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HIGGS MECHANISM AS THE MANIFESTATION OF AN INTRINSIC COUPLING OF "VISIBLE" AND "HIDDEN" DEGREES OF FREEDOM *

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First, we summarize the recently proposed idea of leptons and quarks composed of algebraic degrees of freedom (defined by Clifford algebras) that justifies the existence of three and only three replicas of the fundamental fermions (as *e.g.* e^-, μ^-, τ^-). Then, we find within this algebraic scheme a possible place for Higgs scalar and pseudoscalar, both appearing in two replicas.

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It was suggested recently [1] that the physical difference between fundamental fermions of equal standard-model signature, belonging to three experimentally observed families, may be caused by some "hidden" degrees of freedom involved presumably in the algebraic structure of these fermions. The argument went as follows.

The Dirac algebra

$$\{\Gamma^\mu, \Gamma^\nu\} = 2g^{\mu\nu}\,, \tag{1}$$

determining intrinsic properties of the Dirac equation

$$[\Gamma \cdot (p - gA) - M]\,\psi = 0 \tag{2}$$

in the spacetime, admits the sequence $N = 1, 2, 3, \ldots$ of remarkable representations

$$\Gamma^\mu = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \gamma_i^\mu\,, \tag{3}$$

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where the matrices γ_i^μ , $i = 1, 2, \ldots, N$, are defined by the sequence $N = 1, 2, 3, \ldots$ of Clifford algebras

$$\{\gamma_i^\mu, \gamma_j^\nu\} = 2\delta_{ij} g^{\mu\nu} . \tag{4}$$

As can be readily seen, the representations (3) are reducible except for $N = 1$. It is so, because for any $N > 1$ one can introduce, beside $\Gamma_1^\mu \equiv \Gamma^\mu$ given in Eq. (3), $N - 1$ other Jacobi-type independent combinations $\Gamma_2^\mu, \ldots, \Gamma_N^\mu$ of γ_i^μ, *viz.*

$$\Gamma_2^\mu = \frac{1}{\sqrt{3}} \left(\gamma_1^\mu - \gamma_2^\mu\right) , \Gamma_3^\mu = \frac{1}{\sqrt{6}} \left(\gamma_1^\mu + \gamma_2^\mu - 2\gamma_3^\mu\right) , \ldots , \tag{5}$$

such that

$$\{\Gamma_i^\mu, \Gamma_j^\nu\} = 2\delta_{ij} g^{\mu\nu} . \tag{6}$$

Hence, the representations (3) may be realized in the form

$$\Gamma^\mu \equiv \Gamma_1^\mu = \gamma^\mu \otimes \underbrace{\mathbf{1} \otimes \cdots \otimes \mathbf{1}}_{(N-1) times} . \tag{7}$$

In particular, for $N = 3$ one may write

$$\Gamma_1^\mu = \gamma^\mu \otimes \mathbf{1} \otimes \mathbf{1} \, , \; \Gamma_2^\mu = \gamma^5 \otimes i\gamma^5\gamma^\mu \otimes \mathbf{1} \, , \; \Gamma_3^\mu = \gamma^5 \otimes \gamma^5 \otimes \gamma^\mu . \tag{8}$$

In Eqs. (7) and (8), $\gamma^\mu, \mathbf{1}, \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ are the usual 4×4 Dirac matrices.

With the use of representations (7) the Dirac-type equation (2) for any N can be rewritten as

$$[\gamma \cdot (p - g\, A) - M]_{\alpha_1\beta_1} \psi_{\beta_1\alpha_2\ldots\alpha_N} = 0 \tag{9}$$

if the mass term M does not depend on the matrices Γ_i^μ (though it may depend on N). Here, the wave function (or dynamical field) $\psi = (\psi_{\alpha_1\alpha_2\ldots\alpha_N})$ carries N Dirac bispinor indices $\alpha_i = 1, 2, 3, 4, i = 1, 2, \ldots, N$, of which only the first one is acted on by the Dirac matrices γ^μ and so is coupled to the particle's momentum and to the standard-model gauge fields symbolized by A_μ (among others, to the electromagnetic field). The rest of them are free. Thus, only α_1 may be "visible" in the magnetic field, while $\alpha_2, \ldots, \alpha_N$ are "hidden" in this field (as well as in any other standard-model gauge field). In consequence, a particle described by Eq. (2) or (9) may display in the magnetic field only a "visible" spin $\frac{1}{2}$ though it possesses also $N-1$ "hidden" spins $\frac{1}{2}$. Note that for $N = 1$ Eq. (2) or (9) is the usual Dirac equation, while for $N = 2$ it presents the Dirac form [2] of the Kähler equation [3]. For $N = 3, 4, 5, \ldots$ it gives new Dirac-type equations.

At this point we made in our argument two crucial assumptions:

(i) the physical Lorentz group of the theory of relativity, if applied to Eq. (2) or (9) for any $N > 1$, is generated both by the particle's visible and hidden degrees of freedom, and

(ii) the particle's hidden degrees of freedom (corresponding to $\alpha_2, \ldots, \alpha_N$) are physically undistinguishable and so obey the Fermi statistics along with the Pauli exclusion principle, what implies for any $N > 0$ the antisymmetry of the wave function (or dynamical field) $\psi_{\alpha_1 \alpha_2 \ldots \alpha_N}$ with respect to the hidden bispinor indices $\alpha_2, \ldots, \alpha_N$.

We inferred in Ref. [1] that the assumption *(i)*, if combined with the probability interpretation of $\psi = (\psi_{\alpha_1 \alpha_2 \ldots \alpha_N})$ treated as a wave function, restricts N to its odd values $N = 1, 3, 5, \ldots$, since the Dirac-type equation (2) implies the fully relativistic current- conservation equation

$$\partial_\mu \left(\eta_N \psi^+ \Gamma_1^0 \Gamma_2^0 \ldots \Gamma_N^0 \Gamma_1^\mu \psi \right) = 0 \tag{10}$$

only for N odd. Here, η_N is a phase factor making the matrix $\eta_N \Gamma_1^0 \Gamma_2^0 \ldots \Gamma_N^0$ Hermitian. Then, the assumption *(ii)* restricts N further, to its three values $N = 1, 3, 5$, due to our exclusion principle. Moreover, an interplay of the assumptions *(i)* and *(ii)* with the probability interpretation of $\psi = (\psi_{\alpha_1 \alpha_2 \ldots \alpha_N})$ treated as a wave function causes that — not solely in the case of $N = 1$, but also in the cases of $N = 3$ and $N = 5$ — there exists one and only one Dirac particle of a given standard-model signature, satisfying the Dirac-type equation (2) or (9) (in the cases of $N = 3$ and $N = 5$ the hidden spins $\frac{1}{2}$ sum up to zero). Thus, our assumptions *(i)* and *(ii)* applied to the Dirac-type wave equation (2) or (9) lead to the existence of three and only three versions (replicas) of the Dirac particle with a given color and (weak) flavor. It was tempting for us in Ref. [1] to interpret these versions as three experimentally observed replicas of leptons and quarks, responsible for the phenomenon of three fundamental-fermion families.

Now, let us note that the Dirac-type equation (2) implies the second-order equation of the form

$$\left\{ (p - gA)^2 - M^2 - i\, g \tfrac{1}{4} \left[\Gamma_1^\mu, \Gamma_1^\nu \right] F_{\mu\nu} \right\} \psi = 0\,, \tag{11}$$

where $\Gamma_1^\mu \equiv \Gamma^\mu$ and $F_\mu \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + ig\,[A_\mu\,,\, A_\nu]$ (the inverse implication is not true). Hence, the following fully relativistic current-conservation equation holds:

$$\partial_\mu \left[\eta_N \psi^+ \Gamma_1^0 \Gamma_2^0 \ldots \Gamma_N^0 \left(i \overleftrightarrow{\partial^\mu} - gA^\mu \right) \psi \right] = 0\,, \tag{12}$$

where $\overleftrightarrow{\partial^\mu} = \frac{1}{2} (\partial^\mu - \overleftarrow{\partial^\mu})$, if A_μ are independent of $\Gamma_1^5 = i\Gamma_1^0 \Gamma_1^1 \Gamma_1^2 \Gamma_1^3$, what may be possible only for N even. However, the current in Eq. (12) cannot be satisfactorily interpreted as the probability current, in contrast to the current in Eq. (10) working for N odd. The conclusion we can draw from this observation is that for N even the function $\psi = (\psi_{\alpha_1\alpha_2\ldots\alpha_N})$ escapes from the probability interpretation of a wave function (so, instead, it may be considered as a dynamical field in the spacetime).

Evidently, for N even our exclusion principle restricts N to its two values $N = 2, 4$ corresponding to

$$\psi_{\alpha_1\alpha_2} \equiv \psi^{(2)}_{\alpha_1\alpha_2}, \qquad \psi_{\alpha_1\alpha_2\alpha_3\alpha_4} \equiv \varepsilon_{\beta_2\alpha_2\alpha_3\alpha_4}\psi^{(4)}_{\alpha_1\beta_2}. \tag{13}$$

This implies the existence of two and only two versions (replicas) of 16 spin-(0/1) bosons composed of our algebraic degrees of freedom. We can see from Eq. (13) that in the case of $N = 4$ there are $3! = 6$ equivalent nonzero components (carrying two bispinor indices α_1 and β_2), all equal up to the sign to $\psi^{(4)}_{\alpha_1\beta_2}$.

Formally, both fields $\psi^{(2)}_{\alpha_1\alpha_2}$ and $\psi^{(4)}_{\alpha_1\alpha_2}$ can be expanded into 16 spin-(0/1) fields ,

$$\begin{aligned}\psi^{(2,4)}_{\alpha_1\alpha_2} &= \delta_{\alpha_1\alpha_2}S^{(2,4)} + \gamma^5_{\alpha_1\alpha_2}P^{(2,4)} + \gamma^\mu_{\alpha_1\alpha_2}V^{(2,4)}_\mu \\ &\quad + \left(\gamma^\mu\gamma^5\right)_{\alpha_1\alpha_2}A^{(2,4)}_\mu + i\tfrac{1}{4}\left[\gamma^\mu\gamma^\nu\right]_{\alpha_1\alpha_2}T^{(2,4)}_{\mu\nu},\end{aligned} \tag{14}$$

all having the same standard-model signatures, analogical to those of leptons and quarks. Of these fields, only the color- and charge-neutral components of the scalars $S^{(2,4)}$ and pseudoscalars $P^{(2,4)}$ could develop nonzero vacuum expectation values,

$$\langle\psi^{(2,4)}_{\alpha_1\alpha_2}\rangle = \left(\delta_{\alpha_1\alpha_2}\langle S^{(2,4)00}\rangle + \gamma^5_{\alpha_1\alpha_2}\langle P^{(2,4)00}\rangle\right)(\text{projector on } 00). \tag{15}$$

Thus, only $S^{2,4)00}$ and $P^{(2,4)00}$ might play the role of charge-neutral components of the Higgs doublets

$$\begin{pmatrix} S^{(2,4)0+} \\ S^{(2,4)00}\end{pmatrix}, \qquad \begin{pmatrix} P^{(2,4)0+} \\ P^{(2,4)00}\end{pmatrix}. \tag{16}$$

They would break spontaneously the electroweak symmetry. Of course, in our algebraic scheme there are also other spin-(0/1) fields involved in Eq. (14), but they do not break spontaneously the standard-model symmetry.

Let us tentatively assume that the boson fields (13) couple to pairs of the fundamental-fermion fields of three families, $\bar\psi^{(1)}_{\alpha_1}\psi^{(1)}_{\alpha_2}$, $\bar\psi^{(3)}_{\alpha_1}\psi^{(3)}_{\alpha_2}$, $\bar\psi^{(5)}_{\alpha_1}\psi^{(5)}_{\alpha_2}$, through the simple fully relativistic combination

$$\psi_{\alpha_1\alpha_2} + \lambda\varepsilon_{\alpha_2\beta_2\beta_3\beta_4}\psi_{\alpha_1\beta_2\beta_3\beta_4} \equiv \psi^{(2)}_{\alpha_1\alpha_2} + 6\lambda\psi^{(4)}_{\alpha_1\alpha_2} \tag{17}$$

with $\psi^{(2,4)}_{\alpha_1\alpha_2}$ expanded as in Eq. (14) (as argued in Ref. [1], the fundamental-fermion fields should form in their Yukawa coupling the combination $\bar{\psi}^{(1)}_{\alpha_1}\psi^{(1)}_{\alpha_2} + 4\bar{\psi}^{(3)}_{\alpha_1}\psi^{(3)}_{\alpha_2} + 24\bar{\psi}^{(5)}_{\alpha_1}\psi^{(5)}_{\alpha_2}$). Then, we may expect the following Higgs coupling of the doublets (16) to, say, the leptons ν_e , e^- :

$$h(\overline{\nu_{eL}}\,,\overline{e^-_L})\begin{pmatrix} S^{(2)0+}+\gamma^5P^{(2)0+}+6\lambda S^{(4)0+}+6\lambda\gamma^5P^{(4)0+} \\ S^{(2)00}+\gamma^5P^{(2)00}+6\lambda S^{(4)00}+6\lambda\gamma^5P^{(4)00}\end{pmatrix} e^-_R + \text{h.c.}$$

$$= h(\overline{\nu_{eL}}\,,\overline{e^-_L})\begin{pmatrix} S^{(2)0+}+P^{(2)0+}+6\,\lambda S^{(4)0+}+6\lambda P^{(4)0+} \\ S^{(2)00}+P^{(2)00}+6\,\lambda S^{(4)00}+6\,\lambda\, P^{(4)00}\end{pmatrix} e^-_R + \text{h.c.}\,, \tag{18}$$

where $l_{R,L} = \frac{1}{2}(1 \pm \gamma^5)l$.

Concluding, in the case of Higgs bosons as given in Eq. (16), an intrinsic coupling of visible spin $\frac{1}{2}$ and hidden spin $\frac{1}{2}$ is responsible for the Higgs mechanism, since such a coupling makes possible the formation of 16 spin-(0/1) fields (among them, S and P) involved in Eq. (14).

In contrast, for N odd, when our exclusion principle restricts N to its three values $N = 1, 3, 5$ corresponding to three fundamental-fermion fields $\psi^{(1)}_{\alpha_1}$, $\psi^{(3)}_{\alpha_1}$, $\psi^{(5)}_{\alpha_1}$ [1], no intrinsic coupling of visible and hidden spins $\frac{1}{2}$ manifests itself.

Eventually, it may be interesting to rewrite in the cases of $N = 2$ and $N = 4$ the second-order equation (11) in terms of 16 spin-(0/1) fields appearing in the expansion (14). If for N even there is no Γ^5_1-dependence of A_μ, the result is

$$\begin{aligned}
\left[(p-gA)^2 - M^2_{2,4}\right] S^{(2,4)} &= \tfrac{1}{2} g F^{\mu\nu} T^{(2,4)}_{\mu\nu}\,, \\
\left[(p-gA)^2 - M^2_{2,4}\right] P^{(2,4)} &= -\,\tfrac{1}{2} ig\, \widetilde{F}^{\mu\nu} T^{(2,4)}_{\mu\nu}\,, \\
\left[(p-gA)^2 - M^2_{2,4}\right] T^{(2,4)}_{\mu\nu} &= g\Big[F_{\mu\nu}S^{(2,4)} - i\widetilde{F}_{\mu\nu}P^{(2,4)} \\
&\qquad + i\left(F_\mu{}^\rho T^{(2,4)}_{\rho\nu} - F^\rho{}_\nu T^{(2,4)}_{\mu\rho}\right)\Big]\,, \\
\left[(p-gA)^2 - M^2_{2,4}\right] V^{(2,4)}_\mu &= ig\left(F_\mu{}^\nu V^{(2,4)}_\nu + i\widetilde{F}_\mu{}^\nu A^{(2,4)}_\nu\right)\,, \\
\left[(p-gA)^2 - M^2_{2,4}\right] A^{(2,4)}_\mu &= ig\left(F_\mu{}^\nu A^{(2,4)}_\nu + i\widetilde{F}_\mu{}^\nu V^{(2,4)}_\nu\right)\,,
\end{aligned} \tag{19}$$

where $\widetilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ with $\varepsilon_{0123} = 1$. Here, the mass differences within two 16-plets are ignored. Note that the system of equations (19) would split into two independent subsystems: for S, P, T and for V, A, respectively, if the gauge fields A_μ might be considered as external.

Obviously, the second-order equations (19) can be also obtained from the first-order equations following from the Dirac-type equation (2) when rewritten in the cases of $N = 2$ and $N = 4$ in terms of 16 spin-(0/1) fields involved in the expansion (14). If $A - \mu$ are independent of Γ_1^5, they are

$$
\begin{aligned}
(p - gA)^{\mu} V_{\mu}^{(2,4)} &= M S^{(2,4)} ,\\
(p - gA)^{\mu} A_{\mu}^{(2,4)} &= M P^{(2,4)} ,\\
-i(p - gA)^{\nu} T_{\mu\nu}^{(2,4)} + (p - gA)_{\mu} S^{(2,4)} &= M V_{\mu}^{(2,4)} ,\\
-(p - gA)^{\nu} \widetilde{T}_{\mu\nu}^{(2,4)} + (p - gA)_{\mu} P^{(2,4)} &= M A_{\mu}^{(2,4)} ,\\
-i\left[(p - gA)_{\mu} V_{\nu}^{(2,4)} - (p - gA)_{\nu} V_{\mu}^{(2,4)}\right] &\\
-\varepsilon_{\mu\nu}{}^{\rho\sigma}(p - gA)_{\rho} A_{\sigma}^{(2,4)} &= M T_{\mu\nu}^{(2,4)} , \qquad (20)
\end{aligned}
$$

where $\widetilde{T}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma} T^{\rho\sigma}$ and $p_{\mu} = i\partial_{\mu}$.

Beside the standard-model gauge fields A_{μ}, our spin-(0/1) boson fields are expected to be coupled also to the fundamental-fermion sources (*cf. e.g.* the Higgs coupling). Then, these sources should appear additively on the r.h.s. of Eqs. (19).

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