WHAT IS THE DOMINANT CONTRIBUTION IN A HADRONIC PROCESS?*

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(*Received April 2, 1992*)

The structure and form of the dominant contribution of higher order corrections for a hadronic process in the next-to-leading order with respect to the running coupling α_s are shown explicitly. We demonstrate that this contribution comes from the soft and/or collinear configuration of the particles in the final or initial state. We give the origin of various terms of this contribution and a prescription to derive them.

PACS numbers: 11.15. Bt

1. Introduction

In perturbative QCD, higher order corrections (HOC) have been calculated for many processes. In particular when the Born term is of order α_s or higher, the calculations are very involved and the expressions very complicated. Yet, almost invariably, the result is very simple; an overall inclusive cross section differing from the Born term by a slowly varying factor [1]–[6].

This suggests that there is a relatively simple dominant part of the HOC which, if identified, can perhaps be calculated easily [7]. If so, this could be useful in various directions, as for example in determining approximate HOC for QCD processes of the type $a+b \longrightarrow c+d$ and particularly $a+b \longrightarrow c+d+e$ for which complete HOC are hitherto unknown (due to their complexity), and in going beyond the next-to-leading order in α_s. Moreover, in supersymmetric theories where we are dealing with the standard particles as well as their supersymmetric partners (that is many diagrams involved), the complete calculation of the next-to-leading order is practically impossible. In addition, the recent complete calculation of the Russian group for

* This work is supported by the Algerian Ministry of Education and research under contract No D2501/01/18/90.

the ratio $R_{e^+e^-}$ [8–10] shows that the next-to-leading order in α_s^2 is larger than the one of order α_s. This means that the perturbative serie expansion diverge. This is a serious problem in QCD. Therefore, more tests of the theory are needed and one has to find the simplest and fastest method to determine at least the magnitude and the sign of the corrections.

If one considers for example the standard model of the electroweak theory where massive particles such as the Higgs, Z and $W^{\pm}$ bosons are involved, the complete calculation of HOC will be very complicated and difficult. Thus, it will be very interesting at least from the phenomenological point of view, if one can know easily the magnitude and sign of HOC. This may solve the outstanding problem of the Higgs particle in the sense that: may be the HOC are comparable to the Born term but with opposite sign. This means that the cross section is small, a fact which is compatible with nowadays high energy experiments. This may save the Weinberg–Salam–Glashow theory!

In Section 2 we show the structure of the dominant contribution of processes involving structure and/or fragmentation functions. In Section 3 we give physical as well as theoretical proofs of the dominant of the contribution coming from the soft and/or collinear configuration of the particles at the initial and/or the final state. In Section 4 we determine the origin of each term and establish some rules. Finally, in Section 5 we give some practical proofs of the dominance of such a contribution for some physical processes where the exact HOC is known and draw our conclusions.

2. The structure of higher order corrections

Let us consider a hadronic process of the form

$$A + B \longrightarrow C + D\,, \tag{2.1}$$

where A, B, C and D are hadrons. The cross section can be written as

$$\sigma = \sum_{\substack{a,b\\c,d}} \iiiint dx_a dx_b dx_c dx_d F_{\frac{a}{A}}(x_a, M) F_{\frac{b}{B}}(x_b, M) D_{\frac{C}{c}}(x_c, M')$$

$$\times D_{\frac{D}{d}}(x_d, M')\left\{\frac{\alpha_s(\mu)}{\pi} f_0 \delta\Big(1+\frac{\hat t+\hat u}{\hat s}\Big) + \frac{\alpha_s^2(\mu)}{\pi} f\theta\Big(1+\frac{\hat t+\hat u}{\hat s}\Big)\right\}, \tag{2.2}$$

where $F_{\frac{a}{A}}$, $F_{\frac{b}{B}}$ (resp. $D_{\frac{C}{c}}$, $D_{\frac{D}{d}}$) are structure (resp. fragmentation) functions; a, b, c, d are massless partons, f_0 the Born term and f the HOC.

We introduce the following dimensionless variables

$$v = 1 + \frac{\hat{t}}{\hat{s}}; \quad w = -\frac{\hat{u}}{\hat{t} + \hat{s}}, \tag{2.3}$$

so that

$$\hat{s} + \hat{t} + \hat{u} = v(1 - w)\hat{s} \tag{2.4}$$

$\hat{s}$, $\hat{t}$ and $\hat{u}$ are the usual Mandelstam variables at the partons subprocess.

Now, it follows from a number of explicit results [3–5], [12, 13] and will also become clear in the next sections that the general structure of HOC is as follows:

$$f = f(x_c, x_d, v, w) = f_{\text{sing}}(x_c, x_d, v, w) + \tilde{f}(x_c, x_d, v, w), \tag{2.5}$$

where

$$\begin{aligned} f_{\text{sing}} &= \left[C_1 + \widehat{C}_1 \ln\left(\frac{\hat{s}}{\mu^2}\right) + \widetilde{C}_1 \ln\left(\frac{\hat{s}}{M^2}\right) + \widetilde{\widetilde{C}}_1 \ln\left(\frac{\hat{s}}{M'^2}\right)\right]\delta(1 - w) \\ &+ \left[C_2 + \widetilde{C}_2 \ln\left(\frac{\hat{s}}{M^2}\right) + \widetilde{\widetilde{C}}_2 \ln\left(\frac{\hat{s}}{M'^2}\right)\right]\frac{1}{(1 - w)_+} + C_3\left[\frac{\ln(1 - w)}{1 - w}\right]_+, \end{aligned} \tag{2.6}$$

where μ, M and M' are the renormalization, structure and fragmentation functions factorization points, respectively. Moreover, the distributions $\frac{1}{(1-w)_+}$ and $\left[\frac{\ln(1-w)}{1-w}\right]_+$ are defined as follows:

$$\int_0^1 dw \frac{g(w)}{(1 - w)_+} = \int_0^1 dw \frac{g(w) - g(1)}{(1 - w)}$$

and

$$\int_0^1 dw \left[\frac{\ln(1 - w)}{1 - w}\right]_+ g(w) = \int_0^1 dw \left[g(w) - g(1)\right], \tag{2.7}$$

where $g(w)$ is some regular function.

The function $\tilde{f}$ is smooth as $w \longrightarrow 1$ and in general, very complicated (hundreds of terms); it is the most complicated part of HOC [2, 8, 9]. It receives contributions from hard and non-collinear gluon and/or quark Bremsstrahlung. It should be added that in $\tilde{f}$ the multitude of terms contribute with almost random signs (some are positive, others negative). This means that the resulting cancellations add to the suppression of $\tilde{f}$.

It will be clear in Section 4 that each part of $f_{\rm sing}(C_1,\ \widetilde{C}_1,\ etc.)$ is gauge invariant and receives contributions from soft and/or collinear gluon and/or quark Bremsstrahlung. Each term of $f_{\rm sing}$ is related to the infrared or ultraviolet or mass and collinear singularities which are gauge invariant.

3. Proof of the dominance of $f_{\rm sing}$

Let us state the following:

Theorem:

For processes involving structure and/or fragmentation functions the part $f_{\rm sing}$ dominates and as M (and/or M') increases for a fixed $\hat{s}$, the relative contribution of the part $\tilde{f}$ of HOC is suppressed more and more, and the amount of suppression increases with the softness of the structure and/or fragmentation functions.

Proof:

To simplify matter and keep our arguments transparent, let us deal without fragmentation functions (for the general case see Appendix A). Now, using the property of the heveaside theta function present in Eq. (2.2) we obtain:

$$\hat{s} + \hat{t} + \hat{u} \geq 0\,, \tag{3.1}$$

which implies immediately that:

$$x_a x_b s + x_a t + x_b u \geq 0\,, \tag{3.2}$$

where x_a and x_b are the momentum fraction of the initial partons a and b, respectively. The s, t and u are the Mandelstam variables at the hadronic level. By defining the variables

$$V = 1 + \frac{t}{s}$$

and

$$W = -\frac{u}{t+s} \tag{3.3}$$

Eqs (3.1) and (3.2) give:

$$x_a \geq \frac{x_b V W}{x_b - (1-V)}\,. \tag{3.4}$$

Now, the crucial observation is that $F_{\frac{a}{A}}(x_a, M)$ behaves like $(1-x_a)^n$; with A = parton, n is quite large ($n \sim 3$ for u-quark $n \sim 4$ for $d, n \sim 5$ for gluon). Notice also that the scale violations enhance n as M increases.

The same for $F_{\frac{b}{B}}(x_b, M)$. Thus in order to get a significant contribution for the structure functions, x_b should decrease and x_a takes the minimum value $\frac{x_b VW}{x_b-(1-V)}$. This corresponds to $w \longrightarrow 1$. Then referring to Fig. 1, contributions arising from the region away from $w = 1$ are suppressed by powers of $(1 - x_a)$ and/or $(1 - x_b)$. Now, the terms of f_{sing} of Eq. (2.5) contribute at $w = 1$ or mainly at $w \simeq 1$ (cross hatched region of Fig. 1). However, the multitude of terms of $\bar{f}$ do not mainly contribute at $w \simeq 1$ and they are suppressed. Now, if s increases (t and/or u kept fixed), V (or W) decreases and, therefore, the boundary of the hatched region (Fig. 1) moves towards $x_a = 1$ and $x_b = w$ and the whole region shrinks. Thus, the suppression increases.

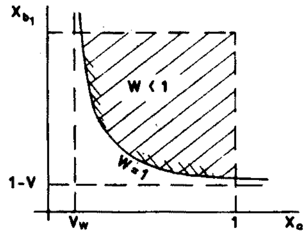

Fig. 1. Kinematic region of the x_a, x_b integration in Eq. (2.2). The boundary corresponds to $\hat{s} + \hat{t} + \hat{u} = 0$ (or $w = 1$), the hatched region corresponds to $\hat{s} + \hat{t} + \hat{u} > 0$ (or $w < 1$), and the cross-hatched region denotes the neighbourhood of the boundary.

Moreover, as we know from phase space arguments that the 2-body final state contribution dominates over that of the 3-body final state. In our case, we are dealing with soft and/or collinear configuration of the partons Bremsstrahlung. This is like having two particles at the final state. Thus, the contribution coming from this configuration dominates.

4. How to get the dominant contribution?

The second term $\widehat{C}_1$ of Eq. (2.6) proportional to $\ln\left(\frac{\hat{s}}{\mu^2}\right)$ is the first term of the expansion of the running coupling constant around the renormalization point times the Born term:

$$\alpha_s(s) = \alpha_s(\mu)\left[1 - \frac{\alpha_s(\mu)}{2\pi}\frac{11N_c - 2N_f}{6}\ln\left(\frac{\hat{s}}{\mu^2}\right) + \ldots\right]. \qquad (4.1)$$

The origin of this logarithmic term comes from the renormalization *i.e.* the subtraction of the ultraviolet divergences.

Similarly, the terms $\widetilde{C}_1$ and $\widetilde{C}_1$ proportional to $\ln\left(\frac{\hat{s}}{M^2}\right)$ gets their origin from the factorization of the structure functions at the scale M. Thus, it is easy to show that for the subprocess $a+b \longrightarrow c+d$ one can write:

$$\widetilde{C}_1 + \widetilde{C}_2 = \tfrac{1}{2}\Big[\sum_i p_{ia} \otimes \hat{\sigma}^B_{ib\to cd} + \sum_j p_{jb} \otimes \hat{\sigma}^B_{aj\to cd}\Big] \tag{4.2}$$

and then obtain $\widetilde{C}_1$ and $\widetilde{C}_2$ separately. Here the symbol $\otimes$ means a convolution product.

For the terms $\widetilde{\widetilde{C}}_1$ and $\widetilde{\widetilde{C}}_2$ (proportional to $\ln\left(\frac{\hat{s}}{M'^2}\right)$ which are the results of the factorization of the fragmentation functions at scale M', one gets:

$$\widetilde{\widetilde{C}}_1 + \widetilde{\widetilde{C}}_2 = \tfrac{1}{2}\Big[\sum_i \hat{\sigma}^B_{ab\to id} \otimes p_{ci} + \sum_j \hat{\sigma}^B_{ab\to cj} \otimes p_{dj}\Big]. \tag{4.3}$$

The quantities p_{ij} (resp. $\hat{\sigma}^B$'s) are the Altarelli-Parisi split functions (resp. Born terms). Thus the expression of $\widetilde{C}_1$, $\widetilde{C}_2$, $\widetilde{\widetilde{C}}_1$ and $\widetilde{\widetilde{C}}_2$ can be obtained easily from Eqs (4.2) and (4.3).

Now for the Bremsstrahlung contribution, the origin of the distributions $\delta(1-w)$, $\frac{1}{(1-w)_+}$ and $\left[\frac{\ln(1-w)}{1-w}\right]_+$ defined in Eq. (2.7) is a factor like $(1-w)^{-1-\alpha\varepsilon}$ (α is some real number) [2]. Thus, one can write the most general expression of the singular Bremsstrahlung contributions as:

$$f^{\rm Bremss}_{\rm sing} = D^\varepsilon\Big[\frac{A}{\varepsilon} + B + \varepsilon C\Big](1-w)^{-1-\alpha\varepsilon}. \tag{4.4}$$

The factor D^ε comes from the 3-body phase space (Appendix A). Using the expansion (up to the $O(\varepsilon)$):

$$(1-w)^{-1-\alpha\varepsilon} = -\frac{1}{\alpha\varepsilon}\delta(1-w) + \frac{1}{(1-w)_+} - \alpha\varepsilon\Big[\frac{\ln(1-w)}{1-w}\Big]_+ + O(\varepsilon^2) \tag{4.5}$$

one gets:

$$\begin{aligned} f^{\rm Bremss}_{\rm sing} = &- \frac{A}{\alpha\varepsilon^2}\delta(1-w) - \frac{1}{\alpha\varepsilon}[A\ln D + B]\delta(1-w) \\ &- \frac{1}{\alpha}[A\ln^2 D + B\ln D + C]\delta(1-w) + \frac{A}{\varepsilon}\frac{1}{(1-w)_+} \\ &+ [A\ln D + B\frac{1}{(1-w)_+} - \alpha A\Big[\frac{\ln(1-w)}{1-w}\Big]_+ + O(\varepsilon). \end{aligned} \tag{4.6}$$

Now, if we assume that the virtual contribution for the subprocess $ab \longrightarrow cd$ is known one can write it as:

$$f^{\text{virt}}_{\text{sing}} = F^{\varepsilon}\left[\frac{A'}{\varepsilon^2} + \frac{B'}{\varepsilon} + C'\right]\delta(1-w)\,. \tag{4.7}$$

The factor F^{ε} comes from the 2-body phase space (Appendix A). After straightforward simplifications we obtain:

$$f^{\text{virt}}_{\text{sing}} = F^{\varepsilon}\left[\frac{A'}{\varepsilon^2} + \frac{(A'\ln F + B')}{\varepsilon} + (A'\ln^2 F + B'\ln F + C')\right]\delta(1-w)\,. \tag{4.8}$$

The Bloch–Nordsieck mechanism tells us that the infrared singularity has to be cancelled, *i.e.*:

$$A = \alpha A'\,. \tag{4.9}$$

Moreover, a consequence of the Slavnov–Taylor identities (or BRS invariance) is that the ultraviolet singularity has to be cancelled. This means that after subtracting the contribution coming from the mass singularity (entering the Altarelli–Parisi split function p_{ij}) and proportional to the Dirac distribution $\delta(1-w)$ we obtain:

$$-\frac{1}{\alpha}(A\ln D + B) - \widetilde{C}_1 = -(A'\ln F + B') \tag{4.10}$$

with

$$B = -\alpha(\widetilde{C}_1 - B' - A'\ln F) - A\ln D\,.$$

Now the remaining part is $(-C/\alpha)$ which is proportional to $\delta(1-w)$. This means, it comes from a collinear and/or soft configuration. The contribution from the latter can be calculated easily [2, 6]. It is important to note that, if one wants to get a physical correction, one has to add the functions F_{ab} and/or D_{ab} given in Appendix C [14] instead of the naive choice $F_{ab} = D_{ab} = 0$ [15] because it reduces the large correction terms of kinematical origin, common to all processes involving gluons and should be absorbed into the structure and fragmentation functions.

Now, if from the F_{ab}'s and D_{ab}'s functions we pick up just the terms proportional to $\delta(1-w)$, $\frac{1}{(1-w)_+}$ and $\left[\frac{\ln(1-w)}{1-w}\right]_+$. Thus, from Eqs (4.9), (4.10) and the arguments given above, we can determine exactly the terms C_1, C_2 and C_3.

Finally, from the above considerations one can get all the terms of f_{sing} (see Eq. (2.6)) without doing a full calculation (hundreds of terms) of the Bremsstrahlung contribution. Thus, in order to obtain the singular contribution of a hadronic process of the form $A + B \longrightarrow C + D$ (which dominates) and up to the next to leading order in α_s, one can state the following rules:

1. Calculate the Born term in $d = 4 - 2\varepsilon$ dimension.
2. Determine $\widehat{C}_1$, $\widetilde{C}_1$, $\widetilde{\widetilde{C}}_1$, $\widetilde{C}_2$ and $\widetilde{\widetilde{C}}_2$ from Eqs (4.1)–(4.2) and (4.3).
3. Write two and three bodies phase space in d dimension.
4. Calculate the virtual contribution and write it in the form:

$$F^\varepsilon\left[\frac{A'}{\varepsilon^2} + \frac{B'}{\varepsilon} + C'\right]\delta(1-w)\,.$$

5. Set the Bremsstrahlung contribution in the form:

$$D^\varepsilon\left[\frac{A}{\varepsilon} + B + \varepsilon C\right](1-w)^{-1-\alpha\varepsilon}\,,$$

where

$$A = \alpha A'$$

$$B = -\alpha(\widetilde{C}_1 - B' - A'\ln F) - A\ln D\,.$$

6. Calculate the Bremsstrahlung contribution coming from soft and/or collinear contribution (easy calculation) *i.e.* proportional to the Born term times $\delta(1-w)$.
7. Write the singular part of F_{ab} and D_{ab} as:

$$F_{ab}^{\text{sing}} = \frac{1}{2}\left[\sum_i f_{ia}^{\text{sing}} \otimes \hat{\sigma}^B_{ib\to cd} + \sum_j f_{jb}^{\text{sing}} \otimes \hat{\sigma}^B_{aj\to cd}\right],$$

$$D_{ab}^{\text{sing}} = \frac{1}{2}\left[\sum_i \hat{\sigma}^B_{ab\to id} \otimes d_{ci}^{\text{sing}} + \sum_j \hat{\sigma}^B_{ab\to cj} \otimes d_{dj}^{\text{sing}}\right],$$

where F_{ab}^{sing} and d_{ab}^{sing} have the following form:

$$f_{ab}^{\text{sing}} = \Omega^1_{ab}\delta(1-w) + \Omega^2_{ab}\frac{1}{(1-w)_+} + \Omega^3_{ab}\left[\frac{\ln(1-w)}{1-w}\right]_+,$$

$$d_{ab}^{\text{sing}} = \omega^1_{ab}\delta(1-w) + \omega^2_{ab}\frac{1}{(1-w)_+} + \omega^3_{ab}\left[\frac{\ln(1-w)}{1-w}\right]_+.$$

8. Determine C_1, C_2 and C_3.

Now, to give an idea how one can calculate easily the Bremsstrahlung contribution coming from the soft and/or collinear configuration and proportional to the Born term times $\delta(1-w)$, let us take as an example the process $q\bar{q} \longrightarrow \gamma g$. The Born (Fig. 2(a),(b),(c)) contribution $|M_B|^2$ (see Fig. 2(d)) summed over spins and colors is of the form:

$$|M_B|^2 \sim \frac{T_B(p_1,p_2)}{\left[(q-p_2)^2\right]^2},$$

where $T_B(p_1,p_2)$ is a Born trace. Now, considering the interference term of the Bremsstrahlung amplitudes M_1 and M_2 (unitary graph, Fig. 2(e)), which gives a contribution to HOC, we have

$$M_1 M_2^+ \sim \frac{T_2(p_1,p_2,k)}{(p_1-k)^2(q-p_2)^2(k-p_2)^2(q-p_2+k)^2},$$

where $T_2(p_1,p_2,k)$ is another trace.

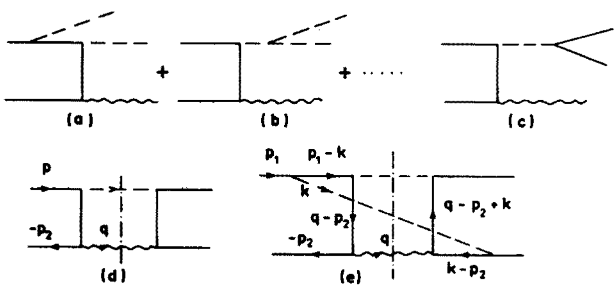

Fig. 2. Unitary graphs contributing to higher order corrections amplitude.

We introduce Sudakov variables as:

$$k = \alpha p_1 + \beta p_2 + l; \quad p_1 \cdot l = p_2 \cdot l = 0; \quad l = (0, \vec{l}_T)$$

so that:

$$(p_1 - k)^2 = -\beta s$$

and

$$(k - p_2)^2 = -\alpha s.$$

Also, we obtain:

$$T_2(p_1,p_2,k) \sim (1-\alpha-\beta) T_B(p_1,p_2).$$

The phase-space of the emitted gluon becomes:

$$\int \frac{d^{d-1}k}{2k_0}\delta_+\left[(p_1+p_2-q-k)^2\right] \sim \int_0^1 d\beta\beta^{-\varepsilon}\delta\left(\beta - \frac{v(1-w)}{1-vw}\right)\int_0^{1-\beta} d\alpha\alpha^{-\varepsilon}$$

notice that for $w \longrightarrow 1$; $\beta \sim 1 - w$. Next, we consider the leading contribution for $\alpha \longrightarrow 0$ (*i.e.* $k \simeq \beta p_2$), and we obtain

$$\frac{d\sigma}{dv dw} \sim \int \frac{d^{n-1}k}{2k_0} \delta_+ \left[(p_1 + p_2 - q - k)^2 \right] M_1 M_2^+$$

$$\sim \int_0^1 d\beta \beta^{-\varepsilon} \delta\Big(\beta - \frac{v(1-w)}{1-vw}\Big) \int_0^{1-\beta} d\alpha \alpha^{-\varepsilon} \frac{T_{\mathrm{B}}(p_1, (1-\beta)p_2)}{\alpha\beta[(q-(1-\beta)p_2)^2]^2}$$

$$\sim \int_0^1 d\beta \beta^{-1-\varepsilon} \delta\Big(\beta - \frac{v(1-w)}{1-vw}\Big) |M_{\mathrm{B}}(p_1, (1-\beta)p_2)|^2 \int_0^{1-\beta} d\alpha \alpha^{-1-\varepsilon} .$$

Notice that $|M_{\mathrm{B}}|^2$ appears with argument p_1, $(1-\beta)p_2$, as it is pertinent to the emission of a gluon with $k \simeq \beta p_2$, thus expressing a factorization property. Now,

$$\int d\alpha \alpha^{-1-\varepsilon} \sim -\frac{1}{\varepsilon}$$

and for

$$w \longrightarrow 1; \quad 1 - \beta = \frac{(1-v)}{(1-vw)} \longrightarrow 1$$

one has:

$$\frac{d\sigma}{dv\, dw} \sim \Big(\frac{v}{1-v}\Big)^{-\varepsilon} (1-w)^{-1-\varepsilon} \Big(-\frac{1}{\varepsilon}\Big) |M_{\mathrm{B}}(p_1, p_2)|^2$$

using Eq. (4.5) and the expression

$$\Big(\frac{v}{1-v}\Big)^{-\varepsilon} = 1 - \varepsilon \ln \Big(\frac{v}{1-v}\Big) + O(\varepsilon^2)$$

one gets for the expression proportional to $\delta(1-w)$

$$\tfrac{1}{2} \ln^2 \frac{v}{1-v} |M_{\mathrm{B}}(p_1, p_2)|^2 .$$

This shows clearly that this contribution is easily obtained.

5. Practical proofs and conclusion

Consider large transverse momenta p_{T} direct photon production and in particular the difference of inclusive cross section

$$\sigma = E \frac{d\sigma}{d^3 p} (\bar{\mathrm{p}}\mathrm{p} \longrightarrow \gamma X) - E \frac{d\sigma}{d^3 p} (\mathrm{pp} \longrightarrow \gamma X) \tag{5.1}$$

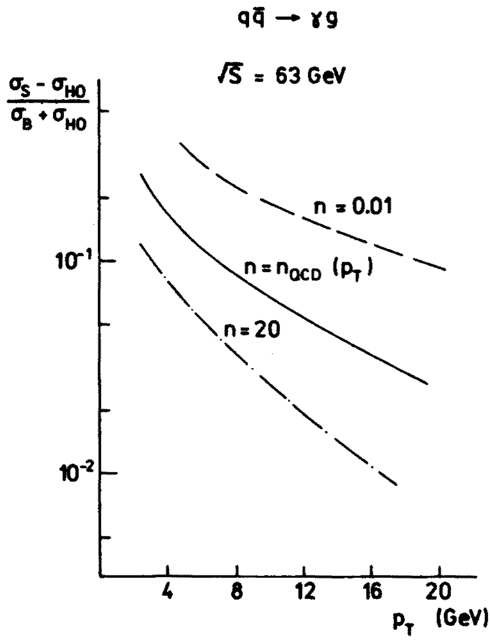

Fig. 3. The ratio $\frac{\sigma_{HO}-\sigma_{sing}}{\sigma_{Born}+\sigma_{HO}}$ for the cross section difference of $\bar{p}p \to \gamma X$ and $pp \to \gamma X$ *versus* P_T at $\sqrt{s} = 63$ GeV and pseudo rapidity $y = 0$. Solid line — results for the d-quark distribution (Duck–Owens set 1). Dashed-dotted line — results with a fictitious distribution of the form $(1-x)^n$, $n = 20$. Dashed line — the same with $n = 0.01$.

dominated by the subprocess $q\bar{q} \longrightarrow \gamma g$.

Now, denoting by σ_{sing} the contribution to the inclusive cross section σ of the part f_{sing} and by σ_{HO} that of the complete f, thus $\sigma_{HO} - \sigma_{sing}$ corresponds to the contribution of $\tilde{f}$. Consider the ratio:

$$\frac{\sigma_{HO} - \sigma_{sing}}{\sigma_{Born} + \sigma_{HO}}. \tag{5.2}$$

Fig. 3 (solid line, denoted by $n = n_{QCD}(p_T)$ [4]) shows that, considered as function of the transverse momenta of the direct photon P_T (here we take $M = p_T$) for fixed s, this ratio is small and decreases with p_T. To test more our ideas, we have carried the ratio (5.2) by writing the structure function in the form:

$$F_{\frac{a}{A}}(x, p_T) = F_{\frac{b}{B}}(x, p_T) = (1-x)^n$$

for the fictitious values $n = 20$ (extremely soft distribution) and $n = 0.01$ (extremely hard). As expected, in the first case (dashed line in Fig. 3) the ratio is significantly smaller than for $n = n_{QCD}(p_T)$; in the second case (long dashed line) it is significantly larger.

Very similar results were obtained for the contribution of the subprocess $qg \longrightarrow \gamma q$ to the inclusive cross section $pp \longrightarrow \gamma + X$ analyzed also in detail (Fig. 4) [4]. The same holds also for the contribution of $\gamma q \longrightarrow \gamma q$ to $\gamma p \longrightarrow \gamma$ (large p_T) + X (Fig. 5). Finally, we got similar results for the contribution of $\gamma\gamma \longrightarrow q\bar{q}$ to $\gamma\gamma \longrightarrow$ hadron (large p_T) + X, involving the fragmentation function $q \longrightarrow$ hadron.

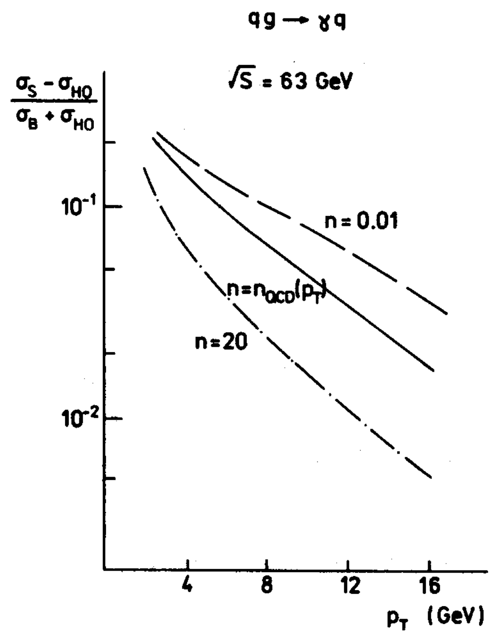

Fig. 4. The ratio $\frac{\sigma_{HO}-\sigma_{sing}}{\sigma_{Born}+\sigma_{HO}}$ for the physical process $pp \to \gamma X$ (contribution of the subprocess $qg \to \gamma q$) *versus* P_T at $\sqrt{s} = 63$ GeV and pseudo rapidity $y = 0$. Solid line — results with a $\tilde{U}$-quark distribution (Duck–Owens set 1). Dashed and dashed-dotted lines — the same as in Fig. 3.

Now, as another example, regarding supersymmetric QCD (SQCD), complete HOC have been determined only for the Drell-Yan type subprocesses [1]:

$$q\bar{\tilde{q}} \longrightarrow \tilde{\gamma}^* \longrightarrow l^+\tilde{l}^- ; \quad \tilde{q}\bar{\tilde{q}} \longrightarrow \gamma^* \longrightarrow l^+l^- ; \tag{5.3}$$

and for the time reversed

$$\tilde{\gamma}^* \longrightarrow q\bar{\tilde{q}} ; \quad e^-e^+ \longrightarrow \gamma^* \longrightarrow \tilde{q}\bar{\tilde{q}} , \tag{5.4}$$

where $\tilde{q}$, $\tilde{\gamma}$ and $\tilde{l}$ are the squark, photino and slepton, respectively. In fact, Ref. [1] considers ultrahigh energies and very large transfers so that all partons (including $\tilde{q}$) can be treated as massless.

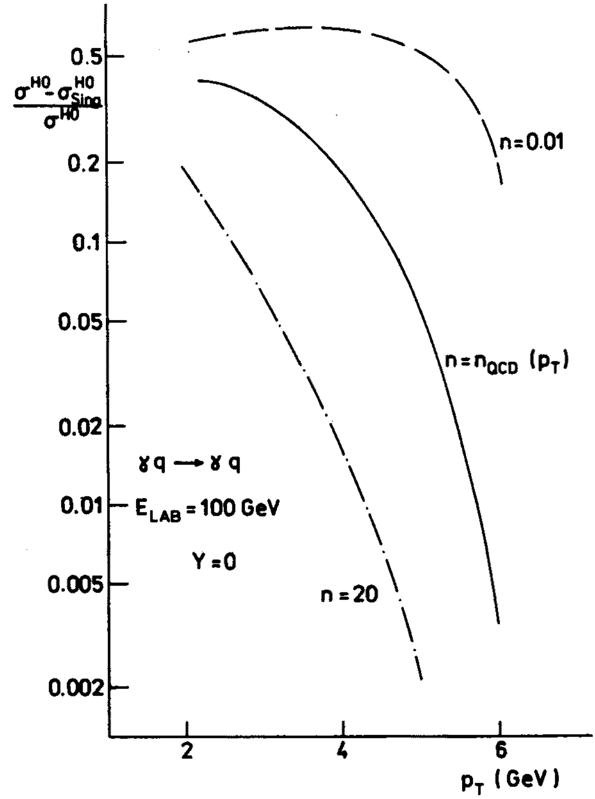

Fig. 5. The ratio $\frac{\sigma_{\rm HO}-\sigma_{\rm sing}}{\sigma_{\rm Born}+\sigma_{\rm HO}}$ for the physical process $\gamma p \to \gamma X$ contribution of the subprocess $\gamma q \to \gamma q$) *versus* P_T at $E_{\rm lab} = 100$ GeV and pseudo rapidity $y = 0$. Solid line — results with a U-quark distribution (Duck–Owens set 1). Dashed and dashed-dotted lines — the same as in Fig. 3.

Since we are interested in processes involving structure (or fragmentation) functions, we consider the two subprocesses (5.3). We denote by Q the four-momentum of $\tilde\gamma^*$ or γ^* and introduce the usual scaling variable $\tau = Q^2/s$. Then the contribution of $q\bar{\tilde q} \to l^+\tilde l^-$ to the inclusive cross section of the physical process $A + B \to l^+ + \tilde l^- + X$ (or of $\tilde q\bar{\tilde q} \to l^+\tilde l^-$ to $A + B \to l^+ + \tilde l^- + X$) can be written as:

$$\sigma = \frac{d\sigma}{dQ^2} \sim \int \frac{dx_a dx_b}{x_a x_b} F_{\frac{a}{A}}(x_a) F_{\frac{b}{B}}(x_b) \Big\{ \delta\Big(1 - \frac{\tau}{x_a x_b}\Big) + \frac{\alpha_s(Q^2)}{\pi} f\Big(\frac{\tau}{x_a x_b}\Big) \theta\Big(1 - \frac{\tau}{x_a x_b}\Big) \Big\}, \tag{5.5}$$

where the term proportional to $\delta \frac{1-\tau}{x_a x_b}$ is the Born term and f stands for the HOC.

The form of f is very similar for the two processes (5.3) [1] and here we present expressions only for the first. One can write:

$$f(\tau) = f_{\text{sing}}(\tau) + \tilde{f}(\tau)\,, \tag{5.6}$$

where

$$f_{\text{sing}}(\tau) = \frac{1}{2} C_F \left\{ \frac{2}{3}\pi^2 \delta(1-\tau) + 8\frac{1}{(1-\tau)_+} + 8\left[\frac{\ln(1-\tau)}{1-\tau}\right]_+ \right\}$$

and

$$\tilde{f}(\tau) = C_F \left\{ \tfrac{3}{2}(1-\tau) - 4 + (1-\tau)\ln(1-\tau) - 4\ln(1-\tau) \right\}. \tag{5.7}$$

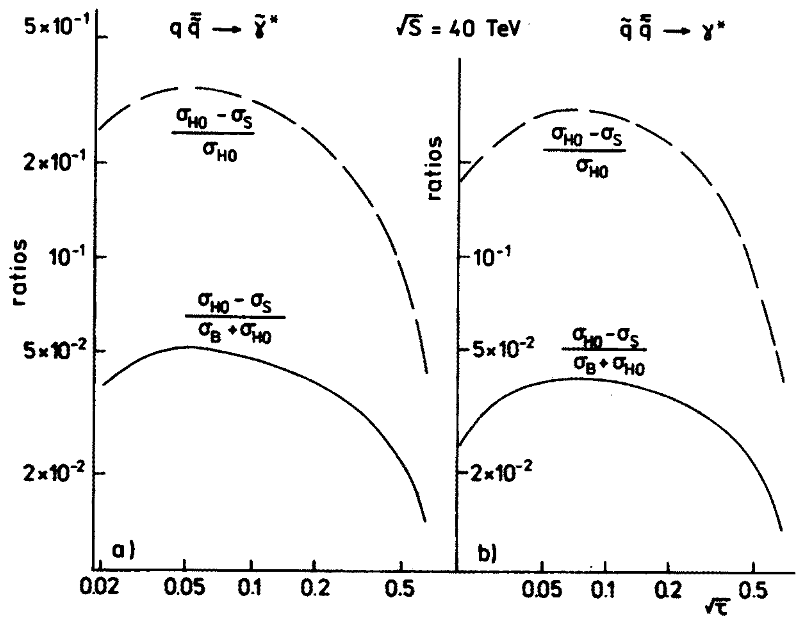

Fig. 6. The ratio $\frac{\sigma_{HO} - \sigma_{sing}}{\sigma_{Born} + \sigma_{HO}}$ at $\sqrt{s} = 40$ TeV, using for the quark and Squark distributions the simple form $(1-x)^n$. (a) — the contribution of the subprocess $q\bar{\tilde{q}} \longrightarrow \tilde{\gamma}^*$ to the physical process $pp \longrightarrow l^+l^- + X$; solid line — results with $n_q = 3$ and $n_{\tilde{q}} = 0.01$. (b) — the contribution of the subprocess to $\tilde{q}\bar{\tilde{q}} \longrightarrow \gamma^*$ to the physical process $pp \longrightarrow l^+l^- + X$; solid line is the result with $n_{\tilde{q}} = 7$. Dashed line is the same as in (a).

As in the first example, we denote by σ_{sing} the contribution of f_{sing} to the inclusive cross section σ, and by σ_{HO} that of the complete f; thus

$\sigma_{\rm HO} - \sigma_{\rm sing}$ is the contribution of $\tilde{f}$. For our SQCD processes we present results for both the ratios:

$$\frac{\sigma_{\rm HO} - \sigma_{\rm sing}}{\sigma_{\rm HO}}; \quad \frac{\sigma_{\rm HO} - \sigma_{\rm sing}}{\sigma_{\rm Born} + \sigma_{\rm HO}}. \tag{5.8}$$

We consider the physical processes $pp \longrightarrow \tilde{l}^+\tilde{l}^- + X$ and $pp \longrightarrow l^+l^- + X$ at $\sqrt{s} = 40$ TeV. For $F_{\frac{a}{A}}$ and $F_{\frac{b}{B}}$, we use simple forms $\sim (1-x)^n$ and take for the quark $n_q = 3$ and for the Spuarks $n_{\tilde{q}} = 7$; this is sufficient, since in particular we are interested in ratios of cross sections. To determine $\alpha_s(Q^2)$ we use six flavours and take the SQCD parameter $\Lambda = 0.2$ GeV.

Figs 6a,b present our results. We see that both ratios (5.8) are small, in particular the second; for $\sqrt{\tau} \geq 0.05$ they decrease with τ.

As for QCD, the explanation lies in the behaviour $\sim (1-x)^n$ of the structure functions. The contributions to $\sigma_{\rm HO}$ arise by integrating in the region $x_a x_b \geq \tau$ with x_a, $x_b \leq 1$. The terms $f_{\rm sing}(\tau)$ give their contribution at our mainly near the boundary $x_a x_b = \tau$, and they dominate the HOC.

We conclude that in order to determine the HOC coming from the singular terms (which dominate), one has just to calculate the Born term and the virtual diagrams contribution. In other words the Bremsstrahlung contribution is easily obtained since its dominant part result from the soft and/or collinear configuration of the particles at the initial or final state.

Thus, knowing the Born and virtual terms, one has just to follow the rules given in Section 4 and get the final contribution. More details and applications of the above prescription will be given in our next paper [16].

We are grateful to Professors M. Le Bellac and G. Clement for fruitful discussions. We would like to thank Professors J.P. Provost and G. Valee for the hospitality extended to us during the time we have spent at the Laboratoire de Physique Theorique de Nice.

APPENDIX A

Let us consider the inclusive physical process $A + B \longrightarrow C + X$ (X means anything). Here we are dealing with a case of two structure functions and one fragmentation function. Denote by x_a, x_b the momentum fraction of the parton a (resp. b) inside the hadron A (resp. B and x_c the momentum fraction of the hadron C from the parton momentum c *i.e.*: $P_a = x_a P_a$, $P_b = x_b P_b$, $P_c = \frac{P_C}{x_c}$. Using the subprocess Mandelstam variables (with massless partons) one gets:

$$\hat{s} + \hat{t} + \hat{u} = x_a x_b s + x_a t + \frac{x_b}{x_c} u, \tag{A.1}$$

where s, t and u are the Mandelstam variables at the hadronic level, defining the variables v, w, V and W such as:

$$\frac{\hat{t}}{\hat{s}} = -(1-v), \quad \frac{\hat{u}}{\hat{s}} = -vw, \quad \frac{t}{s} = -(1-V) \text{ and } \frac{u}{s} = VW$$

we obtain:

$$\hat{s} + \hat{t} + \hat{u} = \begin{cases} s\left[x_a x_b - x_a(1-V) - \frac{x_b}{x_c} VW\right] \\ \text{or} \\ x_a x_b v(1-w) \end{cases} . \tag{A.2}$$

Then, using the property of theta function one gets:

$$x_a \geq \frac{x_b}{x_c} VW[x_b - (1-V)]^{-1} .$$

The minimum value of x_a is obtained when the above equality holds. This means that:

$$\hat{s} + \hat{t} + \hat{u} = 0$$

and Eq. (A.2) implies that $w = 0$ (*i.e.* soft and/or collinear configuration).

APPENDIX B

(B.1) 3-body phase space

Consider the subprocess $a + b \longrightarrow c + d + e$, and the momenta $P_1 = P_a$, $P_2 = P_b$, $P_3 = P_c$, $P_4 = P_d$ and $P_k = P_e$. Introduce the variables $P_{4k} = P_4 + k$ and $s_2 = P_{4k}^2$. Then, the 3-body phase space in $d = 4 - 2\varepsilon$ dimension is given by:

$$(ps)_3 = \int \frac{1}{(2\pi^{5-4\varepsilon}} ds_2 d^d p_3 d^d p_{4k} \delta^+(p_3^2) \delta(p_{4k}^2 - s_2)$$
$$\delta^d(p_1 + p_2 - p_3 - p_{4k}) d^d k d^d p_4 \delta^+(k^2) \delta^+(p_4^2) \delta^d(p_{4k} - p_4 - k)$$

the notation $\delta^+(x)$ means $x > 0$. Working in the rest system of the $P_4 + k$ we orient the vectors P_1, P_2 and P_3 so that they lie in the plane of the d-th and $(d-1)$-th components of the momentum. Thus we have:

$$P_4 = \frac{\sqrt{s_2}}{2}(1, \ldots, \cos\theta_2 \sin\theta_1, \ \cos\theta_1) ,$$
$$k = \frac{\sqrt{s_2}}{2}(1, \ldots, \cos\theta_2 \sin\theta_1, \ -\cos\theta_2) ,$$

where the dots indicate "*d*-3" unspecified momentum which can be integrated over. Defining: $\hat{s} = (P_1 + P_2)^2$, $\hat{t} = (P_1 - P_3)^2$ and $\hat{u} = s_2 - \hat{s} - \hat{t}$ so we obtain:

$$(ps)_3 = \frac{\hat{s}}{2^8\pi^4\Gamma(1-2\varepsilon)}\left(\frac{4\pi}{\hat{s}}\right)^{2\varepsilon}\int_0^1 dv\int_0^1 dw v(1-v)^{-\varepsilon}(1-w)^{-\varepsilon}v^{-2\varepsilon}w^{-\varepsilon}$$
$$\int_0^\pi d\theta_2 \sin^{-2\varepsilon}\theta_2 \int_0^\pi d\theta_1 \sin^{1-2\varepsilon}\theta_1 \,.$$

(B.2) 2-body phase space

Consider the subprocess $ab \longrightarrow cd$. Then, the virtual 2-body phase space is given by:

$$(ps)_2 = \frac{\hat{s}}{2^7\pi\Gamma(1-\varepsilon)}\left(\frac{4\pi}{\hat{s}}\right)^{\varepsilon}\int_0^1 dv\int_0^1 dw v^{-\varepsilon}(1-v)^{-\varepsilon}\delta(1-w)\,.$$

APPENDIX C

For the physical convention of Ref. [14] we have:

$$f_{gg}(x) = 2N_c\left\{x\left[\frac{\ln(1-x)}{1-x}\right]_+ - x\frac{\ln x}{1-x} + \left[\frac{5N_f}{24N_c} - \frac{\pi^2}{6} - \frac{1}{2}\right]\delta(1-x)\right\},$$
$$f_{qg}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]\ln\left(\frac{1-x}{x}\right),$$
$$f_{gq}(x) = C_f\left[\frac{1+(1-x)^2}{x}\ln\left(\frac{1-x}{x}\right) - \frac{4}{3}\right],$$
$$f_{qq}(x) = C_f\left\{(1+x)^2\left[\frac{\ln(1-x)}{1-x}\right]_+ - \frac{3}{2}\frac{1}{(1-x)_+} - \frac{1+x^2}{1-x}\ln x + 3 + 2x - \left(\frac{9}{2} + \frac{\pi^2}{3}\right)\delta(1-x)\right\},$$

$$d_{\mathrm{gg}}(x) = 2N_c\left\{x\left[\frac{\ln(1-x)}{1-x}\right]_+ + 2x\frac{\ln x}{1-x} + \left[\frac{7N_f}{16N_c} + \frac{\pi^2}{3} - \frac{17}{5}\right]\delta(1-x)\right\},$$

$$d_{\mathrm{qg}}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]\ln\left[x^2(1-x)\right],$$

$$d_{\mathrm{gq}}(x) = C_f\left\{\frac{1+(1-x)^2}{x}\ln\left[x^2(1-x)\right] - 2\right\},$$

$$d_{\mathrm{qq}}(x) = C_f\left\{(1+x)^2\left[\frac{\ln(1-x)}{1-x}\right]_+ + 2\frac{1+x^2}{1-x}\ln x - \frac{3}{2}\frac{1}{(1-x)_+} + \frac{3}{2}(1-x) - \left(\frac{9}{2} - \frac{2\pi^2}{3}\right)\delta(1-x)\right\}.$$

Now, for a subprocess $ab \longrightarrow cd$ (a, b, c and d are partons) we have:

$$F_{ab} = \frac{1}{2}\left[\sum_i f_{ia} \otimes \hat{\sigma}^B_{ib\to cd} + \sum_j f_{jb} \otimes \hat{\sigma}^B_{aj\to cd}\right],$$

$$D_{ab} = \frac{1}{2}\left[\sum_i \hat{\sigma}^B_{ab\to id} \otimes d_{ci} + \sum_j \hat{\sigma}^B_{ab\to cj} \otimes d_{dj}\right],$$

where $\hat{\sigma}^B$ is the Born cross section.

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