# MASS DISTRIBUTION OF NUCLEAR FRAGMENTATION BY A PERCOLATION-EVAPORATION PICTURE

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The lattice in percolative simulations of nuclear fragmentation is shown to have a notable effect on the final result from a nucleon clustering, in the case when the total energy is constrained to be conserved.

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## 1. Introduction

The production of complex fragments in nuclear reactions, at intermediate and high energies, is a difficult and far-from-understood area of current nuclear physics research. The possibility of the nuclei to exhibit critical behaviour in their multifragmentation after having been bombarded by high energy protons, has received renewed interest after Purdue's group paper [1]. It was reported by Hirsch *et al.* that the mass distribution of light-to-medium size fragments seems to obey nicely a power-law; $d\sigma/dA_F \sim A_F^{-\tau}$, ($\tau \approx 2.6$); where $A_F$ is the mass of the fragments and $\tau$ is the apparent exponent.

Unfortunately the power-law type mass dependence of $d\sigma/dA_F$ can be obtained using quite different assumptions [1–5]. This somewhat embarrassing situation has been attributed to the inclusive character of the experimental data (see Ref. [6] for the literature).

In particular, the simplest approach for nuclear fragmentation seems to be given by the percolation theory, which is able to reproduce the main features of the nuclear disassembly with a minimum of physical ingredients. Several (dynamical [7] and non-dynamical [8]) percolation models have been proposed to study nuclear fragmentation. As a matter of fact, the relevance of percolation ideas in nuclear multifragmentation has been demonstrated by

Campi [9] and Bauer [10] through cross relations among various moments of the fragments size distribution and more recently by Ploszajczak and Tucholski [11] who studied intermittency in the fragment distribution in nuclear fragmentation and in the results from percolative simulations.

In percolation models, the nucleus is represented by a three-dimensional structure lattice whose sites are occupied by nucleons. The corresponding percolation theory investigates, the properties of subsets of connected sites called clusters (fragments) as a function of either the probability $W$ of a site to be occupied, or of the probability $q$ of a bond to survive, during nucleus excitation. The lattice size $dl$ is always considered irrelevant by the theory but in the case of energy distribution this is no longer true due to dependence on the space coordinates through the Coulomb potential. As a matter of fact, the inclusion on the lattice size of a linear expansion factor has been shown to be essential in describing the experimental data of the energy spectra of fragments [11, 12].

Since the lattice size $dl$ is considered as an ingredient of these percolation models, whereas it actually is a physically determined quantity by previous history of the system, it seems more reasonable to let the lattice size be an independent parameter of the model. Then, one should examine the dependence of the observables, like the mass distribution on this parameter. This is the goal of this work.

We explain the details of this calculation in Sec. 2, in Sec. 3 the connection is made between the percolation model and thermodynamic quantities. Results and discussions are addressed in Sec. 4.

## 2. The percolation approach

We assume the following picture for a collision between a very energetic light projectile and a target nucleus: In first stage, the projectile and the target merge into a compressed compound system of the volume $V_0$. In this stage, the nuclear assembly gains a significant amount of excitation energy and then a fraction of fast nucleons is ejected. The compressed system expands until it reaches the breakup volume $V_b$ and then becomes unstable with respect to a breakup into several fragments. These fragments may be very excited and then, they decay by evaporation or secondary fragmentation processes into the final states.

Of course, the expansion process should be related to the dynamical evolution of the initially hot and compressed nucleus [7]. However, for simplicity, we will not consider the whole dynamical evolution of the system from its initially compressed and heated-up stage,until the moment it splits into fragments, but we will concentrate on this last disassembly stage, when the expanding hot nuclear system reaches the breakup volume $V_b \propto dl^3$. As

a matter of fact, we consider a sphere of expanding hot nuclear matter, with the excitation energy per nucleon $\varepsilon^*$, the volume $V_b$ and the definite charge, $Z_0$, and mass, $A_0$, numbers. This sphere will split into several fragments $(Z_F, A_F)$ giving a certain partition of the system. In order to describe the nuclear fragmentation process, such as pictured above, we adopt the percolation plus evaporation calculation of Ref. [12], which we summarize in the following.

Let us consider a finite sphere containing $A_0$ sites arranged on a simple cubic lattice of size $dl$. A certain fraction of sites $A = WA_0$ is occupied by nucleus, where $W$ is the concentration probability, which can be related to $\varepsilon^*$ [5]. The subgroups formed by those next neighboring sites are the primordial fragments (clusters) which are considered to be formed at normal density $n_0 = 0.153$ fm$^{-3}$. This means the lattice size $dl$ provides us with the breakup volume at the disassembly stage but not with the density of each fragment.

There are two parameters in this model: the site concentration probability $W$ and the lattice size $dl$. They are expected to contain the two most essential elements of the fragmentation process, say, the locality of the breaking process involved in the cluster formation and the short range of the nuclear interaction.

We adopted the following Monte Carlo procedure in order to get a set $\{N_{A_F,Z_F}\}$ of clusters(=partition) of size $A_F$ and charge $Z_F$, from the initial system with $A_0$ nucleons:

1) Allocate randomly $A$ nucleons into $A_0$ sites,
2) Look for clusters and collect them according to their size $A_F$,
3) Assign a charge $q_i \varepsilon \{0, 1\}$ to each nucleon randomly obeying conservation of the total charge: $\sum_i q_i = Z_0$ (we have assumed also, for simplicity, symmetric target $Z_0 = A_0/2$ and symmetric clusters $Z_F = A_F/2$).

The clusters obtained by the percolation theory, in general, are very dilute and ramified and are associated with excited and unstable fragments [13] which one should permit to evaporate. Since such a percolation theory description of nuclear fragmentation is based only on statistical and geometrical ideas, in order to describe evaporation processes, additional information like the temperature of the nuclear system is required. The connection between thermodynamic quantities and percolative simulations was firstly investigated by the Copenhagen group [14], but there, the lattice size was not considered. They assumed that the breakup volume arises only by ejecting nucleons from the target nucleus having normal nuclear matter density $n_0$. However, as we have already mentioned, the expansion factor is essential in the description of energy spectra experimental data. In what follows we describe a simple way to introduce the expansion factor in the percolation model such that the total energy should be conserved.

## 3. The energy conservation

We have adopted the procedure given in Ref. [3], for constraining all the partition to obey the total energy conservation law. There, Bondorf *et al.* wrote the total energy as:

$$E_{\text{total}} = \frac{3}{5}\frac{Z_0^2 e^2}{R_0} + \sum_{A_F Z_F} N_{A_F,Z_F} E_{A_F,Z_F}$$
$$= E_0{}^{\text{ground}} + \varepsilon^* \sum_{A_F} A_F \,, \tag{1}$$

where $E_0{}^{\text{ground}}$ is the ground state energy and $E_{A_F,Z_F}$, the internal plus translational energy of each fragment. The excitation energy per particle $\varepsilon^*$ may be related to the concentration probability $W$ of the percolation model by [5]

$$\varepsilon^* = \varepsilon_0^* \frac{1-W}{W} \,. \tag{2}$$

In Eq. (2), $\varepsilon_0^*$ is interpreted as the mean gain of excitation energy when one nucleon is ejected from the system.

Next the thermal droplet model and the Wigner–Sitz approximation [3] can be used to express the total energy of each fragment $E_{A_F,Z_F}$ as a function of the temperature $T$ of the partition. Namely,

$$E_{A_F,Z_F} = W_0 A_F + \frac{\gamma(A_F - eZ_F)^2}{A_F} + \frac{T^2}{\varepsilon_0} A_F + \frac{3}{2}T$$
$$+ \beta_0 A_F{}^{\frac{2}{3}} \left\{ \left( \frac{T_c^2 - T^2}{T_c^2 + T^2} \right)^{\frac{5}{4}} \left( 1 + \frac{5\left(\frac{T}{T_c}\right)^2}{1 - \left(\frac{T}{T_c}\right)^4} \right) - 1 \right\}$$
$$+ \frac{3}{5}\frac{Z^2 e^2}{R_{A_F,Z_F}} \left( 1 - (1+\chi)^{-\frac{1}{3}} \right) , \tag{3}$$

where $W_0$, $\beta_0$, $\varepsilon_0$, $T_c$ and $\gamma$ have the usual meaning as in Re. [3]. $\chi$ is the factor of expansion and it is related to the initial volume $V_0$ and to the expanding volume $V_b$ by, $V_b = (1+\chi)V_0$. In terms of the percolative parameters one gets $\chi = n_0 dl^3 - 1$. Eq. (3) yields when combined with Eq. (1) and Eq. (2) the temperature $T$ as a function of $dl$. In the following, the results of our calculation are presented.

## 4. Results and discussions

Let us first present the results from a Monte Carlo simulation of 2,000 runs, showing the differences between standard percolation (SP), percolation + energy conservation (PEC) and percolation + evaporation (PEV). The results for $\langle M \rangle$ and $\tau$, at the "critical" probability are summarized in Table I. The "critical" probabilities are 0.41, 0.4 and 0.39 for $A_0 = 57$, 81 and 123, respectively.

TABLE I

| | SP | | | PEC | | | PEV | | |
|---|---|---|---|---|---|---|---|---|---|
| $A_0$ | 57.0 | 81.0 | 123.0 | 57.0 | 81.0 | 123.0 | 57.0 | 81.0 | 123.0 |
| $\tau$ | 1.9 | 1.7 | 1.5 | 1.9 | 1.8 | 2.0 | 2.1 | 2.6 | 2.6 |
| $\langle M \rangle$ | 1.5 | 1.5 | 2.5 | 1.5 | 1.5 | 1.7 | 2.4 | 3.0 | 3.8 |

It should be noted that inclusive experimental results yield $\tau \approx 2.0$ for $p$+$^{58}$Ni [15], $\tau \approx 2.6$ for both $p$+$^{87}$Kr and for $p$+$^{135}$Xe [1] reactions. This means that the PEV calculation gives a value of $\tau$ close to the experimental data. Also it seems that the "critical" exponents are more sensitive to the energy conservation and the evaporation process in the case of bigger systems.

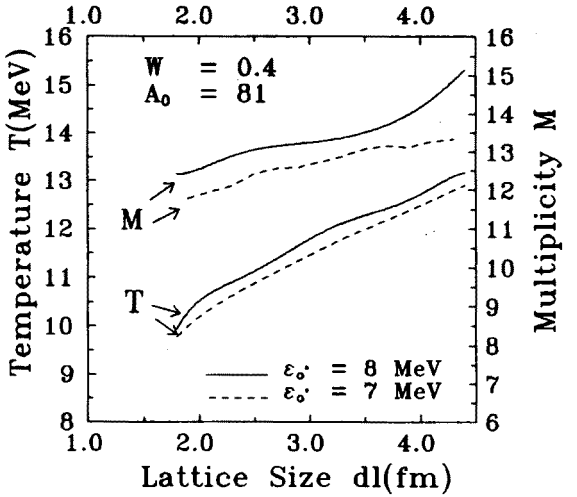


Fig. 1. The temperature $T$ (left-hand scale) and the multiplicity of clusters $M$ (right-hand scale) are plotted against the lattice size $dl$ for two different values of the excitation parameter $\varepsilon_0^*$. The *critical* probability $W$ was taken as 0.4.

In Fig. 1 we plot the temperature $T$ against $dl$ for two different values of the excitation parameter $\varepsilon_0^*$ for a system with $A_0 = 81$ nucleons. The "critical" probability $W$ is assumed to be equal 0.4 for this finite system of particles [12] but may be different if another lattice structure is used [13, 17]. It is clear that the temperature $T$ increases with $dl$ in both cases: $\varepsilon_0^* = 8$ MeV (solid line) and $\varepsilon_0^* = 7$ MeV (dashed line). This is a consequence of the conservation of total energy (Eq. (1)). As a matter of fact, $E_{\text{total}}$ was taken the same in each configuration, so the temperature increases with decreasing Coulomb energy. Similar behaviour is displayed by the corresponding multiplicity $M$ (scale on the right). The clusters at higher temperature evaporate more particles before reaching their ground state. The curves of $T$ and $M$ are shifted upward with increasing $\varepsilon_0^*$.

Of course, $dl$ seems to be no longer a free parameter since it is connected with the breakup density. The values of $dl$=1.88 fm$^{-3}$ and 4.1 fm$^{-3}$ correspond, in a cubic lattice calculation, to $n \sim n_0$ and $0.1n_0$, respectively. These two values are expected to provide brackets on the breakup density [2, 3, 11, 16].

As we have already mentioned, it is usually assumed that the mass distributions have no $dl$ dependence. However, as it is shown in Fig. 2, this is only true if one has not taken into account the energy conservation law. The final mass distributions for $W = 0.4$ and two different values of the lattice size, namely $dl = 2.0$ fm and $dl = 4.0$ fm are displayed in Fig. 2. We may note that if $dl$ is short, heavy fragments ($A_F \geq 30$) may be seen. On the other hand if $dl$ is large, there are only fragments with mass lower than $A_F = 30$. This is again an effect of the temperature since it determines not only the multiplicity of each partition but also the final mass of each cluster. It was assumed that $\varepsilon_0^* = 8$ MeV. Also, the fragment mass five is not depleted in the figure because simultaneous two-neutrons decay is not included in our evaporation code.

The apparent exponent $\tau$ is plotted as a function of $dl$ for two different values of $\varepsilon_0^*$ in Fig. 3. It is seen that the dependence of the apparent exponent $\tau$ on the excitation parameter $\varepsilon_0^*$ is a shift up (or down) of $\tau$, but no significant changes in its qualitative behaviour occur. It is interesting to note that the energy conservation has different effects for different domains of lattice size $dl$. Specifically, for large $dl$-values, $\tau$ is bigger than in the other two cases, but for intermediate $dl$-values, it is smaller becoming larger again for small values of $dl$. This can be understood in the light of the behaviour displayed by the mass distributions in the different ranges of $dl$ (*cf.* Fig. 2).

In Fig. 4 the average temperature $T$ of the partition (Fig. 4(a)) and the average total multiplicity $M$ (Fig. 4(b)) are plotted against $dl$. Two new situations are considered, $A_0 = 57$ and $A_0 = 123$, the results for $A_0 = 81$

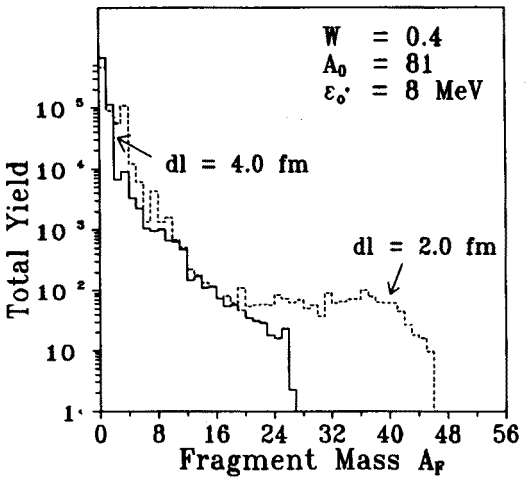


Fig. 2. Yield of fragments by mass, from $A_0 = 81$ and $W = 0.4$ for two different values of $dl$; 2.0 fm (dotted line) and 4.0 fm (solid line). For more details see the text.

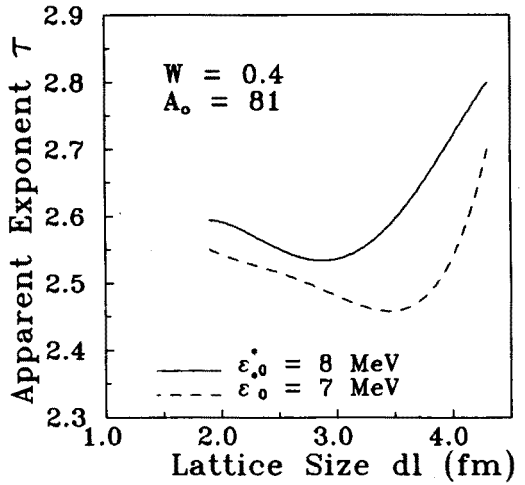


Fig. 3. The apparent exponent $\tau$ in plotted against $dl$. The *critical* probability and the initial mass number is the same as in Fig. 1. $\tau$ is computed after the evaporation process. Two different situations are considered, $\varepsilon_0^* = 8$ MeV (solid line) and $\varepsilon_0^* = 7$ MeV (dashed line).

are again presented for the sake of comparison. We can see that the curves of the temperature exhibit substantially positive slopes. In particular the systems of 81 and 123 sites determine higher $T$ than the smaller system of 57. In this calculation the available initial excitation energy is taken almost

the same irrespective of $A_0$, since the energy needed for cracking becomes smaller with increasing mass, the net energy is larger with increasing mass. Then, the breakup temperature $T$ is smaller in little systems than in the bigger ones. The total multiplicity (Fig. 4(b)) depends strongly on the initial mass number and only slightly on $dl$.

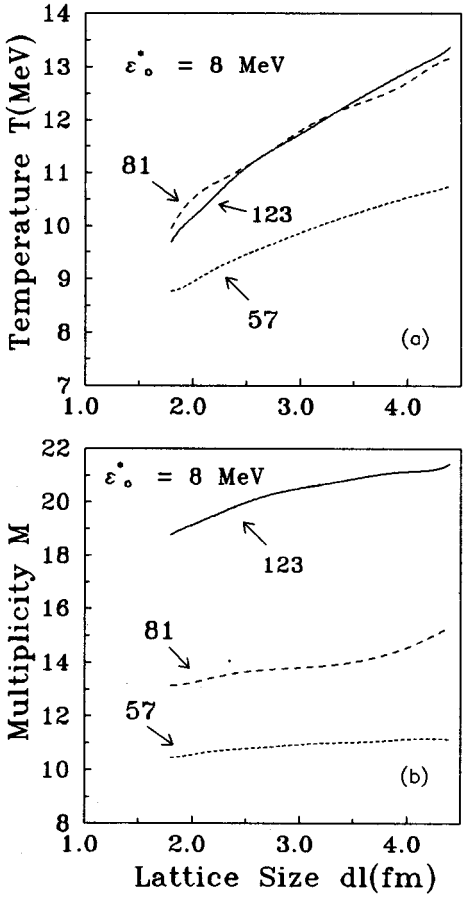


Fig. 4. The temperature $T$ (a) and the total multiplicity $M$ (b) are plotted against $dl$.

For completeness, the energy spectrum of $^{12}$C-fragments is shown in Fig. 5, for two different values of $dl$, namely, $dl$ = 2.0 fm (broken line) and $dl$ = 4.0 fm (dashed line). The results are also compared with the experimental data of Ref. [1] (solid line). We can see that for increasing $dl$ the curves are shifted as discussed in Ref. [12], and the parameter $dl$ = 2.0 fm fits better the experimental data of $^{12}$C energy spectra than $dl$ = 4.0 fm.

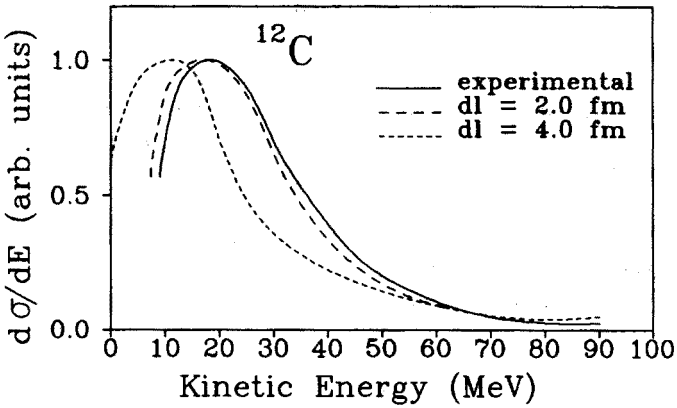


Fig. 5. The energy spectrum of $^{12}$C-fragments of Ref. [1] (solid line) is compared with the prediction of this percolation approach. The broken line represents the energy spectrum for $dl = 2.0$ fm and the dashed line, $dl = 4.0$ fm. The number of runs is 3 000 (see also Ref. [12]).

We would like to warn the reader that our calculations involve several simplifications which should be critically analyzed. In particular, one should note that as our results hardly depend on the Coulomb energy of the partition, and the relaxation of the condition of symmetric nuclei should make the calculation more realistic. Also, we would like to stress that as $\varepsilon^*$ (or $T$) is expected to depend strongly on the concentration probability $W$. The above results may be significantly changed if a different type of lattice (as a fcc lattice [17]) is used. Work on these points is presently in progress.

Summarizing, we have simply investigated the dependence on the lattice size of the main inclusive observables of nuclear multifragmentation, *e.g.*, multiplicity of fragment, the apparent exponent and so on, in the framework of a percolation plus evaporation model. The result of our calculations show that such observables depend strongly on the lattice size $dl$.

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