}{14}\right]\,, \\ |d_3| &\leq \tfrac{49}{30}\,, \\ d_4 &\in \left[-\tfrac{9}{7}, \tfrac{27}{14}\right]\,, \\ |d_5| &\leq \tfrac{50}{21}\,. \end{aligned} \tag{27}$$

Appendix B

In this appendix we describe the I_2 term in the full expression of Eq. (8) for the angular distribution.

This term, which contains the dependence on both angles, can be written as follows:

$$I_2(\theta, \phi) = \frac{1}{4\pi} \sum_{l=1}^{2J} (2l+1) g_l \sum_{m=-l}^{l} R_{lm} d^l_{m0}(\theta)\, e^{im\phi}\,, \tag{28}$$

where the second sum does not include the term with $m = 0$. This part of the angular distribution depends on the off-diagonal elements of the spin density matrix. We set (for $j > k$)

$$\begin{aligned} \varrho_{jk} &= a_{jk} + ib_{jk}\,, \\ \varrho_{kj} &= a_{jk} - ib_{jk}\,. \end{aligned} \tag{29}$$

Due to the hermicity of ϱ there are $J(2J+1)$ real a_{jk} and the same number of real b_{jk}. We decompose I_2 in terms of real orthonormal functions of the angle ϕ. With the help of:

$$d^l_{m0}(\theta) = \sqrt{\frac{(l-m)!}{(l+m)!}}\, P^m_l(\cos\theta) \tag{30}$$

(for $m \geq 0$) and

$$d^l_{m0} = (-1)^m d^l_{-m0}\,, \tag{31}$$

we get

$$I_2(\theta,\phi) = \frac{1}{4\pi}\sum_{l=1}^{2J}(2l+1)g_l \times \sum_{m=1}^{l}\sqrt{\frac{(l-m)!}{(l+m)!}}(R^+_{lm}\cos\phi + R^-_{lm}\sin\phi)P^m_l(x)\,. \tag{32}$$

Coefficients $R^\pm_{lm}$ (for $m \geq 1$) are chosen in the form

$$R^+_{lm} = R_{lm} + (-1)^m R_{l,-m} \tag{33}$$

and

$$R^-_{lm} = i(R_{lm} - (-1)^m R_{l,-m})\,. \tag{34}$$

At such a choice R^+_{lm} depend only on a_{ij}, and R^-_{lm} on b_{ij}. Also R^-_{lm} may be calculated from R^+_{lm} replacing any a_{ij} by the corresponding b_{ij}

$$R^-_{lm} = R^+_{lm}(a_{ij} \to b_{ij})\,. \tag{35}$$

Indeed, in the Eq. (6) for $m \geq 1$ the only non-zero Clebsch–Gordan coefficients are those with $M' > M$, so the sum in (6) contains only the elements $\varrho_{MM'}$ with minus sign in (29). For $m \leq -1$ there are only terms with $M' < M$ *i.e.* those with positive sign in (29). Considering the relation given by Racah [13], namely

$$\langle j_1, m_1; j_2, m_2 | j, m\rangle = (-1)^{j_1 - j + m_2}\sqrt{\frac{2j+1}{2j_1+1}}\langle j, m; j_2, -m_2 | j_1, m_1\rangle \tag{36}$$

we see that adding (subtracting) R_{lm} and $R_{l,-m}$ we get a term dependent on matrix elements a_{jk} ($i\,b_{jk}$) only.

For $J = {}^1\!/_2$ there is only one coefficient R^+_{lm}

$$R^+_{11} = -\frac{4}{\sqrt{6}}a_{\frac{1}{2},-\frac{1}{2}}\,, \tag{37}$$

and one R^-_{lm}

$$R^-_{11} = -\frac{4}{\sqrt{6}}b_{\frac{1}{2},-\frac{1}{2}}\,. \tag{38}$$

For $J = {}^3\!/_2$ there are six R^+_{lm}, namely one for $l = 1$

$$R^+_{11} = -\frac{4}{\sqrt{10}}(a_{\frac{3}{2},\frac{1}{2}} + \frac{2\sqrt{3}}{3}a_{\frac{1}{2},-\frac{1}{2}} + a_{-\frac{1}{2},-\frac{3}{2}}), \qquad (39)$$

two for $l = 2$

$$R^+_{21} = -\frac{4}{\sqrt{10}}(a_{\frac{3}{2},\frac{1}{2}} - a_{-\frac{1}{2},-\frac{3}{2}}),$$
$$R^+_{22} = \frac{4}{\sqrt{10}}(a_{\frac{3}{2},-\frac{1}{2}} + a_{\frac{1}{2},-\frac{3}{2}}), \qquad (40)$$

and three for $l = 3$

$$R^+_{31} = -\frac{4}{\sqrt{35}}(a_{\frac{3}{2},\frac{1}{2}} - \sqrt{3}a_{\frac{1}{2},-\frac{1}{2}} + a_{-\frac{1}{2},-\frac{3}{2}}),$$
$$R^+_{32} = \frac{4}{\sqrt{14}}(a_{\frac{3}{2},-\frac{1}{2}} - a_{\frac{1}{2},-\frac{3}{2}}),$$
$$R^+_{33} = -\frac{4}{\sqrt{7}}a_{\frac{3}{2},-\frac{3}{2}} . \qquad (41)$$

For $J = {}^5\!/_2$ the $R^\pm_{lm}$ coefficients are as follows:
for $l = 1$

$$R^+_{11} = -\frac{4}{\sqrt{70}}\left(\sqrt{5}a_{\frac{5}{2},\frac{3}{2}} + 2\sqrt{2}a_{\frac{3}{2},\frac{1}{2}} + 3a_{\frac{1}{2},-\frac{1}{2}} + 2\sqrt{2}a_{-\frac{1}{2},-\frac{3}{2}} + \sqrt{5}a_{-\frac{3}{2},-\frac{5}{2}}\right), \qquad (42)$$

for $l = 2$

$$R^+_{21} = -\frac{6}{\sqrt{210}}\left(\sqrt{10}a_{\frac{5}{2},\frac{3}{2}} + 2a_{\frac{3}{2},\frac{1}{2}} - 2a_{-\frac{1}{2},-\frac{3}{2}} - \sqrt{10}a_{-\frac{3}{2},-\frac{5}{2}}\right), \qquad (43)$$

$$R^+_{22} = 2\sqrt{\frac{3}{14}}\left(a_{\frac{5}{2},\frac{1}{2}} + \frac{3\sqrt{5}}{5}a_{\frac{3}{2},-\frac{1}{2}} + \frac{3\sqrt{5}}{5}a_{\frac{1}{2},-\frac{3}{2}} + a_{-\frac{1}{2},-\frac{5}{2}}\right), \qquad (44)$$

for $l = 3$

$$R^+_{31} = -\frac{2}{\sqrt{7}}\left(\sqrt{2}a_{\frac{5}{2},\frac{3}{2}} - \frac{\sqrt{5}}{5}a_{\frac{3}{2},\frac{1}{2}} - \frac{2\sqrt{10}}{5}a_{\frac{1}{2},-\frac{1}{2}} - \frac{\sqrt{5}}{5}a_{-\frac{1}{2},-\frac{3}{2}} + \sqrt{2}a_{-\frac{3}{2},-\frac{5}{2}}\right), \qquad (45)$$

$$R^+_{32} = \frac{2}{\sqrt{14}}\left(\sqrt{5}a_{\frac{5}{2},\frac{1}{2}} + a_{\frac{3}{2},-\frac{1}{2}} - a_{\frac{1}{2},-\frac{3}{2}} - \sqrt{5}a_{-\frac{1}{2},-\frac{5}{2}}\right), \qquad (46)$$

$$R^+_{33} = -\frac{2}{\sqrt{21}}\left(\sqrt{5}a_{\frac{5}{2},-\frac{1}{2}} + 2\sqrt{2}a_{\frac{3}{2},-\frac{3}{2}} + \sqrt{5}a_{\frac{1}{2},-\frac{5}{2}}\right), \qquad (47)$$

for $l = 4$

$$R^+_{41} = -\frac{2}{\sqrt{21}}\left(\sqrt{2}a_{\frac{5}{2},\frac{3}{2}} - \sqrt{5}a_{\frac{3}{2},\frac{1}{2}} + \sqrt{5}a_{-\frac{1}{2},-\frac{3}{2}} - \sqrt{2}a_{-\frac{3}{2},-\frac{5}{2}}\right), \quad (48)$$

$$R^+_{42} = \frac{2}{\sqrt{42}}\left(3a_{\frac{5}{2},\frac{1}{2}} - \sqrt{5}a_{\frac{3}{2},-\frac{1}{2}} - \sqrt{5}a_{\frac{1}{2},-\frac{3}{2}} + 3a_{-\frac{1}{2},-\frac{5}{2}}\right), \quad (49)$$

$$R^+_{43} = -\frac{2}{\sqrt{3}}\left(a_{\frac{5}{2},-\frac{1}{2}} - a_{\frac{1}{2},-\frac{5}{2}}\right), \quad (50)$$

$$R^+_{44} = \frac{2}{\sqrt{3}}\left(a_{\frac{5}{2},-\frac{3}{2}} + a_{\frac{3}{2},-\frac{5}{2}}\right), \quad (51)$$

and for $l = 5$

$$R^+_{51} = -\frac{2}{\sqrt{77}}\left(a_{\frac{5}{2},\frac{3}{2}} - \sqrt{10}a_{\frac{3}{2},\frac{1}{2}} + \sqrt{5}a_{\frac{1}{2},-\frac{1}{2}} - \sqrt{10}a_{-\frac{1}{2},-\frac{3}{2}} + a_{-\frac{3}{2},-\frac{5}{2}}\right), \quad (52)$$

$$R^+_{52} = \frac{2}{\sqrt{22}}\left(a_{\frac{5}{2},\frac{1}{2}} - \sqrt{5}a_{\frac{3}{2},-\frac{1}{2}} + \sqrt{5}a_{\frac{1}{2},-\frac{3}{2}} - a_{-\frac{1}{2},-\frac{5}{2}}\right), \quad (53)$$

$$R^+_{53} = -\frac{2}{\sqrt{33}}\left(2a_{\frac{5}{2},-\frac{1}{2}} - \sqrt{10}a_{\frac{3}{2},-\frac{3}{2}} + 2a_{\frac{1}{2},-\frac{5}{2}}\right), \quad (54)$$

$$R^+_{54} = \frac{6}{\sqrt{33}}\left(a_{\frac{5}{2},-\frac{3}{2}} - a_{\frac{3}{2},-\frac{5}{2}}\right), \quad (55)$$

$$R^+_{55} = -\frac{12}{\sqrt{66}}a_{\frac{5}{2},-\frac{5}{2}}. \quad (56)$$

Again R^-_{lm} can be obtained via replacing b_{ik} by a_{ik} in Eq. (35).

After integration of the distribution (8) over $\cos\theta$ we deal with terms coming from I_0 and I_2 only. Adding terms with the same m we get an integrated distribution as a function of the ϕ variable

$$I(\phi) = \frac{1}{2\pi}\left(1 + \sum_{m=1}^{2J}\left(S^+_m \cos m\phi + S^-_m \sin m\phi\right)\right), \quad (57)$$

where (again for $m \geq 1$)

$$S^\pm_m = \sum_{l=m}^{2J}(2l+1)\, g_l \sqrt{\frac{(l-m)!}{(l+m)!}} C^m_l R^\pm_{lm}. \quad (58)$$

C_l^m can be calculated by integrating the associated function of Legendre[6]

$$C_l^m = \int_{-1}^{1} P_l^m(\cos\theta)\, d\cos\theta\,. \tag{59}$$

The S_m^+ and S_m^- coefficients obey a relation analogous to Eq. (35)

$$S_m^- = S_m^+(a_{ij} \to b_{ij})\,. \tag{60}$$

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[6] Integrals of P_l^m vanish for odd $(l+m)$. The remaining ones (with the convention for P_l^m such that $P_1^1 = +\sqrt{1-x^2}$) are $C_1^1 = \pi/2$, $C_2^2 = 4$, $C_3^1 = 3\pi/16$ and $C_3^3 = 45\pi/8$.