COHERENT PION PRODUCTION BY CHERENKOV-LIKE EFFECTS IN NUCLEI*

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The numerical values for the pion refractive index, pion phase velocity and pionic Cherenkov thresholds in heavy nuclei are presented. The π^+-differential cross sections for the coherent pions emitted as *nuclear mesonic Cherenkov radiation* are calculated. Three pionic Cherenkov bands: *CB1* ($\omega = 190 \div 320\,\mathrm{MeV}$), *CB2* ($\omega = 900 \div 960\,\mathrm{MeV}$) and *CB3* ($\omega = 80 \div 1000\,\mathrm{GeV}$) are predicted.

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1. Introduction

The electromagnetic Cherenkov effect is a well known phenomenon in normal dielectrics with large applications in physics and astrophysics. Recently [1], we proposed to extend these considerations to nuclear media where nuclear gamma Cherenkov radiation (NGCR) should be possible to be emitted from charged projectiles moving through nuclei with a velocity larger than the phase velocity of photons. Moreover, the idea of coherent meson production in nuclear reactions via nuclear mesonic Cherenkov radiation (NMCR) has been developed in Refs [2–6].

In this paper following the results of Refs [3–6] and the dispersion relations (DR) predictions [7] for the pion–nucleon forward scattering amplitudes we extend our NMCR investigations [3, 6, 8] up to pion energies in the TeV region. In Sects. 2 and 3 we present the essential formulas and results for the pionic refractive index $n^2(\omega)$, the pion phase velocity $v_{\mathrm{ph}}(\omega)$, the classical and quantum Cherenkov threshold energies for pion

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emission as NMCR in nuclear media. Then, using these results as an input in the quantum theory of NMCR [4, 6], in Sec. 4 numerical results for the NMCR cross sections for π^+ meson production in the nuclear reaction $^{208}Pb(p, n\pi^+)^{208}Pb$ are obtained. Some conclusions and an outlook are presented in Sec. 5.

2. Pion refractive index

The basic ingredient for the NMCR investigation is the refractive index of mesons [9–14] inside the nuclear medium. This problem was systematically discussed by many authors (see *e.g.* Refs [13, 14]). For our numerical calculations we start from the Foldy–Lax formula ($\hbar = c = 1$)

$$n^2(\omega) = \left(\frac{k}{q}\right)^2 = 1 + \frac{4\pi\rho}{\omega^2 - m_M^2} \cdot C\bar{f}^{MN}(\omega), \tag{1}$$

where ω is the meson total energy, m_M the rest mass of the meson, $q = (\omega^2 - m_M^2)^{1/2}$ the free meson momentum, k the meson momentum inside the nuclear medium, ρ the nuclear density. $\bar{f}^{MN}(\omega)$ is the configurational average of the elastic meson-nucleon scattering amplitude in forward direction. The factor C is defined as the ratio between the effective meson field and the coherent field (see *e.g.* Lax [10]). If the nuclear constituents (the nucleons) can be considered to be randomly distributed this factor will be unity.

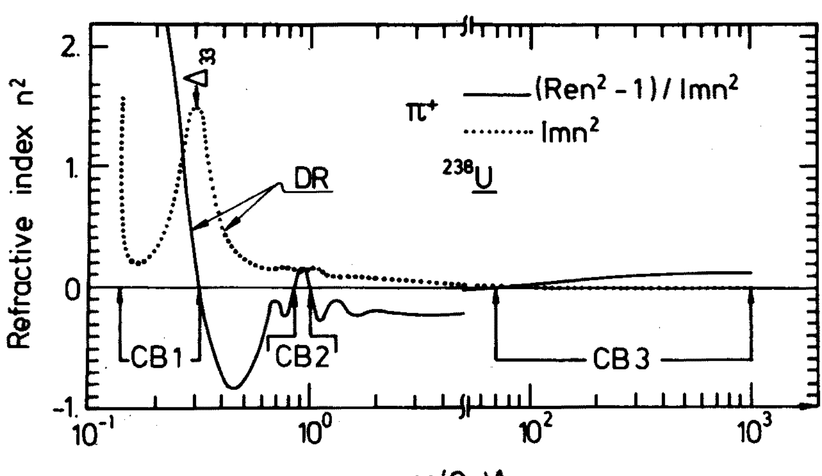

Fig. 1. The pion refractive index $n^2(\omega)$ in ^{238}U.

Now, let us apply Eq. (1), with $C = 1$, to the calculation of the π^+ refractive index $n^2(\omega)$ inside a nucleus with Z protons and $(A-Z)$ neutrons and with $\rho = 0.17\,\mathrm{fm}^{-3}$. Then, the average forward scattering pion-nucleon scattering amplitude $\bar{f}^{\pi^+ N}(\omega)$ is written as $\bar{f}^{\pi^+ N}(\omega) = [Zf^{\pi^+ p}(\omega) + (A - Z)f^{\pi^- p}(\omega)]/A$ since from the isospin invariance $\bar{f}^{\pi^+ n}(\omega) = \bar{f}^{\pi^- p}(\omega)$.

Then, using the numerical results [7] on dispersion relations (DR) for $f^{\pi^\pm p}(\omega)$, the pion index of refraction is calculated and given in Fig. 1. From Fig. 1 (as will be discussed in Sec. 3) we see that three mesonic Cherenkov bands are expected: *CB1, CB2, CB3.*

3. NMCR kinematics

First, we establish the mesonic Cherenkov radiation condition. For this we display schematically in Fig. 2 the Cherenkov emission process for a meson M (with energy ω and momentum $\vec{k} = Re\vec{k}$) that is radiated in the medium from an incident baryon B_1 (with energy E_1 and momentum $\vec{p}_1$) that itself goes over into a final baryon B_2 (with energy E_2 and momentum $\vec{p}_2$). The energy-momentum conservation in the nuclear medium requires

$$E_1 = \omega + E_2\,, \quad \vec{p}_1 = \mathrm{Re}\,\vec{k} + \vec{p}_2\,. \tag{2}$$

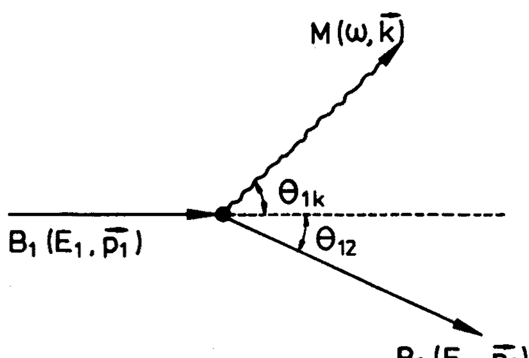

Fig. 2. The NMCR process

For the initial and final baryons B_1 and B_2 we assume that the usual mass-shell relations $E_i^2 - |\vec{p}_i|^2 = M_i^2, i = 1, 2$, are also valid inside the nuclear medium, while for the meson M we take the energy-momentum relation

$$(\mathrm{Re}\,k)^2 = (\omega^2 - m_M^2)(\mathrm{Re}\,n)^2\,. \tag{3}$$

For the NMCR angle θ_{1k} (see Fig. 2) we obtain ($|\vec{p}_1| = p_1$, $|\operatorname{Re}\vec{k}| = \operatorname{Re} k$)

$$\cos\theta_{1k} = \frac{v_{\rm ph}(\omega)}{v_1} + \operatorname{Re} k \cdot \left[1 - v_{ph}^2(\omega) + \frac{(M_2^2 - M_1^2)}{(\operatorname{Re} k)^2}\right](2p_1)^{-1}, \quad (4)$$

where $v_{\rm ph}(\omega)$ is the meson phase velocity defined as

$$v_{\rm ph}(\omega) = \frac{\omega}{\operatorname{Re} k} = \frac{\omega}{q \operatorname{Re} n}. \quad (5)$$

Now, from the condition that θ_{1k} must be a physical angle ($|\cos\theta_{1k}| \leq 1$) we get the *quantum coherence condition*

$$\left|\frac{v_{\rm ph}(\omega)}{v_1} + \frac{\operatorname{Re} k}{2p_1}\left[1 - v_{\rm ph}^2(\omega) + \frac{(M_2^2 - M_1^2)}{(\operatorname{Re} k)^2}\right]\right| \leq 1. \quad (6)$$

At high projectile energy, when $E_1 \gg \omega$ or $p_1 \gg \operatorname{Re} k$, the Eqs (4) and (6) go over into the classical Cherenkov angle, defined by $\cos\theta_{1k} \simeq v_{\rm ph}(\omega)/v_1$, and into the *classical Cherenkov coherence condition* $v_{\rm ph}(\omega) \leq v_1$, respectively. The thresholds for the kinetic projectile energy $T_1 = E_1 - M_1$, for the meson produced as NMCR in the nuclear medium, are given by

$$T_{\rm thr}(\omega) = M_1[(1 - v_{1\rm thr}^2)^{-1/2} - 1]. \quad (7)$$

where $v_{1\rm thr} \equiv v_{\rm ph}(\omega)$ for the *classical threshold* $T_{\rm thr}^C(\omega)$, and

$$v_{1\rm thr} = v_1^0(\omega) \equiv \frac{v_{\rm ph}(\omega)}{1 + F^2} + \frac{F}{(1 + F^2)^{1/2}}\left[1 - \frac{v_{\rm ph}^2(\omega)}{1 + F^2}\right]^{1/2} \quad (8)$$

with

$$F = \frac{\operatorname{Re} k}{2M_1}\left[1 - v_{\rm ph}^2(\omega) + \frac{(M_2^2 - M_1^2)}{(\operatorname{Re} k)^2}\right], \quad (9)$$

for the *quantum threshold* $T_{\rm thr}^Q(\omega)$. We note that $v_1^0(\omega)$ in Eq. (8) is the solution of the equality (6).

Now, using the DR predictions [7] as well as the Pedroni *et al.* data (<u>*P*</u>) [15], we made calculations on $v_{\rm ph}(\omega)$, $T_{\rm thr}^C(\omega)$, $T_{\rm thr}^Q(\omega)$ for the π^+ emission as NMCR in the nuclear reaction ${}^{208}\mathrm{Pb}(p, n\pi^+){}^{208}\mathrm{Pb}$. Our results show clearly the possibility of π^+ emission as NMCR in the following three energy bands: *CB1* in $\omega = 197 \div 313$ MeV, *CB2* in $\omega = 910 \div 960$ MeV, *CB3* in $\omega = 80 \div 1000$ GeV. Similar results are obtained for π^0 emission and π^- emission as NMCR in the nuclear reactions ${}^{208}\mathrm{Pb}(p, p\pi^0){}^{208}\mathrm{Pb}$ and ${}^{208}\mathrm{Pb}(n, p\pi^-){}^{208}\mathrm{Pb}$, respectively, but only for the *CB1* and *CB3* pionic Cherenkov bands. For $A - Z = Z$ nuclei the π^+ *CB2* band is suppressed.

4. π^+ NMCR cross sections

The transition probability (per unit time) for the emission of a pion in a NMCR process (see Fig. 2) in the nuclear medium is given by the golden rule in which $\tilde{H}_{fi}$ are matrix elements of the Hamiltonian describing the interaction between the effective quantized pionic field and initial and final nucleons. The number of physical pions emitted per unit time as NMCR into the energy interval $(\omega, \omega + d\omega)$ is then given by (see Refs [4, 6] for details)

$$\frac{dN^\pi}{d\omega} = \frac{1}{2\pi v_1} \mid \tilde{H}_{fi}|^2 \operatorname{Re} k \frac{d \operatorname{Re} k}{d\omega} \Theta\left(1 - \frac{v_{\rm thr}(\omega)}{v_1}\right), \tag{10}$$

where $\Theta(x)$ is the Heavyside step function. The matrix element $|\tilde{H}_{fi}|^2$ is given by

$$|\tilde{H}_{fi}|^2 = \frac{G^2_{\pi NN}}{\mid n(\omega) \mid^2} \cdot \frac{1}{2\omega} S(E_1, \omega) \cdot F_I = \frac{G^2_{\pi NN}}{|n(\omega)|^2} \frac{k^2 - \omega^2 + (M_1 - M_2)^2}{4\omega E_1 (E_1 - \omega)} \cdot F_I, \tag{11}$$

where $G^2_{\pi NN}/4\pi = 14.6$ is the usual pion–nucleon pseudo-scalar (ps) coupling constant and F_I is the isospin factor (F_I=1 for $N \to \pi^0 N$, and F_I=2

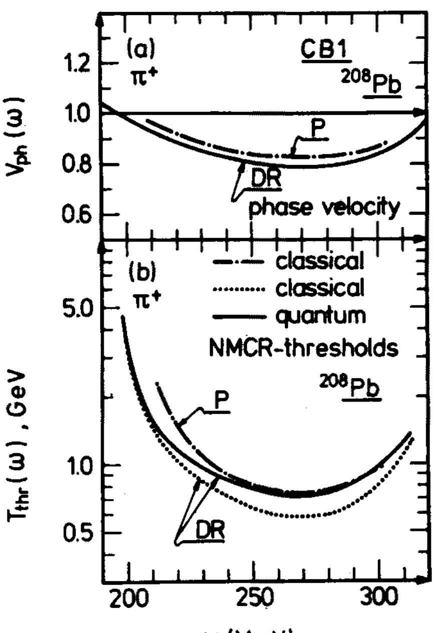

Fig. 3

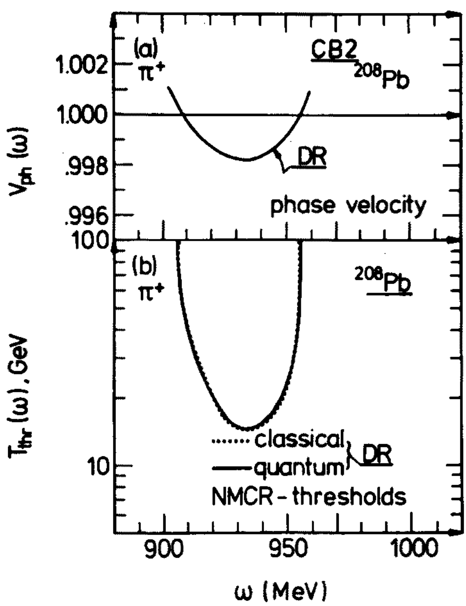

Fig. 4

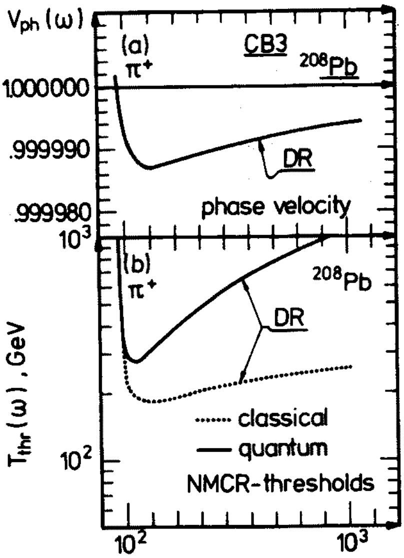

Fig. 5
In Figs 3-5: (a) The pion phase velocity and (b) the NMCR thresholds for π^+ in ^{208}Pb.

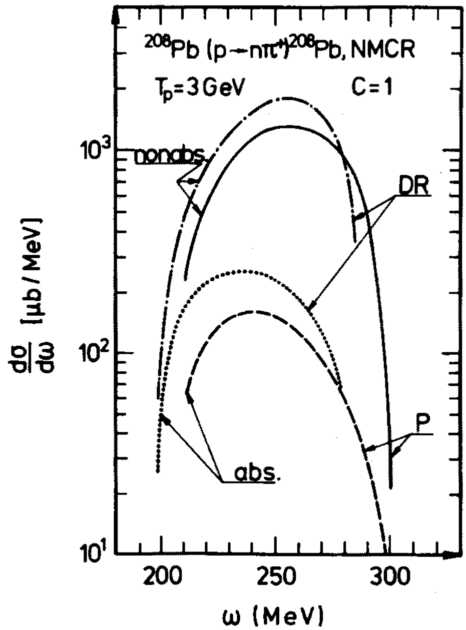

Fig. 6
The π^+-NMCR-differential cross sections in ^{208}Pb.

for $p \to n\pi^+$ or $n \to p\pi^-$ NMCR channels, respectively). Of course, the result (11) was obtained after summing and averaging over initial and final nucleon spin states, respectively. This fact is contained in the spin function: $S(E_1,\omega) = {}^1\!/\!_2 \sum_f [\bar{u}_2(\vec{p}_2)\gamma_5 u_1(\vec{p}_1)] \cdot [\bar{u}_1(\vec{p}_1)\gamma_5 \bar{u}_2(\vec{p}_2)]$, where γ_5 and $u_i(\vec{p}_i)$ are the Dirac matrix and Dirac spinors, respectively. Therefore, combining (11) with (10) we obtain

$$\frac{dN^\pi}{d\omega} = \frac{G^2_{\pi NN}}{4\pi v_1} \cdot \frac{F_I}{|n(\omega)|^2} \cdot \frac{1}{v_{\rm ph}(\omega)} \cdot \frac{d\,{\rm Re}\,k}{d\omega} \cdot \frac{({\rm Re}\,k)^2[1 - v^2_{\rm ph}(\omega)] + (M_1 - M_2)^2}{4E_1(E_1 - \omega)} . \qquad (12)$$

Now, the differential cross section $d\sigma/d\omega$ for the π^+ emission as NMCR in the nuclear reaction $^{208}{\rm Pb}(p,n\pi^+)^{208}{\rm Pb}$ can be obtained from Eq. (12) by multiplying with the factor V/v_1, where V is the collision volume $V = \frac{4\pi}{3} r_0^3 (1 + A_T^{1/3})^3$, $r_0 = 1.12\,{\rm fm}$ and A_T is the mass number of the target nucleus. Numerical values for $d\sigma/d\omega$ for the π^+ emission as NMCR in ^{208}Pb are given in Fig. 6 and Table I for both, absorptive and non-absorptive, cases. The absorption was taken into account in the standard way using the relation

$$\left(\frac{d\sigma}{d\omega}\right)_{\text{abs}} = \left(\frac{d\sigma}{d\omega}\right)_{\text{nonabs}} \cdot \langle F_{\text{abs}} \rangle \,, \tag{13}$$

where

$$\langle F_{\text{abs}} \rangle = \frac{1}{2R} \int_0^{2R} \exp[-Bx]dx = \frac{1 - \exp[-2BR]}{2BR} \tag{14}$$

and the absorption coefficient

$B \equiv 2\,\text{Im}\,k(\omega) = 2q\,\text{Im}\,n(\omega) = (\omega^2 - m_\pi^2)^{1/2} \cdot \text{Im}\,n^2(\omega)/\,\text{Re}\,n(\omega)$.

$R = r_0 A_T^{1/3}$ is the radius of the target nucleus. The values for the derivative $d(\text{Re}\,k)/d\omega$ are obtained from a polynomial fit and are given in Table I. Also in Table I are given the values of $(d\sigma/d\omega)_{\text{nonabs}}$ and $\langle F_{\text{abs}} \rangle$, for π^+ emission in $^{208}\text{Pb}(p, p\pi^0)^{208}\text{Pb}$ at the proton kinetic energy 500 GeV, for both pionic *CB1* and *CB2* bands.

TABLE I

The total pion energy ω, $\text{Re}\,k$, $d\,\text{Re}\,k/d\omega$, $v_{\text{ph}}(\omega)$, $d\sigma/d\omega$ and $\langle F_{\text{abs}} \rangle$ at $T_p =$ 500 GeV.

Band	ω (MeV)	$\text{Re}\,k$ (MeV)	$\frac{d(\text{Re}\,k)}{d\omega}$	$v_{ph}(\omega)$	$(\frac{d\sigma}{d\omega})_{\text{nonabs}}$ (μb/MeV) $T_p = 500$ GeV	$\langle F_{\text{abs}} \rangle$
CB1	197.7	200.6	2.015	0.985270	$3.37 \cdot 10^{-3}$	0.448
	212.3	232.5	2.163	0.913060	$2.88 \cdot 10^{-2}$	0.328
	227.8	264.7	2.161	0.860460	$6.00 \cdot 10^{-2}$	0.226
	243.9	298.5	1.972	0.817090	$9.01 \cdot 10^{-2}$	0.152
	260.5	328.8	1.561	0.792370	$9.75 \cdot 10^{-2}$	0.102
	277.6	350.6	0.897	0.791890	$6.43 \cdot 10^{-2}$	0.070
	295.1	357.0	−0.051	0.826530	—	0.052
	312.9	346.2	−1.313	0.903660	—	0.043
CB2	910.8	911.0	1.137	0.999776	$1.12 \cdot 10^{-3}$	0.133
	930.5	932.2	1.016	0.998240	$8.71 \cdot 10^{-3}$	0.121
	950.3	951.1	0.894	0.999187	$3.69 \cdot 10^{-3}$	0.111

5. Conclusions

In this paper π^+ emissions as NMCR is shown to be an important medium effect that should be detectable via coincidence measurements in high energy nuclear reactions, as *e.g.* $^{208}\text{Pb}(p \to n\pi^+)^{208}\text{Pb}$. The main results are:

(i) The π^+ *refractive index* $n^2(\omega)$ in the energy range $\omega = 0.1 \div 1000$ GeV (Fig. 1), obtained by Eq. (1) and DR data [7], exhibits regions where $\mathrm{Re}\, n^2(\omega) > 1$, which gives rise to three π^+-NMCR bands *CB1* (190-315 MeV), *CB2* (910 – 960 MeV), *CB3* (80 – 1000 GeV) for projectile kinetic energies higher than the *threshold energies* $T_{\rm thr}$ (see Figs 3b–5b).

(ii) The numerical results for the π^+-NMCR *cross sections* show that the spectrum is sufficiently large to be experimentally investigated. The maximum value obtained for $d\sigma/d\omega$ is attained at the energy $\omega \cong 250$ MeV (see Fig. 6).

(iii) For the reaction $^{12}\mathrm{C}(p \rightarrow n\pi^+)^{12}\mathrm{C}$ [17] we find that π^+-NMCR signal $d^2\sigma/d\Omega d\omega$ for $\theta_n = 0^0$ is of the order of magnitude $0.04\,\mu$b/(sr · MeV) at the energy $\omega \cong 245$ MeV. This allows us to conclude that π^+-NMCR is an effect competitive with the "Δ-hole" excitation mode in Ref. [18], even in light nuclei such as ^{12}C.

The Cherenkov pions are radiated as real pions from the projectiles while the Δ-hole pions come from the excitation of the medium. The two kinds of pions belong to different branches of the pion spectrum in the nuclear medium. The Cherenkov pions must be coplanar with the incoming and outgoing projectile, furthermore they must fulfill the coherence condition correlating emission angle and energy.

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