VARIATION OF THE FERMI MATRIX ELEMENTS OF $\Delta J = 0$ $\Delta T = 1$ BETA DECAYS WITH THE SIZE PARAMETER

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Contributions to the Fermi matrix elements M_F of $\Delta J = 0$, $\Delta T = 1$ beta decays arise mainly from the Coulomb and the specifically nuclear charge-dependent effects, for which the short-range phenomenological potential of Blin-Stoyle & Le Tourneux is used. Using jj-coupling shell model with harmonic oscillator wavefunctions, it is found that in the region of interest, the fractional change in M_F is approximately equal but opposite to the fractional change in the size parameter.

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1. Introduction

In nuclear shell model calculations, the many-particle matrix element for the particles outside a closed shell has first to be reduced to two-particle matrix elements, normally by means of fractional parentage coefficients [1] or sometimes by standard methods in which this reduction of the many-particle matrix element could be written in a closed form [2]. Then the two-particle matrix elements are usually calculated using the method of transformation brackets [3], and single-particle wave functions are taken to be simple harmonic type. The radial parts $R_{n\ell}(r)$ of the simple harmonic wave functions are characterized by the so-called size parameter $b = \sqrt{\hbar/m\omega}$, *i.e.*

$$R_{n\ell}(r) = \left(\frac{2^{\ell-n+2}(2\ell+2n+1)!!}{\sqrt{\pi}n!\left((2\ell+1)!!\right)^2 b^{2\ell+3}}\right)^{\frac{1}{2}} r^\ell \exp\left(-\frac{r^2}{2b^2}\right) L^\ell_{n+\ell+1/2}\left(\frac{r^2}{b^2}\right), \quad (1)$$

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where $L^{\ell}_{n+\ell+1/2}(r^2/b^2)$ are the associated Laguerre polynomials. The value of the size parameter adopted in the calculation is usually obtained by using the relation

$$\langle r^2 \rangle = \tfrac{3}{5} R^2 . \tag{2}$$

In this paper, we wish to investigate the changes in the values of the Fermi nuclear matrix elements of $\Delta J = 0$, $\Delta T = 1$ beta-decays due to changes in the values of the size parameter.

2. Calculation and results

The Fermi nuclear matrix elements $M_{\rm F}$ is given by

$$M_{\rm F} = \langle f \,| T_{\pm} |\, i \rangle , \tag{3}$$

where $T_{\pm}$ is the total isospin operator. This obviously leads to the selection rule $\Delta J = 0$, $\Delta T = 0$. Therefore, non-vanishing $M_{\rm F}$ for $\Delta J = 0$, $\Delta T = \pm 1$ transitions should be due to charge-dependent effects $V_{\rm CD}$ given by $V_{\rm CD} = V_{\rm C} + V_{\rm N}$ where the Coulomb potential $V_{\rm C}$ is

$$V_{\rm C} = \frac{e^2}{4} \sum_{i<j} \frac{\left(1 + \tau_z^{(i)}\right)\left(1 + \tau_z^{(j)}\right)}{r_{ij}} \tag{4}$$

and where $V_{\rm N}$ is the phenomenological charge-dependent potential by Blin-Stoyle and Le Tourneux [4]:

$$V_{\rm N} = \sum_{i<j} V_0 \left(\left(\tau_z^{(i)} + \tau_z^{(j)} \right) \left(p + r \bar{\sigma}^{(i)} \cdot \bar{\sigma}^{(j)} \right) + \tau_z^{(i)} \tau_z^{(j)} \left(q + s \bar{\sigma}^{(i)} \cdot \bar{\sigma}^{(j)} \right) \right) \exp \left(-\beta r_{ij}^2 \right) . \tag{5}$$

The parameters p and r measure the deviation from charge symmetry while q and s measure the deviation from charge independence. V_0 is taken as -51.9 MeV and the effective range $\beta^{-1/2} = 1.73$ fm.

Consider a β^+ decay of a nucleus of spin J and isospin $T+1$ into a nucleus of spin J but isospin T. According to the ordinary perturbation theory,

$$\begin{aligned} |i\rangle &= |JM\, T+1, T_z+1\rangle + \alpha' |JM\, T+2, T_z+1\rangle + \ldots , \\ |f\rangle &= |JM\, TT_z\rangle + \alpha |JM\, T+1, T_z\rangle + \ldots . \end{aligned} \tag{6}$$

Neglecting second- and higher-order terms in α, obtain

$$M_{\rm F}(\beta^+) = \langle f | T_- | i \rangle = \alpha \sqrt{(T + T_z + 2)(T - T_z + 1)} , \tag{7}$$

where the isospin impurity amplitude α is given by

$$\alpha = -\frac{\langle JM\,T\,T_z|V_{\rm CD}|JM\,T+1,\,T_z\rangle}{\Delta E} \equiv -\frac{M}{\Delta E}\,. \tag{8}$$

Using jj-coupling nuclear shell model, it can be shown that the only contributions to M come from nucleons within the unfilled shell. Therefore,

$$M = \langle j^k\,JM\,T\,T_z\,\sigma = (s,t)|\sum_{i<j} V_{\rm CD}(i,j)|j^k\,JM\,T+1,\,T_z\sigma = (s,t)\rangle\,. \tag{9}$$

Here $j = 1\ldots k$ refers to the k equivalent nucleons in the last unfilled shell; s the seniority; t the reduced isospin; $\sigma = (s,t)$ denotes the symplectic representation. Using standard methods, for $k = 4$ we obtain

$$\begin{aligned} M = &\sum_{T_{12z}+T_{34z}=T_z} C^{TT_z}_{1T_{12z}1T_{34z}} C^{T+1,T_z}_{1T_{12z}1T_{34z}} \frac{1-4(A_T+A_{T+1})}{\sqrt{(1-4A_T)(1-4A_{T+1})}} \\ &\times \Big(\langle j^2 J_{12} = 0,\, T_{12} = 1|V_{\rm CD}(1,2)|j^2 J_{12} = 0,\, T_{12} = 1\rangle \\ &- \langle j^2_{12} = J,\, T_{12} = 1|V_{\rm CD}(1,2)|j^2 J_{12} = J,\, T_{12} = 1\rangle \Big)\,, \end{aligned} \tag{10}$$

where

$$A_T = \frac{1}{2(2j+1)}(2\delta_{T,2} - \delta_{T,0})\,.$$

The general two-particle matrix elements [3] can be written as

$$M_{12} \equiv \langle n_1\ell_1, n_2\ell_2, \lambda\mu|V|n'_1\ell'_1, n'_2\ell'_2, \lambda\mu\rangle = \sum C_p I_p\,, \tag{11}$$

where the C coefficients are independent of the size parameter b and the Talmi integrals I_p are defined by

$$I_p = \frac{2}{\Gamma(p+\frac{3}{2})}\int_0^\infty r^{2p}{\rm e}^{-r^2} V_{\rm CD} r^2 dr\,. \tag{12}$$

Here the radial distance r is expressed in units of b and because of the way r is defined, the following transformations in the potential $V_{\rm CD}$ should be made:

$$\begin{aligned} {\rm e}^{-\beta r^2} &\to {\rm e}^{-2\beta b^2 r^2}\,, \\ \frac{e^2}{r} &\to \frac{e^2}{\sqrt{2}br}\,. \end{aligned} \tag{13}$$

For the Coulomb potential

$$I_p = \frac{e^2}{b\sqrt{2\pi}}\left(\frac{2^{p+1}p!}{(2p+1)!!}\right) \equiv \frac{B_p}{b}. \tag{14}$$

Eq. (11) becomes

$$M_{12} = \frac{1}{b}\sum C_p B_p\,. \tag{15}$$

From Eqs (7)–(10) and (15), we obtain

$$\frac{\delta M_{\rm F}^{\rm C}}{M_{\rm F}^{\rm C}} = -\frac{\delta b}{b}\,. \tag{16}$$

Therefore a 10% increase in the value of the size parameter would then decrease the value of the Fermi matrix element by the same amount.

For the Blin-Stoyle–Le Tourneux potential, no such simple relation is possible and therefore detailed calculations have to be made. Consider the β^+ decay from the $J = 2^+$, $T = 1$ ground state of ^{44}Sc to the 1.16 MeV, $J = 2^+$, $T = 2$ state of ^{44}Ca. Using Eq. (10),

$$\begin{aligned} M_{\rm F}^{M} &= -2\frac{\langle f_{7/2}^4 J{=}2,T{=}1,T_z{=}-1,\sigma{=}(2,1)|V_{\rm N}|f_{7/2}^4 J{=}2,T{=}2,T_z{=}-1,\sigma{=}(2,1)\rangle}{E(T{=}2)-E(T{=}1)} \\ &= \frac{\sqrt{2}}{\Delta E}\Big(\langle f_{7/2}^2 J_{12} = 0|V_{\rm N}(r_{12}) f_{7/2}^2 J_{12} = 0\rangle \\ &\quad - \langle f_{7/2}^2 J_{12} = 2|V_{\rm N}(r_{12})|f_{7/2}^2 J_{12} = 2\rangle\Big)\,, \end{aligned} \tag{17}$$

where

$$\Delta E = E(T=2) - E(T=1) = 4.56\ \mathrm{MeV}$$

and

$$V_{\rm N}(r_{12}) = V_0\big((p-q) + (r-s)\bar\sigma_1\cdot\bar\sigma_2\big)\exp\big(-\beta r_{12}^2\big)\,.$$

If we write $M_{\rm F}^{\rm N} = (p-q)M_{\rm N} + (r-s)M_{\rm S}$, the values of $M_{\rm N}$ and $M_{\rm S}$ are calculated as a function of b as shown in Fig. 1. It is noted that the contributions of $M_{\rm N}$ and $M_{\rm S}$ are opposite in sign with major contributions coming from $M_{\rm S}$.

Consider another decay with rather different configuration assignments, namely, the β^- decay from the ground state $J = {}^{7^-}\!/_2$, $T = {}^5\!/_2$ of ^{41}Ar to the $J = {}^{7^-}\!/_2$, $T = {}^3\!/_2$ state of ^{41}K. In the jj coupling shell model, we can assign the configuration $(d_{3/2}^{-2}\, J = 0, T = 1, T_z = -1)$ $(f_{7/2}^3\, J = {}^7\!/_2, T = {}^3\!/_2, T_z = -{}^3\!/_2)$ to ^{41}Ar and the configuration $(d_{3/2}^{-2}\, J = 0, T = 1, T_z = -1)$ $(f_{7/2}^3\, J = {}^7\!/_2, T = {}^1\!/_2, T_z = -{}^1\!/_2)$ to ^{41}K. The small admixture of the

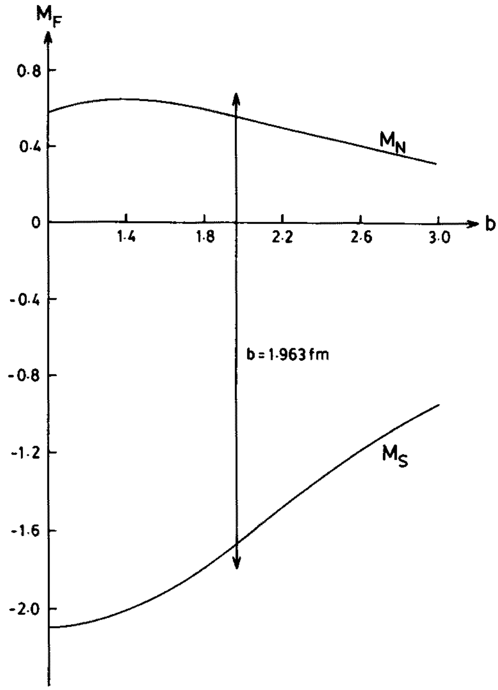

Fig. 1. Variation of the Fermi matrix element $M_{\rm F}$ as a function of the size parameter b for the β^+ decay of ^{44}Sc. $M_{\rm N}$ and $M_{\rm S}$ are defined by $M_{\rm F}^{\rm N} = (p-q)M_{\rm N} + (r-s)M_{\rm S}$ where p and r measure the deviation from charge symmetry and q and s measure the deviation from charge independence.

$T = {}^5\!/_2$ configuration $(d_{3/2}^{-2}\, J = 0, T = 1, T_z = -1)$ $(f_{7/2}^3\, J = {}^7\!/_2, T = {}^3\!/_2, T_z = -{}^1\!/_2)$ in ^{41}K is responsible for its non-vanishing $M_{\rm F}$. Fig. 2 gives the results of the values of $M_{\rm N}$ and $M_{\rm S}$ as a function of b. It is again noted that the contributions of $M_{\rm N}$ and $M_{\rm S}$ are opposite in sign with major contributions coming from $M_{\rm S}$.

Around the value of $b = 1.9$ fm, the calculated results shown in Figs 1 and 2 could be approximated by

$$M_{\rm N} + M_{\rm S} \approx Ab + C\,, \tag{18}$$

where A and C are constants. If b increases by 10 percent, calculation shows that for both cases,

$$\frac{\delta(M_{\rm N} + M_{\rm S})}{M_{\rm N} + M_{\rm S}} \approx -0.10\,. \tag{19}$$

As the parameters [5] p and r of the Blin-Stoyle–Le Tourneux potential are an order of magnitude smaller than q and s and also as $q \approx s$, Eq. (19)

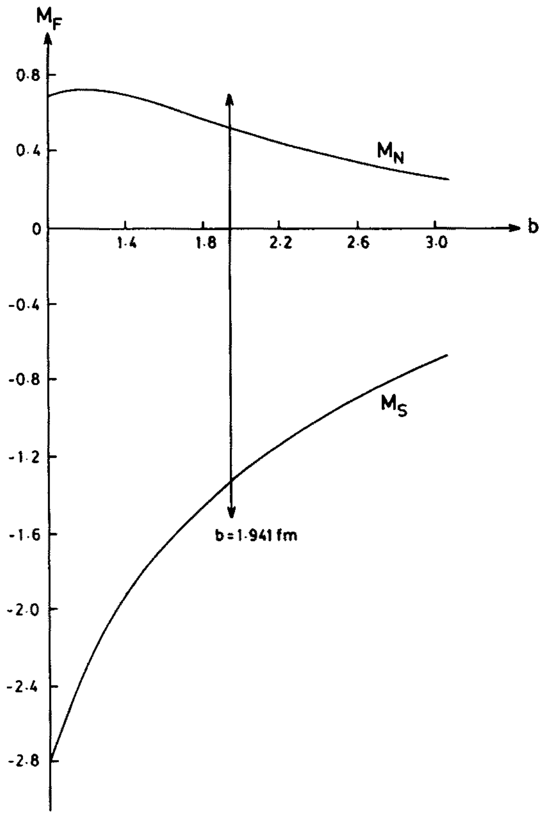

Fig. 2. Variation of the Fermi matrix element M_F as a function of the size parameter b for the β^- decay of ^{41}Ar. M_N and M_S are defined by $M_F^N = (p-q)M_N + (r-s)M_S$ where p and r measure the deviation from charge symmetry and q and s measure the deviation from charge independence.

becomes

$$\frac{\delta M_F^N}{M_F^N} \approx -0.10 \approx -\frac{\delta b}{b}\,. \tag{20}$$

Combining Eqs (16) and (20), we obtain

$$\frac{\delta M_F}{M_F} \equiv \frac{\delta M_F^C + \delta M_F^N}{M_F^C + M_F^N} \approx -\frac{\delta b}{b}\,. \tag{21}$$

Therefore the fractional change in the value of M_F is approximately equal to the negative of the fractional change in the value of the size parameter. It is further noted [6] that Eq. (16) and (20) agree in general with the results of explicit calculations on both the Coulomb and charge symmetry-breaking potentials for the Fermi matrix elements of isospin-forbidden beta decays of ^{20}F and ^{24}Na.

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REFERENCES

[1] A. de-Shalit, I. Talmi, *Nuclear Shell Model*, Academic Press, London, 1963.
[2] C.T. Yap, C.S. Tee, *Nucl. Phys.* **A169**, 609 (1971).
[3] M. Moshinsky, T.A. Brody, *Tables of Transformation Brackets*, Instituto de Fisica, Mexico, 1960.
[4] R.J. Blin-Stoyle, J. Le Tourneux, *Ann. Phys.* **18**, 12 (1962).
[5] D.H. Wilkinson, *Isospin in Nuclear Physics*, North-Holland, 1969.
[6] C.S. Tee, C.T. Yap, *Nucl. Phys.* **A170**, 445 (1971).