HIGGS MECHANISM AS THE MANIFESTATION OF AN INTRINSIC COUPLING OF "VISIBLE" AND "HIDDEN" DEGREES OF FREEDOM: PART TWO*

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In the framework of the model of algebraically composite particles, a new form of the standard-model coupling to gauge fields is introduced, where the "visible" chirality is always included, but via the total chirality comprising two factors: the "visible" chirality and total "hidden" chirality. For leptons and quarks the latter factor assumes its eigenvalue +1, giving the usual standard-model coupling. In the case of the new coupling there exist, beside three and only three replicas of leptons and quarks, two and only two replicas of Yukawa bosons of all colors and weak flavors defined by the standard-model coupling to gauge bosons.

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The concept of algebraically composite particles was recently introduced to provide an explanation to the puzzling phenomenon of three replicas of leptons and quarks [1]. It was also shown that this concept may lead to two replicas of a Higgs scalar and/or a Higgs pseudoscalar (more generally, to two replicas of color-singlet and color-triplet Yukawa bosons with spin/parity $0^{\pm}$ and $1^{\pm}$ and electric charges $0, -1$ and $\frac{2}{3}, -\frac{1}{3}$) [2].

The argument was based on the sequence $N = 1, 2, 3, \ldots$ of Dirac-type equations

$$[\Gamma_1 \cdot (p - gA) - M]\,\psi = 0\,, \tag{1}$$

where the matrices

$$\Gamma_1^{\mu} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \gamma_i^{\mu}\,, \tag{2}$$

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defined by the sequence $N = 1, 2, 3, \ldots$ of Clifford algebras

$$\{\gamma_i^\mu, \gamma_j^\nu\} = 2\delta_{ij} g^{\mu\nu} \qquad (i, j = 1, 2, \ldots, N), \tag{3}$$

form the sequence $N = 1, 2, 3, \ldots$ of composite representations for the Dirac algebra

$$\{\Gamma_1^\mu, \Gamma_1^\nu\} = 2g^{\mu\nu}. \tag{4}$$

Except for $N = 1$, the representations (2) are reducible, since for any $N > 1$ one can introduce, beside the combination Γ_1^μ, $N - 1$ other Jacobi-type independent combinations $\Gamma_2^\mu, \ldots, \Gamma_N^\mu$ of γ_i^μ, *viz.*

$$\Gamma_2^\mu = \frac{1}{\sqrt{2}} \left(\gamma_1^\mu - \gamma_2^\mu\right), \; \Gamma_3^\mu = \frac{1}{\sqrt{6}} \left(\gamma_1^\mu + \gamma_2^\mu - 2\gamma_3^\mu\right), \ldots, \tag{5}$$

such that

$$\{\Gamma_i^\mu, \Gamma_j^\nu\} = 2\delta_{ij} g^{\mu\nu} \qquad (i, j = 1, 2, \ldots, N). \tag{6}$$

Hence, one may always represent Γ_1^μ in the form

$$\Gamma_1^\mu = \gamma^\mu \otimes \underbrace{\mathbf{1} \otimes \cdots \otimes \mathbf{1}}_{(N-1)\,\text{times}}. \tag{7}$$

For instance,

$$\Gamma_1^\mu = \gamma^\mu \otimes \mathbf{1} \otimes \mathbf{1}, \; \Gamma_2^\mu = \gamma^5 \otimes i\gamma^5\gamma^\mu \otimes \mathbf{1}, \; \Gamma_3^\mu = \gamma^5 \otimes \gamma^5 \otimes \gamma^\mu \tag{8}$$

when $N = 3$. Here, $\gamma^\mu, \mathbf{1}$ and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ stand for the usual 4×4 Dirac matrices. In the representation (7) the Dirac-type equations (1) read

$$[\gamma \cdot (p - g\,A) - M]_{\alpha_1\beta_1} \psi_{\beta_1\alpha_2\ldots\alpha_N} = 0, \tag{9}$$

if the mass term M does not depend on the matrices Γ_i^μ (then $M_{\alpha_1\beta_1} = \delta_{\alpha_1\beta_1} M$). In Eqs. (1) and (9), A_μ symbolize the standard-model gauge fields including the coupling matrices of the SU(3)⊗ SU(2)⊗U(1) group i.e., λ's, τ's, Y and $\Gamma_1^5 = i\Gamma_1^0\Gamma_1^1\Gamma_1^2\Gamma_1^3$ (in Eq. (9), the matrices $\Gamma_2^5, \ldots, \Gamma_N^5$ are assumed not to be present in A_μ).

The Dirac-type equations (1) lead to the conserved, fully-relativistic Dirac-type currents

$$j_D^\mu = \eta_N \psi^+ \Gamma_1^0 \Gamma_2^0 \cdots \Gamma_N^0 \Gamma_1^\mu \psi, \tag{10}$$

but only for N odd (the phase factors η_N make $\eta_N \Gamma_2^0 \cdots \Gamma_N^0$ Hermitian). If A_μ did not include Γ_1^5, what might be possible only for N even (corresponding to integer total spins), the conserved, fully-relativistic Klein–Gordon-type currents

$$j_{KG}^\mu = \eta_{N+1} \psi^+ \Gamma_1^0 \Gamma_2^0 \cdots \Gamma_N^0 \left(i \overleftrightarrow{\partial^\mu} - gA^\mu\right) \psi \tag{11}$$

with $\overleftrightarrow{\partial}^{\mu} = \frac{1}{2}(\partial^{\mu} - \overleftarrow{\partial}^{\mu})$ would exist. It would be so, since then Eqs. (1) multiplied by the operators $\Gamma_1 \cdot (p - g\,A) + M$ would give the second-order equations of the form

$$\left\{ (p - gA)^2 - M^2 - i\,g\tfrac{1}{4}\left[\Gamma_1^{\mu}, \Gamma_1^{\nu}\right] F_{\mu\nu} \right\}\psi = 0\,, \tag{12}$$

implying the conserved currents (11) (Eq. (11) in Ref. [2] is valid in this case).

Under the assumption of Pauli exclusion principle requiring that $\psi_{\alpha_1\alpha_2\ldots\alpha_N}$ (*cf. e.g.* Eq. (9)) are fully antisymmetric with respect to the "hidden" bispinor indices $\alpha_2, \ldots, \alpha_N$, it was shown that for N odd there exist three and only three fully-relativistic, probabilistically interpretable states defined by the Dirac-type equations (1) [1]. They correspond to $N = 1, 3, 5$ and all carry total spin $\frac{1}{2}$. In fact, they have the form [1]:

$$\psi^{(1)}_{\alpha_1} \equiv \psi_{\alpha_1}\,, \tag{13}$$

$$\psi^{(3)}_{\alpha_1} \equiv \tfrac{1}{4}\left(C^{-1}\gamma^5\right)_{\alpha_2\alpha_3}\psi_{\alpha_1\alpha_2\alpha_3} = \psi_{\alpha_1\,12}\,, \tag{14}$$

$$\psi^{(5)}_{\alpha_1} \equiv \tfrac{1}{24}\varepsilon_{\alpha_2\alpha_3\alpha_4\alpha_5}\psi_{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5} = \psi_{\alpha_1\,1234}\,, \tag{15}$$

where C is the usual charge-conjugation matrix and the "hidden" bispinor indices are defined in the chiral representation: $\gamma^5 = \mathrm{diag}(1, 1, -1, -1)$ (note that in Eq. (14) the only nonzero components are $\psi_{\alpha_1\,12} = -\psi_{\alpha_1\,21} = \psi_{\alpha_1\,34} = -\psi_{\alpha_1\,43}$). All these three states are eigenstates of the total "hidden" intrinsic parity $\eta_N \Gamma_2^0 \cdots \Gamma_N^0$ (for $N = 1, 3, 5$) with the eigenvalue $+1$, as well as of the total "hidden" chirality $\Gamma_2^5 \cdots \Gamma_N^5$ also with the eigenvalue $+1$. The states (13), (14) and (15) were interpreted in a rather natural way as three replicas of a fundamental fermion (lepton or quark) for any given color and weak flavor defined by the standard-model coupling to gauge bosons [1].

Now, let us introduce a new form of the standard-model coupling, where the gauge fields A_μ always include Γ_1^5, but via the total chirality $\Gamma_1^5 \Gamma_2^5 \cdots \Gamma_N^5$. Note that for three replicas of leptons and quarks corresponding to $N = 1, 3, 5$ such a coupling reduces to the usual standard-model coupling, since then $\Gamma_2^5 \cdots \Gamma_N^5$ assumes the eigenvalue $+1$.

The Dirac-type equations (1) multiplied by the operators $\Gamma_1 \cdot (p - g\widetilde{A}) + M$, where $A_\mu \to \widetilde{A}_\mu$ with $\Gamma_1^5 \to -\Gamma_1^5$, give generally the second-order equations of the form

$$\left\{ (p - gA)^2 - M^2 - i\,g\tfrac{1}{4}\left[\Gamma_1^{\mu}, \Gamma_1^{\nu}\right] F_{\mu\nu} - g\,M \Gamma_1^{\mu}(A_\mu - \widetilde{A}_\mu) \right\}\psi = 0. \tag{16}$$

In the case of our new coupling, where $A_\mu \to \widetilde{A}_\mu$ with $\Gamma_1^5 \Gamma_2^5 \cdots \Gamma_N^5 \to -\Gamma_1^5 \Gamma_2^5 \cdots \Gamma_N^5$, it is not difficult to show that Eqs. (16) imply the conserved, fully-relativistic Klein–Gordon-type currents (11), but only for N even. The conserved, fully-relativistic Dirac-type currents (10) still exist in consequence of Eqs. (1), but only for N odd.

The density j^0_{KG} of the Klein–Gordon-type current (11), relevant for N even, is obviously not positive-definite, in a similar way as the density of usual Klein–Gordon current related to the familiar Klein–Gordon equation.

Concluding, in the case of our new coupling there exist two and only two replicas of color-singlet and color-triplet Yukawa bosons with spin/parity $0^\pm$ and $1^\pm$ and electric charges $0, -1$ and $\frac{2}{3}, -\frac{1}{3}$ (two possible replicas of a Higgs scalar and/or a Higgs pseudoscalar are among them). These replicas correspond to $N = 2, 4$ and have the form [2]

$$\psi^{(2)}_{\alpha_1\alpha_2} \equiv \psi_{\alpha_1\alpha_2} \,, \tag{17}$$

$$\psi^{(4)}_{\alpha_1\alpha_2} \equiv \frac{1}{6}\varepsilon_{\beta_2\beta_3\beta_4\alpha_2}\psi_{\alpha_1\beta_2\beta_3\beta_4} \,. \tag{18}$$

Here, on the rhs, our Pauli exclusion principle does not apply to the bispinor indices of different sorts (i.e., "visible" α_1 and "hidden" α_2). Note that $\widehat{\psi}^{(2,4)}_{\alpha_1\alpha_1}$ are scalars, where $\widehat{\psi}^{(2,4)}_{\alpha_1\alpha_2} = (C^{-1}\gamma^5)_{\beta_2\alpha_2}\psi^{(2,4)}_{\alpha_1\beta_2}$ (in Eq. (14) of Ref. [2] there should stand these $\widehat{\psi}^{(2,4)}_{\alpha_1\alpha_2}$).

The double-bispinors (17) and (18), transformed to $\widehat{\psi}^{(2,4)}_{\alpha_1\alpha_2} = (C^{-1}\gamma^5)_{\beta_2\alpha_2}\psi^{(2,4)}_{\alpha_1\beta_2}$, can be expanded in terms of 16 spin-(0/1) Yukawa fields,

$$\begin{aligned}\widehat{\psi}^{(2,4)}_{\alpha_1\alpha_2} =& \delta_{\alpha_1\alpha_2}S^{(2,4)} + \gamma^5_{\alpha_1\alpha_2}P^{(2,4)} + \gamma^\mu_{\alpha_1\alpha_2}V^{(2,4)}_\mu \\ &+ \left(\gamma^\mu\gamma^5\right)_{\alpha_1\alpha_2}A^{(2,4)}_\mu + i\tfrac{1}{4}\left[\gamma^\mu,\gamma^\nu\right]_{\alpha_1\alpha_2}T^{(2,4)}_{\mu\nu} \,,\end{aligned} \tag{19}$$

for any color and weak flavor defined by the standard-model coupling to gauge fields. In particular, there are two replicas $S^{(2,4)1Q}$ of a color-singlet scalar Yukawa boson with electric charges $Q = 0, -1$ (of them, the $N = 2$ and/or $N = 4$ may happen to be realized as Higgs scalars breaking spontaneously the electroweak symmetry by a negative mass squared). But, in particular, there should exist also two replicas $S^{(2,4)3Q}$ of a color-triplet scalar Yukawa boson with electric charges $Q = \frac{2}{3}, -\frac{1}{3}$.

The latter quark-like scalar bosons, being strongly interacting particles, may be easy to produce in (virtual) pairs at the Large Hadron Collider and Superconducting Super Collider. These copiously produced pairs may then rescatter electroweakly into pairs of Higgs bosons, the latter forming longitudinal parts of weak gauge bosons. Thus, there may be, for example, the process intiated by gluon pairs:

$$gg \xrightarrow{\text{strongly}} S^{(2,4)\bar{3}\bar{Q}} S^{(2,4)3Q} \xrightarrow{\text{electroweakly}} S^{(2)1+} S^{(2)1-} \equiv W_L^+ W_L^- , \tag{20}$$

giving a large additional rate for $W_L^+ W_L^-$ production at a hadron supercollider. Such a mechanism for pair production of longitudinally polarized weak intermediate bosons was discussed previously in detail [3,4] on the base of models operating with colored pseudo-Goldstone bosons as *e.g.* colored technipions. In our model, they are replaced by the algebraically composite quark-like scalar bosons (in particular).

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