COSMOLOGICAL BLACK HOLES AND THE FINITENESS OF THE UNIVERSE*

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We have found that trapped surfaces due to spherical inhomogeneities can be formed easier in closed universes than in open Friedmann universes. That opens, in principle at least, a new way to resolve the old standing question concerning the openness of the Universe by performing quasi-local experiments.

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The question whether the Universe in which we live is open or closed, can be reduced to the relation between the average energy density $\hat{\rho}$, Hubble constant $H(t)$ and a scale coefficient $a(t)$:

$$\hat{\rho} = \frac{3}{8\pi}\left[H(t)^2 + \frac{\kappa}{a^2(t)}\right], \tag{1}$$

where κ equals 1, 0 or -1, for a closed, open and flat or open and curved, respectively, Friedman model. (We put $\mathcal{G} = c = 1$.) In closed cosmologies a is a radius of the Universe. Einstein pointed out in his popular book [1] that the conclusive answer of the above problem might be achieved only if the Universe is closed. That is because one can always argue that the real energy density is greater than the observed one. In fact, one of the open cosmological problems of nowadays cosmology is that of "dark matter" that is needed to have the equality $\hat{\rho} = 3H^2/(8\pi) = \rho_{\mathrm{critical}}$, which would mean that the Universe is open and flat. The amount of matter that astronomers observe directly gives rise to about 1 percent of ρ_{critical}. The use of indirect methods, like the virial theorem, improves that estimation to about 15 percents. Dark matter is blamed for the lacking 85 percents that are needed to have a flat and open Universe.

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Below we will propose an approach that might improve the estimation or even solve the problem, in principle at least. The starting remark is that even if the Universe is isotropic and homogeneous in the large (present data give a scale d between 30 to 300 megaparsecs), it still can contain large inhomogeneities, of the size of order d. We call such inhomogeneities as cosmological and trapped surfaces (black holes) that might be formed by them as cosmological trapped surfaces (black holes). Next, there has been found a set of criteria governing the formation of outer trapped surfaces due to some classes of inhomogeneities present in otherwise homogeneous and isotropic universes [2, 3, 4]. Of those, the necessary conditions are relevant for us. Let us describe the conditions that are assumed. We suppose that an initial Cauchy hypersurface t = const is locally inhomogeneous but spherical, with the energy density contrast $\delta\rho = \rho - \hat{\rho}$ where ρ is the actual energy density; we assume also that the gravitational momentum K^{ij} is not changed by the perturbations, which means that matter does nor change (initially) its momentum. Let $\delta M(L) = \int_{V(L)} \delta\rho dV$ be the amount of energy contrast inside a ball of a proper radius L centered at a center of the deviation from spherical symmetry. Let us recall that S is an outer trapped surface if a bundle of light emanating orthogonally to S in the outward direction is focused, at least initially [5]. Thus, trapped surfaces (I omit below the adjective "outer") are measurable, in principle at least. They might be imagined as closed reflectors that have the ability of focusing outgoing light in the full solid angle. Then one can prove the following.

Theorem. Under the above conditions, if a surface S of an area S, proper radius L and a volume V inside S is trapped then

$$\delta M(L) \geq \frac{L}{2} + \frac{HS}{4\pi a^2} - \frac{3\kappa V}{8\pi a^2} \,. \tag{2}$$

The case of $\kappa = 0$ (open flat universes) of (2) has been investigated in [2] while that of $\kappa = 1$ (closed universes) has been done in [3] and that of $\kappa = -1$ in [4].

The above estimation is not sufficient to draw conclusions concerning the closeness of the Universe. In order to know what is the contrast energy δM one has to know the average energy density $\hat{\rho}$, which is not available to us. But still, there are some possibilities to overcome that difficulty. We will investigate two of them.

Let us stress that astronomers attempt to find the scale d of homogeneity and isotropy on the basis of data concerning only visible (directly or indirectly) matter. They make an implicit assumption (call it, for the use of present article, the *scale principle*) that the scale of homogeneity of dark (unobserved) matter is the same as of matter that can be detected.

We will keep that condition and in addition we assume that, for cosmological inhomogeneities, the fraction

$\lambda =$ *(directly visible matter density)/(total matter density)*

is constant on a Cauchy slice. Formula (2) might be written as follows, replacing the third term in it by the use of (1) and using $M = \delta M + \hat{\rho} V$:

$$M \geq \frac{L}{2} + \frac{HS}{4\pi} + \frac{3H^2V}{8\pi}. \tag{3}$$

This formula allows one to bound from below the amount of total mass inside a trapped surface S. The amount of visible matter inside S can be measured directly, thus the fraction λ inside a trapped surface is in principle measurable. Using it, and available to us information about the visible matter density (from observation, astronomers found that the bright matter density constitutes about 1 percent of the critical density ρ_{critical}) one might conclude that the ratio $\rho/\rho_{\text{critical}}$ is equal to $0.01/\lambda$. In the case of sufficiently small λ that would allow one to claim *that the Universe is closed.*

The next case consistent with the *scale principle* is that in which dark matter density is exactly isotropic and homogeneous in regions of size greater than d. Thus the formation of trapped surfaces would require large concentrations of visible matter and δM might be identified with the amount od visible matter inside a surface S. (That is a justified identification, if one takes into account, that the ratio of visible matter to dark matter is probably about one hundredth. One can be more precise and include the fact that outside black holes there is also a small contribution of bright matter, but this lowers the value of excess energy only slightly.) *In such a case, if S is trapped and the amount of bright matter δM satisfies the inequality*

$$\delta M(L) < \frac{L}{2} + \frac{HS}{4\pi a^2}, \tag{4}$$

then the Universe must be closed.

A characteristic feature of the above statements is that they are formulated in terms of quasilocal quantities: Hubble constant H, a proper radius L of a ball, its volume V and area S of a boundary. The total mass inside a ball of a radius L, $M(L)$, might be estimated from below by the use of inequality (3), which allows one to estimate from above the quantity λ. One would need also the mass of bright matter inside a ball of a radius L. The Hubble function H in practise is determined using information coming to us from a small part of the whole Universe. Thus, the above statements are "Machian" [6] in the sense, that quasilocal information allows to infer a global property of the Universe. That is not surprising, since we perform our analysis on the surface of initial data of Einstein equations and these satisfy elliptic equations that are naturally "Machian"; a local change of coefficients of an elliptic equation may change the global behaviour of a solution.

The next interesting observation is that once again, if there is any conclusive statement, then it says that the Universe is closed. Not only our criterion bases on local properties of a Cauchy slice but also the (eventually obtained) knowledge about the finiteness of our Universe is probably local one. The reason is that trapped surfaces are enclosed by event horizons, assuming the validity of cosmic censorship [7]. Any explorer that enters into a black hole in order to get information about its energy content and its proper radius, may be able to draw the conclusion that the Universe is closed, but he will not be able to communicate that to his folks, unless they follow him.

A final remark is that the above results are true assuming spherically symmetric deviations from homogeneity, but I expect they should be valid also for a class of nonspherical lumps in Friedman geometry. The first step in this direction has been made in [8], where a necessary condition has been formulated in the case of conformal flat deviations from an open and flat Friedman model.

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