COLLECTIVE MOTION IN VERY EXCITED NUCLEI

P. F. Bortignon[a,b], F. V. De Blasio[a], W. Cassing[c], M. Tohyama[d] and R. A. Broglia[a,b,e].

a) *Dipartimento di Fisica, Università di Milano, Italy*
b) *I.N.F.N., sezione di Milano, Italy*
c) *Institut für Teoretische Physik, Universität Giessen, Germany*
d) *School of Medicine, Kyorin University, Tokyo, Japan*
e) *The Niels Bohr Institute, Univ. of Copenhagen, Denmark*

1. INTRODUCTION

Giant resonances are the nuclear zero sounds. Once excited it takes a few periods before the oscillation is relaxed. Empirical evidence from excited nuclei testifies to the fact that the properties of giant dipole resonance (GDR) are remarkably stable with temperature [1]. In fact, the centroid seems not to change with excitation energy, as for a quantity determined by the mean–field properties, and the same is true for the strength in terms of the classical sum rule fixed by the number of nucleons in the nucleus. Conspicuous changes are observed in the total width Γ as a function of temperature, but a saturation at a value of about 10 MeV for excitation energies $E^*/A \geq 1$ MeV and the mass value $A \sim 100$ has been experimentaly established [2]. The strong increase from the zero temperature value ~ 5 MeV is understood as arising from deformation effects induced by the angular momentum of the compound nucleus and from the connected thermal fluctuations [3]. The saturation of the value of the total width seems to imply a mild temperature dependence of the other contribution to Γ, that is of the spreading width $\Gamma^{\downarrow}$. In what follows, we will discuss this important feature of the nucleus as a finite many–body system. It will be concluded that theory provides a simple explanation of the observations based on the fact the progressive loss of definition of the Fermi surface as a function of temperature is accompanied by a progressive loss in the definition of the nuclear surface. While the first phenomenon makes collisions more prolific the second makes each of them less effective.

2. A THEORETICAL FRAMEWORK

A variety of studies have identified the coupling to the nuclear surface as the main damping mechanism contributing to $\Gamma^{\downarrow}$ [4]. Ultimately, this coupling can be traced back to collisions among the nucleons. However, central questions remain unanswered concerning the nature of these processes, the most pressing being its temperature dependence, and we shall address this question in a systematic way, starting from the mean–field physics.

The Hartree-Fock approximation provides a natural description of the single-particle motion in atomic nuclei. Its time-dependent extension (TDHF) constitutes a powerful tool to study collective motion in many-body systems. Also for going beyond the collisionless regime.

A general formulation of mean-field theories supplemented by collisions has been given in refs. [5,6], resulting in a set of coupled equations for the one-body density matrix $\rho(1,1';t)$ and the two-body correlation function $c_2(12,1'2';t)$. Because of technical limitations these equations can, at present, only be solved accurately for small-amplitude nuclear motion. In this case one can expand both ρ and c_2 in terms of single-particle states ψ_α fulfilling the TDHF equations

$$(i\hbar\frac{\partial}{\partial t} - h(1))\psi_\alpha(1,t) = 0. \tag{1}$$

according to

$$\rho(1,1',t) = \sum_{\alpha,\beta} n_{\alpha\beta}\psi^*_\beta(1',t)\psi_\alpha(1,t) \tag{2}$$

and

$$c_2(1,2,1',2';t) = \sum_{\alpha,\beta\alpha',\beta'} C_{\alpha\beta\alpha'\beta'}(t)\psi_\alpha(1,t)\psi_\beta(2,t)\psi^*_{\alpha'}(1',t)\psi^*_{\beta'}(2',t) \tag{3}$$

The quantity $h(i) = t(i) + U(i)$ is the one-body hamiltonian, i.e. the sum of a kinetic energy term and the mean field

$$U(i;t) = Tr_{2=2'}[v(i2)A_{i2}\rho(22';t)]. \tag{4}$$

The quantity $v(12)$ is the two-body interaction acting among nucleons while $A_{12} = 1 - P_{12}$, P_{12} denotes the permutation operator between nucleons.

The equations of motion for the occupation matrix $n_{\alpha\beta}(t)$ and $C_{\alpha\beta\alpha'\beta'}(t)$ are

$$i\hbar\frac{\partial}{\partial t}n_{\alpha\beta} = \sum_{\gamma\delta\sigma}[C_{\gamma\delta\beta\sigma} < \alpha\sigma|v|\gamma\delta > - C_{\alpha\delta\gamma\sigma} < \gamma\sigma|v|\beta\delta >] \tag{5}$$

and

$$i\hbar\frac{\partial}{\partial t}C_{\alpha\beta\alpha'\beta'} = B_{\alpha\beta\alpha'\beta'} + P_{\alpha\beta\alpha'\beta'} + H_{\alpha\beta\alpha'\beta'}. \tag{6}$$

where

$$B_{\alpha\beta\alpha'\beta'} = \sum_{\lambda_1\lambda_2\lambda_3\lambda_4} < \lambda_1\lambda_2|v|\lambda_3\lambda_4 >_A$$

$$[(\delta_{\alpha\lambda_1} - n_{\alpha\lambda_1})(\delta_{\beta\lambda_2} - n_{\beta\lambda_2})n_{\lambda_3\alpha'}n_{\lambda_4\beta'} \tag{7}$$

$$-n_{\alpha\lambda_1}n_{\beta\lambda_2}(\delta_{\lambda_3\alpha'} - n_{\lambda_3\alpha'})(\delta_{\lambda_4\beta'} - n_{\lambda_4\beta'})]$$

represents the lowest order contribution of collisions in the particle-particle channel (Born approximation), while the term P represents the higher order particle-particle (and hole-hole) contributions

$$
\begin{aligned}
P_{\alpha\beta\alpha'\beta'} = & \sum_{\lambda_1\lambda_2\lambda_3\lambda_4} < \lambda_1\lambda_2|v|\lambda_3\lambda_4 > \\
& [\delta_{\alpha\lambda_1}\delta_{\beta\lambda_2}C_{\lambda_3\lambda_4\alpha'\beta'} - \delta_{\lambda_3\alpha'}\delta_{\lambda_4\beta'}C_{\alpha\beta\lambda_1\lambda_2} \\
& -\delta_{\alpha\lambda_1}n_{\beta\lambda_2}C_{\lambda_3\lambda_4\alpha'\beta'} - \delta_{\lambda_2\beta}n_{\alpha\lambda_1}C_{\lambda_4\lambda_3\beta'\alpha'} \\
& +\delta_{\lambda_3\alpha'}n_{\lambda_4\beta'}C_{\alpha\beta\lambda_1\lambda_2} + \delta_{\lambda_4\beta'}n_{\lambda_3\alpha'}C_{\alpha\beta\lambda_1\lambda_2}].
\end{aligned} \tag{8}
$$

The last term

$$
\begin{aligned}
H_{\alpha\beta\alpha'\beta'} = & \sum_{\lambda_1\lambda_2\lambda_3\lambda_4} < \lambda_1\lambda_2|v|\lambda_3\lambda_4 > \\
& [\delta_{\alpha\lambda_1}(n_{\lambda_3\alpha'}C_{\beta\lambda_4\beta'\lambda_2} - n_{\lambda_3\beta'}C_{\beta\lambda_4\alpha'\lambda_2} - n_{\lambda_4\alpha'}C_{\lambda_3\beta\lambda_2\beta'} - n_{\lambda_4\beta'}C_{\lambda_3\beta\alpha'\lambda_2}) \\
& +\delta_{\beta\lambda_2}(n_{\lambda_4\beta'}C_{\alpha\lambda_3\alpha'\lambda_1} - n_{\lambda_4\alpha'}C_{\alpha\lambda_3\beta'\lambda_1} - n_{\lambda_3\beta'}C_{\alpha\lambda_4\alpha'\lambda_1} - n_{\lambda_3\alpha'}C_{\alpha\lambda_4\lambda_1\beta'}) \\
& -\delta_{\beta'\lambda_4}(n_{\beta\lambda_2}C_{\alpha\lambda_3\alpha'\lambda_1} - n_{\alpha\lambda_2}C_{\beta\lambda_3\alpha'\lambda_1} - n_{\beta\lambda_1}C_{\alpha\lambda_3\alpha'\lambda_2} - n_{\alpha\lambda_1}C_{\beta\lambda_3\lambda_2\alpha}) \\
& -\delta_{\alpha'\lambda_3}(n_{\alpha\lambda_1}C_{\beta\lambda_4\beta'\lambda_2} - n_{\alpha\lambda_2}C_{\alpha\lambda_4\beta'\lambda_2} - n_{\alpha\lambda_2}C_{\beta\lambda_4\beta'\lambda_1} - n_{\beta\lambda_2}C_{\alpha\lambda_4\lambda_1\beta'})]
\end{aligned} \tag{9}
$$

is the contribution to the equations of motion of collisions in the particle-hole channel, which we loosely address as density fluctuations. The set of coupled equations (5) and (6) provides a non-perturbative description of nuclear motion, known as Time-Dependent-Density-Matrix (TDDM) theory, which takes into account collisions among nucleons to all orders in the interaction, and therefore effectively takes into account the coupling to the surface collective vibrations. In more general terms, we are considering a truncation of the BBGKY hierarchy [7] on the two–body level. Examples of the contributing processes are illustrated by the diagrams of Fig. 1. We note that the theory fulfills the conservation of particle number, momentum and energy.

The interaction v appearing in the mean-field potential of eq. (4) may be approximated by a Skyrme–type force and we use $v(12)=V_0\delta(\mathbf{r}_1-\mathbf{r}_2)$ for the residual interaction appearing in eq. (7-9). In keeping with the fact that the matrix elements of $v(1,2)$ at full density are very small due to Pauli blocking - therefore making the nuclear surface the main source of damping of giant resonances - the strength of the interaction V_0 has been determined from the strength of the used Skyrme force calculated at density $\rho = \rho_0/2 \approx 0.08 fm^{-3}$. In modifying the strength of V_0 by 30 % we did not find any significant change in the results presented below.

The coupled equations (5) and (6) have been solved for the case of the isovector dipole and the isoscalar quadrupole vibrations in the nuclei ^{16}O and ^{40}Ca as a function of temperature [8]. The correlated ground state is boosted at a certain time (typically after $0.2 \cdot 10^{-21}$s) by applying appropriate phase factors proportional to a strength factor α. In this way the system acquires a well defined collective energy proportional to α^2. In the case of isoscalar quadrupole motion we have used the BKN [9] force in the calculation of the self-consistent field, eq. (4), and V_0=-300 MeV for the strength of the interaction v responsible for collisions. In the description of the isovector dipole motion the SK2 [9] interaction was adopted in eq. (4) and V_0=-420 MeV. The set of single-particle levels used

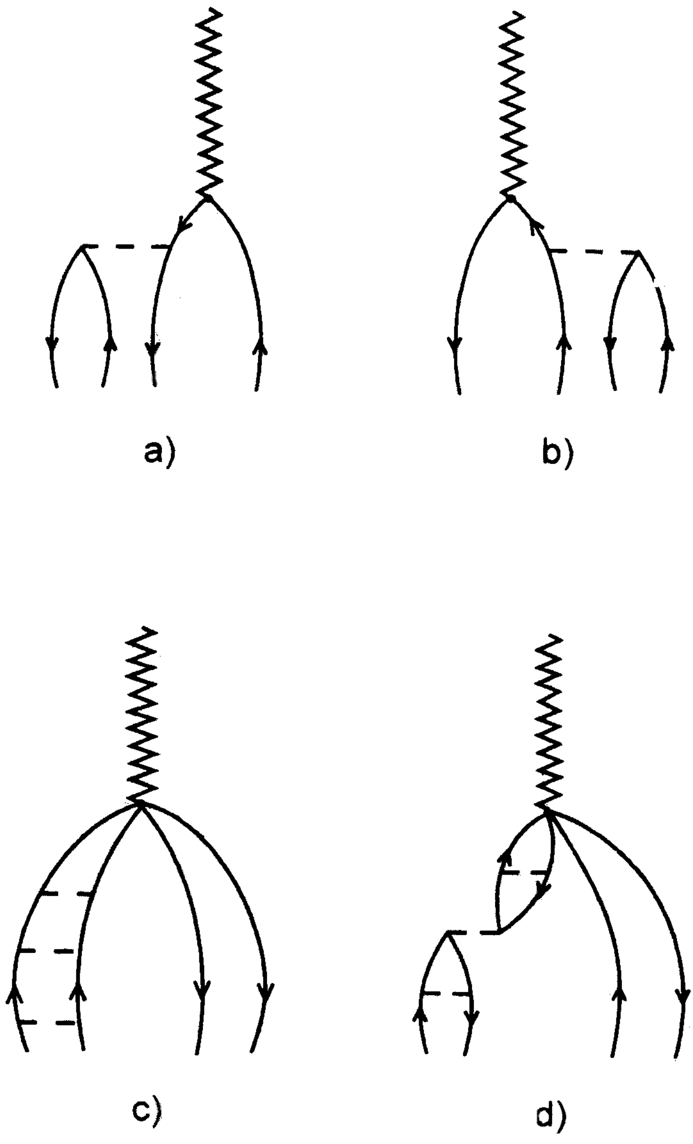

Fig. 1. Examples of processes contributing to the damping of giant resonances (wavy lines) in TDDM.

in the calculations includes the 1s, 1p, 2s and 1d in the case of ^{16}O and 1s, 1p, 2s, 1d, 2p and 1f in the case of ^{40}Ca.

The evolution of the system, both in TDHF and in TDDM, is followed in time by calculating the mean-square radius, the quadrupole moment $Q_2(t)$ and the dipole moment $Q_1(t)$ of the system. The total energy and number of particles are found to be conserved within 7 % and 1 % respectively. The reason for this is to be found in the fact that the single–particle basis used in the calculation has been truncated. In fact, decreasing the energy given to the system in the boosting process, the conservation of energy and particle number are better satisfied. On the other hand, as long as the response of the system is linear, the properties extracted from it concerning giant resonances, are independent on the value of the boosting parameter [8]. To carry out calculations at finite temperature, we replace the initial occupation numbers by appropriate Fermi distributions and propagate the system in time to build up its correlations before applying the collective boost of interest. The temperature is no dynamical constraint but merely enters as a parameter for the initial conditions in the equations of motion which are solved at constant energy.

In Fig. 2a are shown the time-dependent evolution of the dipole moment of ^{40}Ca calculated at T=0 in TDHF and TDDM approximations, while in 2b the result for the two temperatures (T=0 and T=4 MeV) are directly compared in the full TDDM theory.

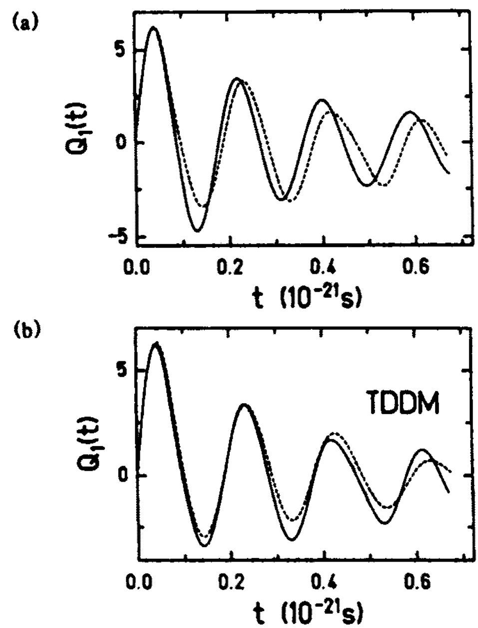

Fig. 2a. The dipole moment $Q_1(t)$ in ^{40}Ca at T=0 in the limits TDHF (solid line) and TDDM (dashed line). **Fig. 2b.** The dipole moment $Q_1(t)$ in ^{40}Ca in the limit TDDM at temperature T=0 (solid line) and T=4 MeV (dashed line).

Two cospicuous features emerge from simple inspection of these results. First, the vibration displays a single frequency mode which is strongly damped already in TDHF, the inclusion of fluctuations and collisions essentially not affecting this behaviour. Consequently the relaxation of the giant dipole resonance of ^{40}Ca seems to be determined both by the decay into single-particle motion (like Landau damping in infinite systems) and by the coupling to the continuum, the so-called escape width $\Gamma^{\uparrow}$. Second, the properties of the resonance are uneffected by temperature. This is a natural consequence of the fact that damping is in this case controlled by mean-field effects.

Fitting the main frequency with a damped oscillator whose coordinate is parametrized according to

$$Q_{test}(t) = A_0 \sin(\omega \cdot t) \cdot e^{-\gamma t/\hbar} \tag{10}$$

one obtains a width $\Gamma = 2\gamma$ which in all cases is about 4 MeV to be compared with the experimental value of 5 MeV. Similar results are obtained in the case of ^{16}O where the calculated width is $\Gamma = 5$ MeV essentially independent of the temperature or of the presence of collisions (cf. Table 1).

	T=0 MeV				T=4 MeV			
	TDHF	p-p	p-h	TDDM	TDHF	p-p	p-h	TDDM
2^+ ^{40}Ca	0.80	1.86	5.60	5.50	0.52	1.97	2.02	2.89
1^- ^{40}Ca	3.56	/	/	4.12	4.37	/	/	4.33
1^- ^{16}O	6.00	/	/	5.53	5.00	/	/	4.90

Table 1. Theoretical values for the width of quadrupole and dipole modes as extracted (as in eq. 10) from the decay constant relative to the first three maxima. The columns p-p and p-h contain the results obtained respectively by putting $H = 0$ and $P = 0$ in eq. (6) as discussed in the text. The corresponding columns in the case of the dipole motion are left free as the corresponding damping arizes manly from mean field effects.

In Fig. 3 the time dependent evolutions of the quadrupole moment associated with ^{40}Ca calculated in mean field and including fluctuations and collisions are shown for two temperatures (T=0 and T=4 MeV) for TDHF (a), $H = 0$ (b), $P = 0$ (c) and TDDM. A simple inspection allows to extract the main features of the results. The giant quadrupole resonance (GQR) corresponds in TDHF (Fig. 3a) to a single mode which is only slightly damped and whose properties are independent on temperature. The presence of collisions and fluctuations changes this picture in a qualitative way by producing a strongly damped vibration which now displays a complicated beating pattern, pointing to the existence of a variety of normal modes. These effects are mainly controlled by fluctuations (cf. Fig 3(c)) while collisions in the particle-particle and hole-hole channels (Fig. 3(b)) play a minor role. In any case, the role of these channels cannot be neglected in a quantitative description of

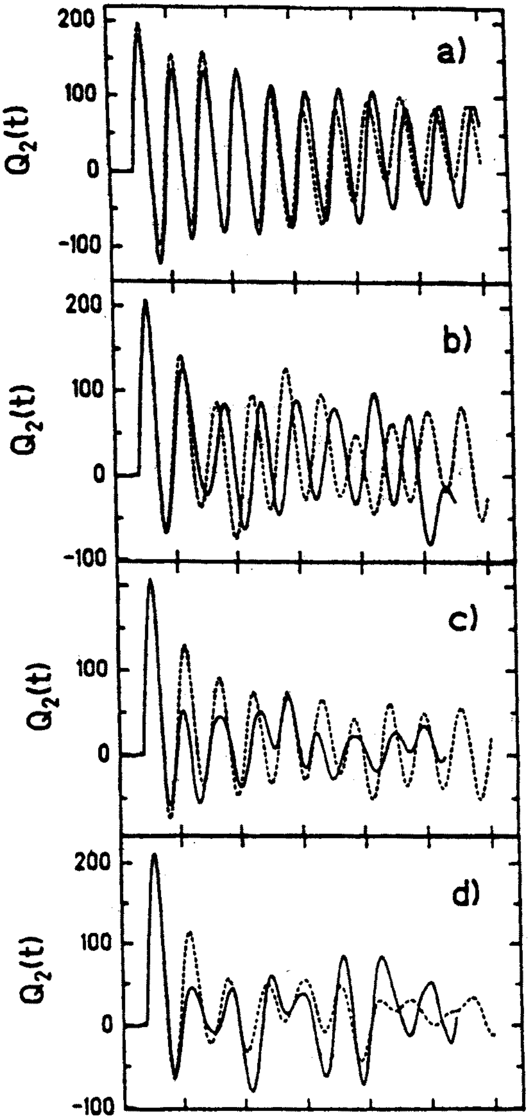

Fig. 3. The quadrupole moment $Q_2(t)$ in ^{40}Ca in the limits TDHF (a); TDHF plus residual "two-body collisions", $H = 0$, (b); TDHF plus fluctuations, $P = 0$, (c); and TDDM (d) at temperatures T = 0 (solid lines) and T = 4 MeV (dashed lines).

the damping processes, because of the interference with the particle-hole channel (cf. Fig 3(d)).

Making use of a fit as in eq. (10) for the main peak, representative damping widths for the GQR in ^{40}Ca were obtained. The results are displayed in Table 1. We have also studied the collective dipole and quadrupole response of ^{16}O and ^{32}S obtaining results which are consistent with those of ^{40}Ca.

We conclude that the giant dipole and quadrupole resonances of light nuclei provide textbook examples of the workings of the main relaxation mechanisms found in many-body systems, namely: escaping of particles in the continuum, Landau damping and the collisional damping arizing from density fluctuations and particle collisions. The first two damping mechanism are found not to vary with temperature, as expected for mean-field phenomena. Remarkably, the last mechanism is also found to be essentially independent on temperature. This is because temperature leads to a simultaneous and progressive smearing of the Fermi surface (thus enhancing the role of collisions by neutralizing the limitations imposed by the Pauli principle) as well as of the nuclear surface (thus making the role of collisions, which essentially all take place at low density, less effective). The empirically found almost total cancellation between these two effects does not appear necessary, and some temperature dependence may occasionally be found for other nuclei or collective modes. Only the first effect is included in the approach of ref. [10], and a strong temperature dependence of $\Gamma^{\downarrow}$ is obtained.

This calculation is fully microscopic, without an a–priori choice of the doorway states for the damping mechanism [4], but the resulting physics agrees with what was obtained in heavy nuclei in the surface coupling model of ref. [11]. This is intimately connected with the finite size of the atomic nucleus and the special role that the nuclear surface plays in the relaxation processes.

We may now consider the role in the damping of the compound nucleus many particle–many hole states. Recently it has been pointed out that, in the regime of chaotic intrinsic dynamics, this contribution to $\Gamma^{\downarrow}$ is independent of temperature and equal to few times a typical matrix element of the residual effective interaction, coupling mean field states [12]. The key assumption is to consider a "democratic" wave function for the compound nucleus states $|\alpha>$ in terms of unperturbed mean–field states $|k>$ belonging to a certain exciton class (number of $p-h$ excitations), that is

$$|\alpha>=\sum_{k} C_{k}^{\alpha}|k> \tag{11}$$

All the mixing amplitudes C_k^α are expected to be of the order of magnitude $N^{-1/2}$ where N is the dimension of the exciton space. The actual value of N is not important for later discussion, only that N $>>$ 1. This is an alternative formulation of an assumption of chaotic dynamics when the components of a complicated state cover uniformly the available domain of the Hilbert space and the memory of the original $p-h$ structure of individual states is smeared away. Then, typical matrix elements of the residual interaction between the chaotic states $|\alpha>$ are smaller than the matrix elements between the simple states $|k>$ by a random walk factor $N^{-1/2}$. As a consequence, applying the Fermi golden rule a contribution to $\Gamma^{\downarrow}$ independent of the level density (of temperature) and equal to few times a typical matrix element of the residual effective is found.

3. SUMMARY

Summing up, the results presented above answer a longstanding problem found in the study of collective motion in hot nuclei, answer which can be summarized simply as follows: the damping width $\Gamma^{\downarrow}$ of giant resonances associated with quantal small-amplitude fluctuations is independent of the temperature of the system. As explained in [1], this result may be also related to the experimental features of the multiplicity and angular distribution of the GDR gamma rays as a function of the excitation energy of the compound nucleus.

4. REFERENCES

1. J. J. Gaardhøje, Ann. Rev. Nucl. Part. Sci. **42**, 483 (1992); J. J. Gaardhøje *et al.*, contribution to this conference.
2. A. Bracco *et al.*, Phys. Rev. Lett., **62** 2080 (1989); G. Enders *et al.*, Phys. Rev. Lett., **69** 249 (1992).
3. A. Bracco, P. F. Bortignon and R. A. Broglia, Proc. of Int. School of Phys. "E. Fermi", CXII Course, Varenna 1989, P. Kienle and C. Detraz eds., Editrice Compositori, Bologna, 1991.
4. G. F. Bertsch, P.F. Bortignon and R.A. Broglia, Rev. Mod. Phys. **55**, 287 (1983); J. Wambach, Rep. Prog. Phys. **51**, 989 (1988).
5. S. J. Wang and W. Cassing, Ann. Phys. **159**, 328 (1985).
6. W. Cassing and S. J. Wang, Z. Phys. **A328**, 423 (1987); W. Cassing and A. Pfitzner, Z. Phys. **A337**, 175 (1990) and Z. Phys. **A342**, 161 (1992).
7. See, e. g., L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics*, 1962; R. Balescu, *Equilibrium and Nonequilibrium Statistical Mechanics*, 1975.
8. F. V. De Blasio, W. Cassing, M. Tohyama, P. F. Bortignon and R. A. Broglia, Phys. Rev. Lett., 68 (1992) 1663. Previous applications had been done for the case of quadrupole vibrations at zero temperature by M. Gong and M. Tohyama, Z. Phys. **A335**, 153 (1990).
9. P. Bonche, S. E. Koonin and J. W. Negele, Phys. Rev. **C13**, 1226 (1976); M. Beiner *et al.*, Nucl. Phys. **A238**, 29 (1976).
10. A. Smerzi, A. Bonasera and M. DiToro, Phys. Rev. **C44**, 1713 (1991).
11. P. F. Bortignon, R. A. Broglia, G. F. Bertsch and J. M. Pacheco, Nucl. Phys. **A460**, 149 (1986).
12. V. G. Zelevinsky and P. von Brentano, Nucl. Phys. **A529**, 141 (1991); V. G. Zelevinsky, P. F. Bortignon and R. A. Broglia, submitted .