THE TEMPERATURE DEPENDENCE OF THE LEVEL DENSITY PARAMETER

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Using the Thomas–Fermi model and the leptodermous expansion in powers of $A^{1/3}$, we derive a formula for nuclear level density parameter "a" at finite temperature. The level density parameter was found to depend on the nuclear potential only through the effective mass m^*/m. The experimental data of "a" at zero temperature favours a value of $m^*/m \approx 1$. The level density parameter was found to decrease with increasing temperature in a qualitative agreement with experiment.

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1. Introduction

An expression for the level density $\rho(A, E^*)$ of a Fermi system of A particles at an excitation energy E^* is useful starting point for the discussion of the thermodynamic properties of highly excited nuclei [1–5]. A satisfactory description of the nuclear level densities is a necessary test for methods used to calculate an equation of state for nuclear matter at high temperatures [6].

At temperature low when compared to the Fermi energy, the key quantity entering expression for ρ is the level density parameter. Since the surface thickness of a heavy nucleus is small compared to its size, we can expand the level density parameter in powers of $A^{1/3}$. This expansion gives volume, surface and curvature level density parameters [6–8] and can reproduce the experimental value of $a = (A/8)$ MeV^{-1}.

Experimental evidence for a temperature dependence of the level density parameter has been reproduced [9]. Whereas the low temperature data are consistent with the value $a = (A/8)$ MeV^{-1}, at high temperature ($T \approx 6$ MeV) a smaller value $a = (A/13)$ MeV^{-1} has to be adopted. A very precise determination of the level density parameter is also obtained in experiments devoted to the study of the properties of giant dipole resonances in highly excited nuclei [10]. The available data are limited to temperatures below 2 MeV and consistent with the value of $a = (A/8)\ \mathrm{MeV}^{-1}$.

On the theoretical side considerable effort has been devoted to study the level density parameter at finite temperature. Suraud *et al.* [11] calculated the level density of a hot nucleus plus confining gas system and subtracted the level density of the gas, they found that the parameter (a/A) stays constant up to $T \approx 5$ MeV, then increases by about 6% for $T = 8$ MeV. This result is consistent with conclusions from Hartree–Fock [2] and Extended Thomas–Fermi [12] calculations on hot nuclei. Dean and Mosel [13] calculated the entropy S of a nuclear system at excitation energy E^* thereby taking the continue into account *via* the Levinson theorem. By relating the level density parameter to the entropy, $S^2 = 4aE^*$, they found an opposite trend, namely that a/A remains constant $= (1/13.5)$ MeV up to $T = 5$ MeV and then decreases by about 8% at $T = 10$ MeV.

This paper is organized as follows: In Section 2 we derive the analytical expression for the level density parameter at finite temperature using the Thomas–Fermi model. We employ the leptodermous expansion to get the temperature dependence of the volume, surface and curvature level density parameters. Results and discussion are presented in Section 3. Finally, Section 4 is devoted to some concluding remarks.

2. Theory

The single particle level density parameter "a" is defined by [1]

$$E^* = aT^2\,, \tag{1}$$

where E^* is the excitation energy per nucleon and T is the nuclear temperature. We write the energy of the system per nucleon as

$$E(\rho, T) = E(\rho, T = 0) + E^*(\rho, T)\,, \tag{2}$$

where ρ is the nuclear density. This equation gives the specific heat at constant volume as

$$C_{\mathrm{V}} = \frac{\partial E^*}{\partial T}\,,$$

but the specific heat is related to the single particle entropy S by

$$C_{\mathrm{V}} = T\frac{\partial S}{\partial T}\,.$$

This gives

$$a = \frac{1}{T^2}\int_0^T T'\left(\frac{\partial S}{\partial T'}\right) dT'\,. \tag{3}$$

Equation (3) shows that the key quantity to calculate the level density parameter, or the thermal properties of the system, is the single particle entropy S.

If we treat the nucleus as a Fermi liquid containing N neutrons and Z protons, the entropy density is given by

$$\sigma = \sum_{\tau} \left(\frac{5}{3} \frac{j_{3/2}(\eta_\tau)}{j_{1/2}(\eta_\tau)} - \eta_\tau \right) \rho_\tau \,, \tag{4}$$

where τ is the isospin and j_α are the Fermi integrals defined by

$$j_\alpha(\eta) = \int_0^\infty \frac{X^\alpha}{1+\exp(X-\eta)} dX \,.$$

The parameter η is given by

$$\eta_\tau = \frac{1}{T}(\mu_\tau - V_\tau)\,;$$

μ is the chemical potential and V_τ is the single particle potential. If the temperature is sufficiently small, the parameter η becomes large and we can expand the Fermi integrals in powers of η.

We follow the method used by Landau [16] to calculate this expansion up to the order T^6. The result is

$$\begin{aligned} j_\alpha(\eta) =& \frac{\eta^\alpha + 1}{\alpha+1}\Big(1 + \frac{\pi^2}{6}\alpha(\alpha+1)\eta^{-2} \\ &+ \frac{4\pi^4}{360}\alpha(\alpha^2-1)(\alpha-2)\eta^{-4} \\ &+ \frac{11\pi^6}{8400}\alpha(\alpha^2-1)(\alpha-2)(\alpha-3)(\alpha-4)\eta^{-6} + \ldots\Big)\,. \end{aligned} \tag{5}$$

The total entropy of the system is

$$S = \int_0^\infty (\sigma_{\mathrm{n}} + \sigma_{\mathrm{p}}) dr \,.$$

So,

$$\begin{aligned} S =& \frac{1}{6}\left(\frac{3\pi^2}{2}\right)^{1/3}\left(\frac{2m^*T}{\hbar^2}\right) \int \left(\rho_{\mathrm{n}}^{1/3} + \rho_{\mathrm{p}}^{1/3}\right) dr \\ &- \frac{7}{1080}\left(\frac{2m^*T}{\hbar^2}\right)^3 \int \left(\rho_{\mathrm{n}}^{-1} + \rho_{\mathrm{p}}^{-1}\right) dr \\ &- \frac{22}{13035}\left(\frac{2m^*T}{\hbar^2}\right)^5 \int \left(\rho_{\mathrm{n}}^{-7/3} + \rho_{\mathrm{p}}^{-7/3}\right) dr \,, \end{aligned} \tag{6}$$

where m^* is the nucleon effective mass.

To calculate the integrals in equation (6), we make use of two simplifications: 1) in the asymptotic limit of a very heavy nucleus the curvature effect becomes negligible and the density profile across the surface is that of the so-called semi-infinite nuclear matter. This system has a one dimensional geometry; the density varies only along one axis (say r). 2) We parameterize the density distribution according to the Fermi form

$$\rho_\tau = \frac{\rho_{0\tau}}{1+\exp\left(\frac{r-R}{d}\right)} .$$

This form is found to give a good approximation to the Hartree–Fock density distribution [17].

Applying these simplifications, the integrations in equation (6) reduce to the form

$$I(q) = \int_0^\infty \left(1+\exp\left(\frac{r-R}{d}\right)\right)^q r^2 dr . \tag{7}$$

Sirvastava [18] calculated $I(q)$ for negative values of q, the results is

$$I(q) = \frac{1}{3}R^3 - R^2 d A_1(q) + 2Rd^2 A_2(q) - 2d^3 A_3(q) , \tag{8}$$

where the coefficients $A_n(q)$ are given by:

$$A_n(q) = \frac{1}{(n-1)!}\int_0^\infty \left(1 - \left(1+\mathrm{e}^{-X}\right)^{-q} + (-)^n\left(1+\mathrm{e}^{X}\right)^{-q}\right) X^{n-1} dX .$$

We calculate the integral $I(q)$ for positive q, the result is:

$$I(q) = \frac{1}{3}R^3 - R^2 d B_1(q) + 2Rd^2 B_2(q) - 2d^3 B_3(q) , \tag{9}$$

where the coefficients $B_n(q)$ are given by:

$$B_1(q) = \frac{-1}{q}\left(1 - e^{Cq}\right) - \sum_{m=1}^{\infty} f(m,q)\left[\frac{1 - e^{C(q-m)}}{q-m} - \frac{1}{m}\right],$$

$$B_2(q) = \frac{1}{q^2} + \left(\frac{C}{q} - \frac{1}{q^2}\right)e^{Cq} + \sum_{m=1}^{\infty} f(m,q)\left[\left(\frac{1}{(q-m)^2} - \frac{1}{m^2}\right)\right.$$

$$\left. + \left(\frac{C}{q-m} - \frac{1}{(q-m)^2}\right)e^{C(q-m)}\right], \quad \text{and}$$

$$B_3(q) = -\frac{1}{q^3} + \left(\frac{C^2}{2q} - \frac{C}{q^2} + \frac{1}{q^3}\right)e^{Cq}$$

$$+ \sum_{m=1}^{\infty} f(m,q)\left[\frac{1}{m^3} - \frac{1}{(q-m)^3}\right.$$

$$\left.\left(\frac{C^2}{2(q-m)} - \frac{C}{(q-m)^2} + \frac{1}{(q-m)^3}\right)e^{C(q-m)}\right],$$

where C is a small finite constant, such that $\rho(R+C) = 0$, and $f(m,q)$ is defined by

$$f(m,q) = \frac{q(q-1)\dots(q-m+1)}{m!}.$$

For the radius parameter, we used the relation [19]

$$R = \left(\frac{3A}{4\pi\rho}\right)^{1/3} - \frac{\pi^2 d^2}{3}\left(\frac{4\pi\rho}{3A}\right)^{1/3}.$$

Performing the integrations in equation (6), we get the leptodermous expansion for the level density parameter in the form

$$a = a_V A + a_S A^{2/3} + a_C A^{1/3}, \tag{10}$$

where a_V, a_S and a_C are the volume, surface and curvature level density parameters, respectively. The volume level density parameter is:

$$a_V = \frac{1}{6}\frac{2m^*}{\hbar^2}\left(\frac{3\pi^2}{2}\right)^{1/3}\rho^{-2/3}\left(1 - \frac{X^2}{9}\right)$$

$$- \frac{7}{4320}\left(\frac{2m^*}{\hbar^2}\right)^3 T^2\rho^{-2}(1 + X^2)$$

$$- \frac{11}{39105}\left(\frac{2m^*}{\hbar^2}\right)^5\left(\frac{2}{3\pi^2}\right)^{1/3} T^4\rho^{-10/3}\left(1 + \frac{35}{9}X^2\right).$$

The surface level density parameter is

$$\begin{aligned} a_S = &- \pi d A_1 \left(\frac{1}{3}\right) \frac{2m^*}{\hbar^2} \left(\frac{1}{4}\right)^{1/3} \rho^{-1/3} \left(1 - \frac{X^2}{9}\right) \\ &- \frac{7d}{1080} B_1(1) \left(\frac{2m^*}{\hbar^2}\right)^3 \left(\frac{3}{4\pi}\right)^{2/3} T^2 \rho^{-5/3} (1 + X^2) \\ &- \frac{22d}{39105} B_1 \left(\frac{7}{3}\right) \left(\frac{2m^*}{\hbar^2}\right)^5 \left(\frac{3}{\pi}\right)^{1/3} T^4 \rho^{-3} \left(1 + \frac{35}{9} X^2\right) . \end{aligned}$$

The curvature level density parameter is:

$$\begin{aligned} a_C = &\frac{\pi d^2}{3} (9\pi)^{1/3} \left(\frac{2m^*}{\hbar^2}\right) \left(2A_2 \left(\frac{1}{3}\right) - \frac{\pi^2}{9}\right) \left(1 - \frac{X^2}{9}\right) \\ &- \frac{7\pi d^2}{1080} \left(\frac{3}{4\pi}\right)^{1/3} \left(\frac{2m^*}{\hbar^2}\right)^3 \left(2B_2(1) - \frac{\pi^2}{9}\right) T^2 \rho^{-4/3} (1 + X^2) \\ &- \frac{44\pi d^2}{39105} \left(\frac{1}{2}\right)^{1/3} \left(\frac{2m^*}{\hbar^2}\right)^5 \left(2B_2 \left(\frac{7}{3}\right) - \frac{\pi^2}{9}\right) T^4 \rho^{-8/3} \left(1 + \frac{35}{9} X^2\right) . \end{aligned}$$

We expand the level density parameter in X up to X^2 (X is the neutron excess parameter defined by $X = [N - Z]/A$).

These equations show that the dependence of the level density parameter (also the thermal properties of the system) on the nuclear interaction is only through the effective mass.

3. Results and discussion

The comparison between our results (at zero temperature and for symmetric nuclear matter) and experimental data is shown in Fig. 1. Although the parameter "a" shows a strong shell structure, its average dependence on the mass number A is of the order of $a = (A/8)$ MeV^{-1}, the best fit being obtained with $m \approx m^*(a = A/9.4$ MeV$^{-1})$. This result seems to be in contradiction with the value $m^* \approx 0.8m$, given by quadrupole resonance analysis [14]. We think that this is due to shell effects, so that the average curve of the level density parameter should lie under the experimental values.

Asymmetry effects have been found negligible. For $X = 0.3$ the level density parameter decreases only by 1%. The temperature effects have also been found negligible. For the T^2 term we found that the level density parameter decreases by 1–2%, while for the T^4 term it decreases by 0.5–1.0% (for $T = 5$–8 MeV). Fig. 2 shows the inverse level density parameter A/a versus temperature. It is seen that qualitative agreement with experiment is obtained. The difference at low temperature is due to shell effects (which disappears for $T \geq 4$ MeV) and at high temperature is due to neglecting the dependence of the effective mass on temperature. For comparison we plot the nonlocal Hartree–Fock potential based on the Gogny force and the local Woods–Saxon potential results [15]. Blin and Brack [5] have used the Extended Thomas–Fermi calculations and the density parametrized as a Fermi like function, to calculate the entropy of hot nuclei. The level density parameter deduced from their results ($a = (S/A)/2T$) has the same behaviour as ours.

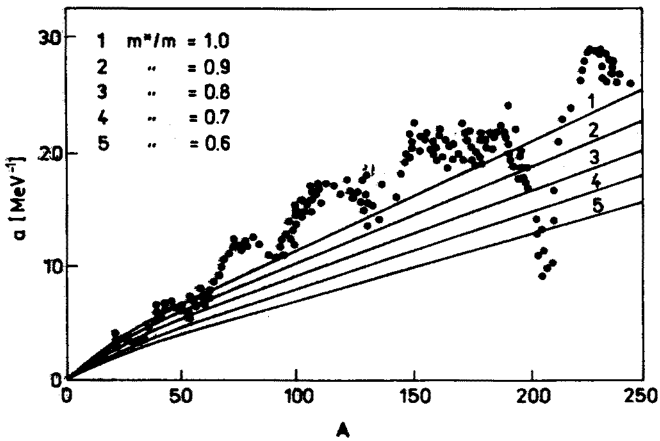

Fig. 1. The level density parameter (a), for symmetric semi-infinite nuclear matter at zero temperature, versus the mass number (A). Experimental values are taken from Ref. [1].

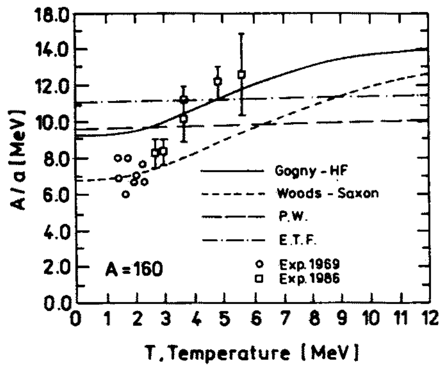

Fig. 2. The inverse nuclear level density parameter A/a versus temperature. Experimental data (1969) and (1986) are taken from Refs [1, 9].

4. Conclusion

A simple analytic expression for the level density parameter at finite temperature has been derived. The level density parameter appears to contain the volume, surface and curvature terms. One thus confirmed in a quantitative way the particular role played by the nuclear surface and curvature. The level density parameter does not depend explicitly on the nuclear interaction, but only through the effective mass. We found that the level density parameter gives an independent constraint on the value of the effective mass at zero temperature. The best fit favours the value $m \approx m^*$. At finite temperature it is necessary to take into account the temperature dependence of the effective mass. This can be done if accurate enough experimental data are available.

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