THE PHYSICAL SPACETIME AS A CHRONOSTAT DEFINING TIME
(Prolegomena to a Future Chronodynamics)*

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Dedicated to Andrzej Trautman on the occasion of his 60th birthday

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The familiar analogy, appearing in the quantum theory, between the time evolution of an isolated system and the thermal equilibrium of a system with a thermostat, is taken at its face value. This leads us to the phenomenological conjecture that, in reality, the so called isolated system may remain in a "temporal equilibrium" with the physical spacetime which plays then the role of a "chronostat" defining time equal at all space points (in a Minkowski frame of reference). Such a conjecture suggests virtual deviations from this equilibrium and so seems to imply an extension of the first law of thermodynamics as well as of the state equation in the quantum theory.

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1. Introduction

As is well known, a formal analogy appears in the quantum theory between the time-evolution operator [1] for states of an isolated system,

$$\sim \exp\left(-\frac{iHt}{\hbar}\right), \tag{1}$$

and the temperature-distribution operator [2] for states of a nonisolated system remaining in the thermal equilibrium with a thermostat,

$$\sim \exp\left(-\frac{H}{kT}\right). \tag{2}$$

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In both cases H is the energy operator of the system treated as isolated. This analogy has suggested in the past the familiar formalism of the temperature Green functions [3].

In the present note we would like to put forward a bold conjecture that the above analogy is *not* only formal. Namely, we conjecture that, in reality, any so called isolated system (as *e.g.* a hydrogen atom) is nonisolated: it always interacts with the physical spacetime and often remains with it in an equilibrium of a new kind which we will call *temporal*. We further conjecture that in such a temporal equilibrium the physical spacetime (treated as surroundings of the system) behaves as a *chronostat* allowing the parameter called *time* to be introduced into the description of states of the system (much like a thermostat allows the parameter called temperature to be used for the system; however, an important difference is that temperature may be experimentally fixed, while time is always *running*).

Thus, in the temporal equilibrium of the system with the physical spacetime (behaving as a chronostat), *equal* running time t is ascribed to all space points $\vec{r}$ (in a Minkowski frame of reference): then $t \equiv t_0$ where t_0 is time running (*e.g.*) at the space point $\vec{r} = 0$. Such a conventional concept of time has been used, for instance, in the relativistic equal-time many-body problem of the quantum field theory [4].

2. Time field and thermodynamics

However, one may expect that the physical spacetime cannot always behave as a chronostat defining time equal at all space points. In fact, the temporal equilibrium with a chronostat is a concept corresponding to a special physical situation (like the concept of thermal equilibrium with a thermostat). Thus, in general, time equal at all space points (like equal temperature) *cannot* be defined for the system: the perturbation of the temporal equilibrium caused by the system may be nonnegligible in some cases. One may imagine that these are the cases when large enough excitations can occur in the system (as, perhaps, for an energetic cosmic ray colliding with a heavy nucleus or for colliding beams in the planned Superconducting Super Collider).

Nevertheless, if processes in the system run not too far from its temporal equilibrium with the physical spacetime (behaving as a chronostat), one may define (in place of time equal at all space points) a parameter-valued *time field* $t(\vec{r})$, such that (*e.g.*) at $\vec{r} = 0$: $t(0) = t_0$ (much like the temperature field $T(\vec{r})$ with $T(0) = T_0$; however, an important difference is that $t(\vec{r})$ is always *running* at any $\vec{r}$). Then, in the framework of thermodynamics there should appear a new Onsager-type flow $\vec{j}_T(\vec{r})$ satisfying, in the simple case of a homogeneous matter medium, the transport equation

$$-i\vec{j}_\Gamma(\vec{r}) = -\sigma_\Gamma \frac{\hbar}{i} \,\mathrm{grad}\, \left[t^{-1}(\vec{r}) - t_0^{-1}\right] \tag{3}$$

analogous to the familiar Fourier law

$$\vec{j}_Q(\vec{r}) = -\sigma_Q\, k \,\mathrm{grad}\, [T(\vec{r}) - T_0] \tag{4}$$

for the heat flow $\vec{j}_Q(\vec{r})$ (here, the analogy ($it/\hbar \leftrightarrow 1/kT$) is exploited). The constant σ_Γ should be positive like σ_Q. Note that the flow $\vec{j}_\Gamma(\vec{r})$, as appearing in Eq. (3), depends (functionally) also on the running time hypersurface σ given by $t = t(\vec{r})$, so that, in general, $\vec{j}_\Gamma(\vec{r}) \equiv \vec{j}_\Gamma(\vec{r}, t(\vec{r}))$.

In Eq. (3) (as well as further in Eq. (7)) we consider the space points at which $t(\vec{r}) \neq 0$. As to the time hypersurface $t = t(\vec{r})$ one might expect that it should be *space-like* within the considered system: $|\, c\,\mathrm{grad}\, t(\vec{r})\, | < 1$. Then, this property would be shared by our time hypersurface with another running time hypersurface σ which took over the role of conventional time t = const in the Tomonaga–Schwinger state equation for quantum fields [5]. Since, in contrast to our time hypersurface, the other σ can be always *arbitrarily* deformed (in particular, made flat: t = const), not contradicting the motion, the operation of Tomonaga–Schwinger derivative $\delta/\delta\,\sigma(x)$ is well-defined during the motion in that case.

In the framework of thermodynamics, the transport equation (3) suggests that the first law of thermodynamics [2]

$$dU = \delta W + \delta Q \tag{5}$$

should be now extended to the *new* form

$$dU = \delta W + \delta Q - i\delta\Gamma \tag{6}$$

including an *imaginary* term $-i\delta\Gamma$. Here, $\delta\Gamma$ is the infinitesimal increase of a new thermodynamic quantity Γ whose flow $\vec{j}_\Gamma(\vec{r})$ has appeared in Eq. (3). Such a quantity may be called the *energy width* transferred to the system from its surroundings including the physical spacetime (or being the physical spacetime in the case of a so called isolated system).

Thus, in general, the internal energy of the system, U, is *complex.* It becomes real when the system reaches the temporal equilibrium with the physical spacetime (behaving as a chronostat): then $\delta\Gamma = 0$ since $t(\vec{r}) \equiv t_0$ and, consequently, $\vec{j}_\Gamma(\vec{r}) \equiv 0$ by Eq. (3). Until this equilibrium is reached, states of the system *are not* evolved by the operator (1) as they are in the case of the conventional quantum theory of isolated systems. But, after the temporal equilibrium is established, this conventional theory works exactly.

Concluding the second part of this note, we would like to stress that we avoided to discuss the physical nature of spacetime which should be connected, to some extent at least, with the theory of gravitation (or supergravitation) and its desired *quantization*. From the fundamental point of view, such a quantization would be *necessary* for the consistency of our picture (postulating the interaction of matter with physical spacetime), unless the gravitation would be a statistical phenomenon of a quantum theory standing behind it. Instead, in this note, we restricted ourselves to the rather *phenomenological* conjecture that, in reality, the so called isolated system may remain in a temporal equilibrium with the physical spacetime which plays then the role of a chronostat defining time equal at all space points (in a Minkowski frame of reference). Such a conjecture suggested virtual deviations from this equilibrium and the phenomenon of transporting them in space.

3. Chronodynamics in homogeneous matter media

In the third part of this note we will formulate a tentative *equation of motion* for our time field $t(\vec{r})$ appearing in the transport equation (3) in the homogeneous matter medium.

To this end, as a first step, let us notice that the flow $\vec{j}_\Gamma(\vec{r}) \equiv \vec{j}_\Gamma(\vec{r}, t(\vec{r}))$ should correspond to a density $\rho_\Gamma(\vec{r}) \equiv \rho_\Gamma(\vec{r}, t(\vec{r}))$ satisfying, in the simple case of a homogeneous matter medium, the equation

$$-i\delta\rho_\Gamma(\vec{r}) = c_\Gamma \frac{\hbar}{i} d\left[t^{-1}(\vec{r}) - t_0^{-1}\right] \tag{7}$$

analogous to the familiar formula

$$\delta\rho_Q(\vec{r}) = c_Q k\, d\left[T(\vec{r}) - T_0\right] \tag{8}$$

for the heat density $\rho_Q(\vec{r})$ (here, the analogy $it/\hbar \leftrightarrow 1/kT$ is used again). The constant c_Γ should be positive like c_Q .

Since the time field $t(\vec{r})$ fulfils the boundary condition (*e.g.*) at $\vec{r} = 0$: $t(0) = t_0$, we can write $t(\vec{r}) = t(\vec{r}, t_0)$. Then, in Eqs. (3) and (7) $\vec{j}_\Gamma(\vec{r}) \equiv \vec{j}_\Gamma(\vec{r}, t(\vec{r})) = \vec{j}_\Gamma(\vec{r}, t(\vec{r}, t_0)) \equiv \vec{j}_\Gamma(\vec{r}, t_0)$ and $\rho_\Gamma(\vec{r}) \equiv \rho_\Gamma(\vec{r}, t(\vec{r})) = \rho_\Gamma(\vec{r}, t(\vec{r}, t_0)) \equiv \rho_\Gamma(\vec{r}, t_0)$ (in a nonrigorous notation).

As a second step, let us assume for our matter medium that the thermodynamic quantity Γ contained in a fixed space region V,

$$\Gamma_V(t_0) \equiv \int_V d^3\vec{r}\, \rho_\Gamma(\vec{r}, t(\vec{r}, t_0)), \tag{9}$$

(and functionally depending on the time hypersurface σ given by $t = t(\vec{r}, t_0)$ with $\vec{r} \in V$) is locally balanced at any space point $\vec{r}$ through the equality

$$\frac{d}{dt_0} \int\limits_{dV} d^3\vec{r'}\, \rho_\Gamma(\vec{r'}, t(\vec{r'}, t_0)) = - \int\limits_{\partial(dV)} d^2\sigma'\, \vec{n}(\vec{r'})\vec{j}_\Gamma(\vec{r'}, t(\vec{r'}, t_0)) \tag{10}$$

(except, possibly, isolated points $\vec{r}_S$). Hence we get the local conservation law :

$$\mathrm{div}\vec{j}_\Gamma(\vec{r}, t_0) + \frac{\partial \rho_\Gamma(\vec{r}, t_0)}{\partial t_0} = 0 \tag{11}$$

(at any $\vec{r} \neq \vec{r}_S$), where $\vec{j}_\Gamma(\vec{r}, t_0) \equiv \vec{j}_\Gamma(\vec{r}, t(\vec{r}, t_0))$ and $\rho_\Gamma(\vec{r}, t_0) \equiv \rho_\Gamma(\vec{r}, t(\vec{r}, t_0))$, and so

$$\mathrm{div}\vec{j}_\Gamma \equiv \mathrm{div}_{t-\mathrm{fixed}}\, \vec{j}_\Gamma + \frac{\partial \vec{j}_\Gamma}{\partial t} \cdot \mathrm{grad}\, t\,, \quad \frac{\partial \rho_\Gamma}{\partial t_0} \equiv \frac{\partial \rho_\Gamma}{\partial t}\frac{\partial t}{\partial t_0}\,. \tag{12}$$

The three equations (3), (7) and (11) formulate a thermodynamic theory that may be called the *chronodynamics* in homogeneous matter media. From these formulae we readily deduce for the parameter-valued field $\phi(\vec{r}, t_0) \equiv t(\vec{r}, t_0)^{-1} - t_0^{-1}$ the conductivity equation playing here the role of an equation of motion :

$$\left(\Delta - \frac{1}{\kappa_\Gamma}\frac{\partial}{\partial t_0}\right) \phi(\vec{r}, t_0) = 0\,, \tag{13}$$

where the new constant $\kappa_\Gamma = \sigma_\Gamma / c_\Gamma > 0$ is an *inverse-time conductivity.* Two constants $\sigma_\Gamma > 0$ and $c_\Gamma > 0$ characterize in a measure the interaction of the homogeneous matter medium with the physical spacetime represented by the time field (c_Γ is, in addition, proportional to the constant mass density $\rho_M > 0$ of the medium). Note that

$$t(\vec{r}, t_0) \equiv \frac{t_0}{1 + \phi(\vec{r}, t_0)t_0}\,. \tag{14}$$

In Eq. (13), the field $\phi(\vec{r}, t_0)$ is required to fulfil the boundary condition (*e.g.*) at $\vec{r} = 0$: $\phi(0, t_0) = 0$, since $t(0, t_0) = t_0$. In the special case of temporal equilibrium we have $t(\vec{r}, t_0) \equiv t_0$ and so $\phi(\vec{r}, t_0) \equiv 0$, thus $\vec{j}_\Gamma(\vec{r}, t_0) \equiv 0$ through Eq. (3).

Now, let us observe that making use of the ansatz

$$\phi(\vec{r}, t_0) = \phi_0(\vec{r}) \exp(-\gamma_\Gamma t_0)\,, \tag{15}$$

where $\gamma_\Gamma > 0$ is a constant, we obtain from Eq. (13) the oscillatory equation

$$\left(\Delta + \frac{\gamma_\Gamma}{\kappa_\Gamma}\right) \phi_0(\vec{r}) = 0 . \tag{16}$$

Hence, as a pedagogical example for $\phi_0(\vec{r})$, we may consider its spherical oscillatory solution

$$\begin{aligned} \phi_0(\vec{r}) &= \mathrm{Re}\Phi_0(\vec{r}) , \\ \Phi_0(\vec{r}) &= G_\Gamma c \mid \vec{r} - \vec{r}_S \mid^{-1} \exp\left(i\sqrt{\frac{\gamma_\Gamma}{\kappa_\Gamma}} \mid \vec{r} - \vec{r}_S \mid\right) \end{aligned} \tag{17}$$

at any $\vec{r} \neq \vec{r}_S$, where $\vec{r}_S$ is an arbitrary constant vector and $G_\Gamma > 0$ a small dimensionless coupling constant. In the solution (17) there is a singularity at $\vec{r} = \vec{r}_S$, which introduces the point-source term $-4\pi G_\Gamma c \delta^3(\vec{r} - \vec{r}_S) \exp(-\gamma_\Gamma t_0)$ to the rhs of the conductivity equation (13). Such a source term may be interpreted as an exponentially waning external perturbation switched on at $t_0 = 0$ in our matter medium at the point $\vec{r} = \vec{r}_S$.

We can see from Eqs. (14), (15) and (17) that $\phi(\vec{r}, t_0) =\!\!\rightarrow 0$ and hence $t(\vec{r}, t_0) =\!\!\rightarrow t_0$ for $t_0 \rightarrow \infty$ at any space point $\vec{r} \neq \vec{r}_S$. So, the homogeneous matter medium, perturbed at $t_0 = 0$ (locally in space at $\vec{r} = \vec{r}_S$) tends to attain at $t_0 \rightarrow \infty$ the temporal equilibrium with the physical spacetime. Similarly, $\phi(\vec{r}, t_0) =\!\!\rightarrow 0$ and hence $t(\vec{r}, t_0) =\!\!\rightarrow t_0$ at $r \equiv \mid \vec{r} \mid \rightarrow \infty$. Thus, in the case of our example for $\phi(\vec{r}, t_0)$, the boundary condition (*e.g.*) at $\vec{r} = 0$: $t(0, t_0) = t_0$ is replaced by the asymptotic condition at $r \rightarrow \infty$: $t(\vec{r}, t_0) \rightarrow t_0$.

Taking $\rho_\Gamma(\vec{r}, t_0) = c_\Gamma \hbar \phi(\vec{r}, t_0)$ on the base of formula (7), we get from Eqs. (9), (15) and (17) that, in the case of our example, the energy width Γ contained in a fixed space region V of the homogeneous matter medium is

$$\begin{aligned} \Gamma_V(t_0) &= c_\Gamma \hbar \int_V d^3\vec{r}\, \phi(\vec{r}, t_0) \\ &= c_\Gamma G_\Gamma \hbar c \mathrm{Re} \left[\int_V d^3\vec{r} \mid \vec{r} - \vec{r}_S \mid^{-1} e^{i\sqrt{\frac{\gamma_\Gamma}{\kappa_\Gamma}} \mid \vec{r} - \vec{r}_S \mid} \right] e^{-\gamma_\Gamma t_0} , \end{aligned} \tag{18}$$

thus $\Gamma_V(t_0) =\!\!\rightarrow 0$ for $t_0 \rightarrow \infty$ (as it should be in the asymptotic temporal equilibrium at $t_0 \rightarrow \infty$). For instance, if V is a spherical region of radius R, centered at $\vec{r} = \vec{r}_S$, the real part of the integral in Eq. (18) turns out to be $4\pi(\kappa_\Gamma/\gamma_\Gamma)(x \sin x + \cos x - 1)$, where $x = \sqrt{\gamma_\Gamma/\kappa_\Gamma}\, R$, and is positive for $0 < R \leq (\pi/2)\sqrt{\kappa_\Gamma/\gamma_\Gamma}$, though for some larger R it may alternate the sign (however, the integral $=\!\rightarrow 2\pi R^2 > 0$ with $\kappa_\Gamma/\gamma_\Gamma \rightarrow \infty$ for any R).

It is easy to check by means of Eqs. (14), (15) and (17) that for $t_0 \rightarrow \infty$

$$\frac{\partial t}{\partial t_0} \equiv \frac{1}{1+\phi t_0}\left[1 - \frac{\phi t_0}{(1+\phi t_0)^2}(1-\gamma_\Gamma t_0)\right] =\!\rightarrow 1 > 0 \tag{19}$$

and

$$|\, c\,\mathrm{grad}\, t\,| \equiv c\frac{t_0^2}{(1+\phi t_0)^2}\,|\,|\,\vec{r}-\vec{r}_S\,|^{-1}\mathrm{Re}\,\Phi + \sqrt{\frac{\gamma_\Gamma}{\kappa_\Gamma}}\mathrm{Im}\,\Phi\,| =\!\rightarrow 0 < 1\,, \tag{20}$$

where $\Phi = \Phi_0 \exp(-\gamma_\Gamma t_0)$ and $\phi = \mathrm{Re}\,\Phi = \phi_0 \exp(-\gamma_\Gamma t_0)$. Thus, in the case of our pedagogical example for $\phi(\vec{r}, t_0)$, we can be sure that, at least asymptotically at $t_0 \rightarrow \infty$, the time field $t(\vec{r}, t_0)$ *is growing* with t_0 and the time hypersurface $t = t(\vec{r}, t_0)$ *is space-like*, at any space-point $\vec{r} \neq \vec{r}_S$. Note that the smaller the coupling constant $G_\Gamma > 0$, the larger is the asymptotic interval of t_0 where these conditions are met at a fixed $\vec{r}$ (in the limit of $G_\Gamma \rightarrow 0$ the interval becomes $0 \leq t_0 < \infty$ at any $\vec{r}$). At this point it is worthwhile to remark that, in contrast to time $t = t_0$ (corresponding to the temporal equilibrium), the time-field $t = t(\vec{r}, t_0)$ (appearing in the case of a not–too–large deviation from the temporal equilibrium) is a very important but essentially approximate notion: the better, the smaller is this deviation (an analogical situation occurs for the temperature field $T = T(\vec{r}, T_0)$).

Of course, in general, any source term introduced to the rhs of Eq. (13) perturbs the temporal equilibrium of our matter medium with the physical spacetime. Then, the same source term multiplied by $-\hbar\sigma_\Gamma$ appears on the rhs of the local–conservation equation (11) for the thermodynamic quantity Γ, spoiling its local and global conservation in the homogeneous matter medium.

In conclusion of the third part of this note, let us recall that our argument leading to the field equation (13) is expected to work if processes in the homogeneous matter medium proceed not too far from its temporal equilibrium with the physical spacetime. Then, there exists the time field $t(\vec{r}) = t(\vec{r}, t_0)$ which is running at any space point $\vec{r}$ because t_0 runs. Such a time field provides for our matter medium a part of its thermodynamic description, much like the temperature field. Let us emphasize that both fields are not well defined in the general situation, when the system (the matter medium in our case) is far away from the temporal and thermal equilibrium with the physical spacetime and a heat reservoir, respectively.

4. Time field and quantum theory

In the fourth part of this note, a *mixed theory* will be suggested, where the matter is treated on a quantum level, while both parameter fields: the time field as well as the temperature field are described on a thermodynamic level as, in our case, by the chronodynamic equation (13). Then, in general, there are two types of coupling between the quantal matter and both parameter fields: operator — parameter and operator expectation value (or material coefficient) — parameter. The case of the conventional quantum theory of isolated systems, interpreted as a theory of so called isolated quantum systems remaining in the temporal equilibrium with the physical spacetime, is a trivial example of such a mixed theory. Then, Eq. (1) gives the time-evolution operator, where $t = t(\vec{r}, t_0) \equiv t_0$.

Looking for such a mixed theory, we would like to make a guess as to the state equation for so called isolated quantum systems remaining in no temporal equilibrium with the physical spacetime, but still not too far from it. Namely, basing on the experience collected from our pedagogical example, we tentatively propose (in the Schrödinger picture) the following *new* form of the state equation for such a system, say, of quantum fields (*e.g.* of those considered in QED):

$$i\hbar \frac{d\Psi(t_0)}{dt_0} = [H - i\mathbf{1}\Gamma(t_0)]\,\Psi(t_0)\,, \tag{21}$$

where $\mathbf{1}$ stands for the unit operator and

$$\Gamma(t_0) \equiv g_\Gamma \hbar \int d^3\vec{r}\, \rho(\vec{r}, t_0)\phi(\vec{r}, t_0) \tag{22}$$

is the total energy width of the system. Here, the parameter field $\phi(\vec{r}, t_0) \equiv t(\vec{r}, t_0)^{-1} - t_0^{-1}$ satisfies the *chronodynamic equation*

$$\left(\Delta - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t_0}\right)\phi(\vec{r}, t_0) = 4\pi g_\Gamma \lambda_\Gamma \left[\frac{\partial}{\partial t_0}\rho(\vec{r}, t_0)\right] \exp\frac{-ct_0}{2\lambda_\Gamma} \tag{23}$$

and the boundary condition at an $\vec{r} = \vec{r}_0$ (*e.g.* at $\vec{r} = 0$): $\phi(\vec{r}_0, t_0) = 0$ or $t(\vec{r}_0, t_0) = t_0$ (as to the form of source term in Eq. (23) — that turns out to be valid if $\operatorname{div}\vec{j} = 0$ for the current $\vec{j}$ corresponding to ρ — *cf.* Appendix A). In Eqs. (22) and (23),

$$\rho(\vec{r}, t_0) \equiv \frac{1}{c}\langle \Psi(t_0) \mid J^0(\vec{r}) \mid \Psi(t_0)\rangle_{\rm av} \tag{24}$$

is the spin-averaged expectation value in the state $\Psi(t_0)$ of the operator of total field density $c^{-1}J^0(\vec{r})$ with $J^0(\vec{r})$ being the time component of the total field-current four-vector $(J^{\mu}(\vec{r}))$ (in the Schrödinger picture). Thus, Eqs. (21) and (23) for $\Psi(t_0)$ and $\phi(\vec{r}, t_0)$ *are coupled*, and together describe $\Psi(t_0)$ and $\phi(\vec{r}, t_0)$. In these equations $g_{\Gamma} > 0$ is a small dimensionless coupling constant, while $\lambda_{\Gamma} > 0$ is a conductivity constant of the length dimension.

When constructing such equations, we tentatively conjectured that $\sigma_{\Gamma} = \sigma_{\Gamma}(\vec{r}, t_0) = g_{\Gamma}\lambda_{\Gamma}\, c\,\rho(\vec{r}, t_0)$ as well as $c_{\Gamma} = c_{\Gamma}(\vec{r}, t_0) = g_{\Gamma}\,\rho(\vec{r}, t_0)$, what results into $\kappa_{\Gamma} = \sigma_{\Gamma}/c_{\Gamma} = \lambda_{\Gamma}\, c$. Then, we neglected in Eq. (23) the terms $\rho^{-1}\mathrm{grad}\rho \cdot \mathrm{grad}\phi$ and $\rho^{-1}(\partial\rho/\partial t_0)\phi$ (so assuming small changes of $\ln\rho$ in space and time).

Note that for $\Gamma(t_0) \neq 0$ the operator $H - i\,1\;\Gamma(t_0)$ is not Hermitian (but has always a diagonal anti-Hermitian part). This causes that the Schrödinger picture, in which Eq. (21) is postulated, is not unitary equivalent to the Heisenberg picture where $d\Psi_H/dt_0 = 0$, though it is unitary equivalent to the interaction picture in which

$$i\hbar\,\frac{d\Psi_I(t_0)}{dt_0} = \left[H_I^{\mathrm{int}}(t_0) - i\,1\Gamma(t_0)\right]\Psi_I(t_0) \tag{25}$$

with $H = H^{\mathrm{free}} + H^{\mathrm{int}}$. On the other hand, unitary equivalent to the Heisenberg picture is the modified Schrödinger picture or *equilibrium picture* where

$$i\hbar\,\frac{d\Psi_E(t_0)}{dt_0} = H\,\Psi_E(t_0)\,. \tag{26}$$

Evidently, the relations

$$\Psi(t_0) = \exp\left(-\frac{i}{\hbar}H^{\mathrm{free}}t_0\right)\Psi_I(t_0) = \Psi_E(t_0)\exp\left[-\frac{1}{\hbar}\int_0^{t_0} dt_0'\,\Gamma(t_0')\right] \tag{27}$$

and

$$\Psi_E(t_0) = \exp\left(-\frac{i}{\hbar}H\,t_0\right)\Psi_H \tag{28}$$

hold.

In the case of our example, where

$$g_{\Gamma}\,\lambda_{\Gamma}\left[\frac{\partial}{\partial t_0}\rho(\vec{r}, t_0)\right]\exp\frac{-ct_0}{2\lambda_{\Gamma}} = -G_{\Gamma}\,c\,\delta^3(\vec{r} - \vec{r}_S)\exp(-\gamma_{\Gamma}\,t_0)$$

represents in Eq. (23) an external point-source and $\Gamma(t_0)$ is given as in Eq. (18), we get

$$\int_0^{t_0} dt_0'\Gamma(t_0') = \Gamma(0)\,\gamma_{\Gamma}^{-1}\left[1 - \mathrm{e}^{-\gamma_{\Gamma}\,t_0}\right] =\!\!\longrightarrow \Gamma(0)t_0 \;\text{ and }\; =\!\!\longrightarrow \Gamma(0)\gamma_{\Gamma}^{-1}$$

for $t_0 \to 0$ and $t_0 \to \infty$, respectively. In this example, Eq. (23) becomes decoupled from Eq. (21), moreover, for $t_0 \to \infty$ any influence of this external source on $\Psi(t_0)$ disappears since $\Gamma(t_0) =\to 0$ in Eq. (21) (for an extension of our example *cf.* Appendix B).

As to the coupled equations (21) and (23), it should be stressed that they have a nonrelativistic form and are valid in a *special* Minkowski frame of reference, where the temporal equilibrium of the system with the physical spacetime is defined by the identity $t(\vec{r}, t_0) \equiv t_0$ or $\phi(\vec{r}, t_0) \equiv 0$ (with $\phi(\vec{r}, t_0) \equiv t(\vec{r}, t_0)^{-1} - t_0^{-1}$) which is relativistically noncovariant. Notice, that a relativistically *covariant* definition of this equilibrium may be expressed, in the geometrical language, by the requirement that our time hypersurface described by $t = t(\vec{r}, t_0)$ has to be then identical with the Tomonaga–Schwinger time hypersurface σ which, not contradicting the motion, can be *arbitrarily* deformed at any spacetime point $x = (ct, \vec{r})$, provided $\sigma + \delta\sigma$ remains space-like. This may suggest the following relativistically covariant definition for the inverse-time field:

$$\phi_{\rm rel}(\vec{r}, t_0) \equiv c\,[n(x) \cdot x]^{-1} - c\,[n(x_0) \cdot x_0]^{-1} ,$$

where $x = (c\,t(\vec{r}, t_0), \vec{r})$ and $x_0 = (c\,t_0, \vec{r}_0)$, while $n(x)$ is the unit four-vector perpendicular to our (space-like) time hypersurface $t = t(\vec{r}, t_0)$ at the spacetime point x and directed to the future (for a further discussion *cf.* Appendix A).

Observe, by the way, that time $t = t_0$ running at the point $\vec{r} = \vec{r}_0$ may *always* be *formally* extended to the Tomonaga–Schwinger time hypersurface $\sigma = \sigma_0$ and then

$$\frac{\delta\Gamma[\sigma_0]}{\delta\sigma_0(x_{t_0})} = \partial_{\mu t_0}\, j^\mu_\Gamma(x_{t_0}) \ \text{ with } \ \Gamma[\sigma_0] \equiv \int_{\sigma_0} d^3\sigma_{\mu 0}(x_{t_0}) j^\mu_\Gamma(x_{t_0}) ,$$

where $(j^\mu_\Gamma) = (c\rho_\Gamma, \vec{j}_\Gamma)$, $x_{t_0} = (ct_0, \vec{r})$ and $\partial_{\mu t_0} = \partial/\partial x^\mu_{t_0}$, while $j^\mu_\Gamma(x_{t_0}) \equiv j^\mu_\Gamma(\vec{r}, t_0) \equiv j^\mu_\Gamma(\vec{r}, t(\vec{r}, t_0))$ (in the already used nonrigorous notation). Here, $t = t(\vec{r}, t_0)$ is our time hypersurface (in general, not in temporal equilibrium).

From Eqs. (21), (22) and (24) it follows that the new state equation (21) is, strictly speaking, *nonlinear* (and *nonlocal*) with respect to $\Psi(t_0)$. However, let $\rho(\vec{r}, t_0)$ be perturbatively replaced in the zeroth order by

$$\rho^{(0)}(\vec{r}, t_0) \equiv \frac{1}{c} \langle \Psi^{(0)}(t_0) \mid J^0(\vec{r}) \mid \Psi^{(0)}(t_0) \rangle_{\rm av} , \tag{29}$$

where $\Psi^{(0)}(t_0)$, assumed to be known, corresponds to the temporal equilibrium *i.e.*, to $\phi^{(0)}(\vec{r}, t_0) \equiv 0$, and so satisfies the conventional state equation

$$i\hbar \frac{d\Psi^{(0)}(t_0))}{dt_0} = H\Psi^{(0)}(t_0) . \tag{30}$$

Then, in the first perturbation order, we obtain from Eqs. (21) and (22) the following approximate equations :

$$i\hbar\,\frac{d\Psi^{(1)}(t_0))}{dt_0} = \left[H - i\,1\Gamma^{(1)}(t_0)\right]\Psi^{(1)}(t_0)\,, \tag{31}$$

where

$$\Gamma^{(1)}(t_0) \equiv g_\Gamma \hbar \int d^3\vec{r}\,\rho^{(0)}(\vec{r},t_0)\phi^{(1)}(\vec{r},t_0)\,, \tag{32}$$

and

$$\left(\Delta - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t_0}\right)\phi^{(1)}(\vec{r},t_0) = 4\pi g_\Gamma \lambda_\Gamma \left[\frac{\partial}{\partial t_0}\rho^{(0)}(\vec{r},t_0)\right]\exp\frac{-ct_0}{2\lambda_\Gamma} \tag{33}$$

with $\rho^{(0)}(\vec{r},t_0)$ as given in Eq. (29). Here, the approximate chronodynamic equation (33) is independent of $\Psi^{(1)}(t_0)$, while the approximate state equation (31), though depending on $\phi^{(1)}(\vec{r},t_0)$, is linear (and local) with respect to $\Psi^{(1)}(t_0)$. Thus, the system of Eqs. (31) and (33) may be solved in two steps, starting from the second equation that can be used to find the inverse-time field $\phi^{(1)}(\vec{r},t_0)$ in terms of the matter density $\rho^{(0)}(\vec{r},t_0)$, known from Eq. (29), and of imposed boundary conditions (among these the boundary condition at an $\vec{r}=\vec{r}_0$: $\phi^{(1)}(\vec{r}_0,t_0)=0$ or $t(\vec{r}_0,t_0)=t_0$ which defines the meaning of t_0). Then, the energy width $\Gamma^{(1)}(t_0)$ calculated from Eq. (32), when inserted into the state equation (31), enables us to find readily the state vector $\Psi^{(1)}(t_0)$. In fact, Eq. (27) takes then the form

$$\Psi^{(1)}(t_0) = \Psi_E(t_0)\exp\left[-\frac{1}{\hbar}\int_0^{t_0} dt_0'\,\Gamma^{(1)}(t_0')\right]\,, \tag{34}$$

where $\Psi_E(t_0)$ satisfies the exact state equation in the equilibrium picture (26), identical with Eq. (30) for $\Psi^{(0)}(t_0)$. Thus,

$$\Psi_E(t_0) = \Psi^{(0)}(t_0) \tag{35}$$

is known by our perturbative assumption, and such is also $\Psi^{(1)}(t_0)$ as given in Eq. (34).

We can see that the norm of the state $\Psi^{(1)}(t_0)$ is, in general, *slightly changing* (sometimes jumping up, but most of the time diminishing towards relaxed values), what is a new, perhaps surprising, effect of our theory of small deviations from the temporal equilibrium of the so called isolated systems (for an example *cf.* Appendix B). In consequence, a tiny *unitarity defect* generally occurs for the S matrix.

In our theory, valid only for not too large deviations from the temporal equilibrium, the approximation around the temporal equilibrium, leading to Eqs. (31) and (33), would be expected to work excellently in most cases.

On the ground of the chronodynamic equation (23) considered in the whole space, it can be easily seen that the temporal equilibrium, where $\phi(\vec{r}, t_0) \equiv 0$, is *necessarily* realized if $g_\Gamma = 0$ or $\partial\rho/\partial t_0 = 0$ (the latter always occurs for *stationary* state vectors $\Psi(t_0) = \Psi(0)\exp(-i\,E\,t_0/\hbar)$), while $\phi(\vec{r}_0, t_0) = 0$ and, in addition, $\mathrm{grad}\phi(\vec{r}_0, t_0) = 0$ at an $\vec{r} = \vec{r}_0$. Indeed, in these cases, Eq. (23) reduces to its particular form (13) (with $\kappa_\Gamma = \lambda_\Gamma c$),

$$\left(\Delta - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t_0}\right)\phi(\vec{r}, t_0) = 0\,, \tag{36}$$

which for $\phi(\vec{r}, t_0)$ *regular everywhere* (as implied by $g_\Gamma = 0$ or $\partial\rho/\partial t_0 = 0$) shows that $\phi(\vec{r}, t_0) \equiv 0$ or $t(\vec{r}, t_0) \equiv t_0$ *since* $\phi(\vec{r}_0, t_0) = 0$ and $\mathrm{grad}\phi(\vec{r}_0, t_0) = 0$ at $\vec{r} = \vec{r}_0$ for any t_0. This follows from the fact that in the complete set of independent solutions to Eq. (36) (all finite at $t_0 \to \infty$),

$$\phi_\nu(\vec{r}, t_0) = \mathrm{Re}\left[A_\nu \exp\left(i\,\vec{e}_\nu\sqrt{\frac{\gamma_\nu}{\lambda_\Gamma c}}\cdot\vec{r} - \gamma_\nu t_0\right)\right] \tag{37}$$

with $|\,\vec{e}_\nu\,| = 1$ and $\gamma_\nu \geq 0$, all complex amplitudes A_ν must vanish because of $\phi_\nu(\vec{r}_0, t_0) = 0$ and $\mathrm{grad}\phi_\nu(\vec{r}_0, t_0) = 0$.

Obviously, in the temporal equilibrium, where $\phi(\vec{r}, t_0) \equiv 0$, we get $\Gamma(t_0) \equiv 0$ from Eq. (22), thus the state equation (21) reduces to its *conventional* form

$$i\,\hbar\,\frac{d\Psi(t)}{dt} = H\Psi(t) \tag{38}$$

which implies the time-evolution operator (1). This conventional state equation may be valid in any Minkowski frame of reference, as it is in the conventional, relativistic quantum field theory.

Concluding the fourth part of this note, we would say that the system of coupled equations (21) and (23) (or, rather, (31) and (33)) could show, how the virtual reality of small deviations from the temporal equilibrium might be realized.

APPENDIX A

Relativistic chronodynamic equation

Let us assume the inverse-time field in the relativistically covariant form suggested in Section 4 :

$$\phi_{\rm rel}(\vec{r}, t_0) \equiv \frac{c}{n(x)\cdot x} - \frac{c}{n(x_0)\cdot x_0}, \tag{A.1}$$

where $x = (c\,t(\vec{r}, t_0), \vec{r})$ and $x_0 = (c\,t_0, \vec{r}_0)$ with $t(\vec{r}_0, t_0) = t_0$, while $n(x)$ is the unit four-vector perpendicular to our (space-like) time hypersurface $t = t(\vec{r}, t_0)$ at the spacetime point x and directed to the future. Hence,

$$c^{-1}n(x)\cdot x \equiv \frac{c^{-1}\,n(x_0)\cdot x_0}{1 + \phi_{\rm rel}(\vec{r}, t_0)c^{-1}n(x_0)\cdot x_0} \tag{A.2}$$

is a relativistically covariant time field.

Now, for reasons which will be apparent later, let us consider the modified inverse-time field

$$\chi_{\rm rel}(\vec{r}, t_0) \equiv \phi_{\rm rel}(\vec{r}, t_0)\exp\frac{n(x_0)\cdot x_0}{2\lambda_\Gamma} \tag{A.3}$$

and tentatively impose on it the following *relativistic chronodynamic equation* in the coordinates $\vec{r}, t_0$:

$$\left(\Box_{t_0} + \frac{1}{4\lambda_\Gamma^2}\right)\chi_{\rm rel}(\vec{r}, t_0) = 4\pi\, g_\Gamma \lambda_\Gamma\, \partial_{t_0}\cdot j(\vec{r}, t_0), \tag{A.4}$$

where $\Box_{t_0} \equiv \Delta - c^{-2}(\partial/\partial t_0)^2$ and $\partial_{t_0}\cdot j \equiv c^{-1}\partial j_0/\partial t_0 + {\rm div}\vec{j}$, while

$$j^\mu(\vec{r}, t_0) \equiv \langle \Psi(t_0) \mid J^\mu(\vec{r}) \mid \Psi(t_0)\rangle_{\rm av} \tag{A.5}$$

with $(J^\mu(\vec{r}))$ being the operator of total matter-current four-vector (in the Schrödinger picture). Of course, this current is not conserved in the Heisenberg picture, since the total number of bosons may change. Notice the plus sign at $(4\lambda_\Gamma^2)^{-1}$ in Eq. (A.4), so the covariant parameter-valued field $\chi_{\rm rel}(\vec{r}, t_0)$ in the free case has a *tachyonic* character.

The chronodynamic equation (A.4) is really relativistic if $j = (j^\mu) = (c\,\rho, \vec{j})$ as defined in Eq. (A.5) is a true four-vector, what depends on the actual covariance of the state vector $\Psi(t_0)$ involved in the definition (A.5). In order to guarantee this covariance we propose to replace in the state equation (21) the energy width $\Gamma(t_0)$, given in Eq. (22), by the following actually covariant form (making use of $\phi_{\rm rel}(x_{t_0}) \equiv \phi_{\rm rel}(\vec{r}, t_0)$ and $j^\mu(x_{t_0}) \equiv j^\mu(\vec{r}, t_0)$ introduced in Eqs. (A.1) and (A.5)):

$$\Gamma(t_0) \equiv g_\Gamma \hbar \int d^3\vec{r} n(x) \cdot j(x_{t_0}) \phi_{\rm rel}(x_{t_0}) \,. \tag{A.6}$$

Indeed, its evidently covariant version,

$$\Gamma[\sigma_0] \equiv \frac{g_\Gamma \hbar}{c} \int_{\sigma_0} d^3\sigma_0 n(x) \cdot j(x_{t_0}) \phi_{\rm rel}(x_{t_0}) \,, \tag{A.7}$$

may be constructed by means of the Tomonaga–Schwinger time hypersurface $\sigma = \sigma_0$ generalizing covariantly the time hyperplane $t = t_0$, where t_0 is time running at the particular space point $\vec{r}_0$. In Eqs. (A.6) and (A.7) the coordinates of the spacetime point $x = (ct(\vec{r}, t_0), \vec{r})$ in $n(x)$ are expressed by the coordinates of the spacetime point $x_{t_0} = (ct_0, \vec{r})$ used in both integrations. Note that the normal four-vector field $n_0(x_{t_0})$ corresponding to σ_0 is time-like but otherwise arbitrary, while the normal four-vector field $n(x)$ of our time hypersurface $t = t(\vec{r}, t_0)$, appearing in Eqs. (A.1) and (A.7), is dynamically determined.

Then, the state equation (21) with $\Gamma(t_0)$ given in Eq. (A.6) is valid in any Minkowski frame of reference (of course, in the case of relativistic quantum field theory). In fact, through the unitary transformation

$$\Psi_M(t_0) = \exp\left(-\frac{i}{\hbar} H t_0\right) \Psi(t_0) \tag{A.8}$$

to a new modified Heisenberg picture, Eq. (21) transits into the form

$$i\hbar \frac{d\Psi_M(t_0)}{dt_0} = -i\Gamma(t_0)\Psi_M(t_0) \tag{A.9}$$

which is easily seen to be actually covariant. Indeed, due to Eq. (A.7), it may be presented in the evidently covariant version

$$i\hbar \frac{\delta \Psi_M[\sigma_0]}{\delta\sigma_0(x_{t_0})} = -\frac{ig\hbar}{c} n(x) \cdot j(x_{t_0}) \phi_{\rm rel}(x_{t_0}) \Psi_M[\sigma_0] \tag{A.10}$$

by means of the Tomonaga–Schwinger time hypersurface $\sigma = \sigma_0$. Again, x in $n(x)$ is expressed through x_{t_0} lying on σ_0. Here, $j^\mu(x_{t_0}) \equiv j^\mu(\vec{r}, t_0)$ as defined in Eq. (A.5) involves $\Psi_M[\sigma_0]$: $j^\mu(x_{t_0}) \equiv \langle \Psi_M[\sigma_0] \mid J^\mu_M(x_{t_0}) \mid \Psi_M[\sigma_0] \rangle_{\rm av}$ with x_{t_0} lying on σ_0. The dynamical character of the unit four-vector $n(x)$ (with x expressed through x_{t_0}) makes the covariance of Eq. (A.10) real. The coupling $n(x) \cdot j(x_{t_0}) \phi_{\rm rel}(x_{t_0})$ in Eq. (A.10) may be also rewritten in the form $j(x_{t_0}) \cdot A^{\text{time}}(x_{t_0})$, where $A^{\text{time}}_{\mu}(x_{t_0}) = n_{\mu}(x)\phi_{\text{rel}}(x_{t_0})$ gives us a (parameter-valued) dynamical four-vector of the inverse time.

Defining the noncovariant parameter-valued field

$$\phi(\vec{r}, t_0) \equiv \chi_{\text{rel}}(\vec{r}, t_0) \exp \frac{-ct_0}{2\lambda_{\Gamma}} \equiv \phi_{\text{rel}}(\vec{r}, t_0) \exp \frac{n(x_0) \cdot x_0 - c\, t_0}{2\lambda_{\Gamma}}, \quad \text{(A.11)}$$

we can rewrite Eq. (A.4) in the equivalent form :

$$\left(\triangle - \frac{1}{\lambda_{\Gamma} c} \frac{\partial}{\partial t_0} - \frac{1}{c^2} \frac{\partial^2}{\partial t_0^2} \right) \phi(\vec{r}, t_0) = 4\pi\, g_{\Gamma} \lambda_{\Gamma}\ [\partial_{t_0} \cdot j(\vec{r}, t_0)] \exp \frac{-c\, t_0}{2\lambda_{\Gamma}}. \quad \text{(A.12)}$$

The definitions (A.11) and (A.1) show that for small deviations from the temporal equilibrium, where $t(\vec{r}, t_0) \equiv t_0$ and so $n(x) \equiv (1, 0, 0, 0)$, we have approximately

$$\phi(\vec{r}, t_0) \simeq \phi_{\text{rel}}(\vec{r}, t_0) \simeq \frac{1}{t(\vec{r}, t_0)} - \frac{1}{t_0}. \quad \text{(A.13)}$$

Neglecting in Eq. (A.12) the second time derivative $c^{-2}\partial^2\phi/\partial t_0^2$ in comparison with the first, we obtain the following nonrelativistic equation for $\phi(\vec{r}, t_0)$:

$$\left(\triangle - \frac{1}{\lambda_{\Gamma} c} \frac{\partial}{\partial t_0} \right) \phi(\vec{r}, t_0) = 4\pi\, g_{\Gamma} \lambda_{\Gamma}\ [\partial_{t_0} \cdot j(\vec{r}, t_0)] \exp \frac{-c\, t_0}{2\lambda_{\Gamma}}. \quad \text{(A.14)}$$

In Eqs. (A.4), (A.12) and (A.14) the four-divergence on their rhs can be expressed as follows:

$$\partial_{t_0} \cdot j(\vec{r}, t_0) = \langle \Psi(t_0) \mid \frac{i}{\hbar} [P_{\mu}, J^{\mu}(\vec{r})] - \frac{2}{\hbar c} \Gamma(t_0) J^0(\vec{r}) \mid \Psi(t_0) \rangle_{\text{av}}, \quad \text{(A.15)}$$

where $(P_{\mu}) = (H/c, \vec{P})$ is the operator of total matter-momentum four-vector (in the Schrödinger picture). In fact,

$$\langle \Psi(t_0) \mid J^{\mu}(\vec{r}) \mid \Psi(t_0) \rangle_{\text{av}} = \langle \Psi(0) \mid e^{\frac{i}{\hbar} P \cdot x_{t_0}} J^{\mu}(0) e^{-\frac{i}{\hbar} P \cdot x_{t_0}} \mid \Psi(0) \rangle_{\text{av}} \times e^{-\frac{2}{\hbar} \int_0^{t_0} dt_0' \Gamma(t_0')}, \quad \text{(A.16)}$$

where $x_{t_0} = (c\, t_0, \vec{r})$, while $\partial_{t_0} = (c^{-1}\partial/\partial t_0, \partial/\partial \vec{r})$.

In the case, when $\Psi(t_0)$ are *eigenstates* of the total *matter momentum* $\vec{P}$, we get from Eq. (A.15)

$$\partial_{t_0} \cdot j(\vec{r}, t_0) = \langle \Psi(t_0) \mid \frac{i}{\hbar c} [H, J^0(\vec{r})] - \frac{2}{\hbar c} \Gamma(t_0) J^0(\vec{r}) \mid \Psi(t_0) \rangle_{\text{av}} = \frac{\partial}{\partial t_0} \rho(\vec{r}, t_0) \quad \text{(A.17)}$$

with $j^0 = c\rho$. Thus, inserting Eq. (A.17) into Eq. (A.14) we deduce in this case our previous nonrelativistic chronodynamic equation (23) for the field $\phi(\vec{r}, t_0)$ as given in Eq. (A.13) for small deviations from the temporal equilibrium. Generally, Eq. (23) holds if $\mathrm{div}\vec{j} = 0$.

When, in addition, $\Psi(t_0)$ are *eigenstates* of the total *matter energy H* (they are *stationary states* if $\Gamma(t_0) \equiv 0$), we obtain from Eq. (A.17)

$$\partial_{t_0} \cdot j(\vec{r}, t_0) = \frac{\partial}{\partial t_0} \rho(\vec{r}, t_0) = -\frac{2}{\hbar} \Gamma(t_0)\, \rho(\vec{r}, t_0)\,, \tag{A.18}$$

where

$$\rho(\vec{r}, t_0) \;= \rho(\vec{r}, 0)\, \exp \left[-\frac{2}{\hbar} \int_0^{t_0} dt'_0\, \Gamma(t'_0) \right] . \tag{A.19}$$

In general, if $\partial_{t_0} \cdot j(\vec{r}, t_0) = 0$, Eq. (A.4) (with the boundary condition at an $\vec{r} = \vec{r_0}$: $\chi_{\rm rel}(\vec{r}_0, t_0) = 0$ and, in addition, $\mathrm{grad}\chi_{\rm rel}(\vec{r}_0, t_0) = 0$) implies the relativistically covariant temporal equilibrium : $\chi_{\rm rel}(\vec{r}, t_0) \equiv 0$ or $\phi_{\rm rel}(\vec{r}, t_0) \equiv 0$, and hence $n(x)\cdot x \equiv n(x_0)\cdot x_0$, where $x = (c\, t(\vec{r}, t_0), \vec{r})$. Of course, in such a temporal equilibrium — in agreement with its definition in Section 4 — the normal four-vector field $n(x)$ is not determined, so we can always put $n(x) \equiv (1, 0, 0, 0)$, what gives $t(\vec{r}, t_0) \equiv t_0$.

On the base of Eq. (A.14), the perturbative chronodynamic equation (33) should now be replaced by

$$\left(\triangle - \frac{1}{\lambda_\Gamma c} \frac{\partial}{\partial t_0} \right) \phi^{(1)}(\vec{r}, t_0) = 4\pi\, g_\Gamma \lambda_\Gamma \left[\partial_{t_0} \cdot j^{(0)}(\vec{r}, t_0) \right] \exp \frac{-c\, t_0}{2\lambda_\Gamma}\,, \tag{A.20}$$

where

$$j^{\mu(0)}(\vec{r}, t_0) \equiv \langle \Psi^{(0)}(t_0) \mid J^\mu(\vec{r}) \mid \Psi^{(0)}(t_0) \rangle_{\rm av} \tag{A.21}$$

with the zeroth-order state vector $\Psi^{(0)}(t_0)$ satisfying the conventional state equation (30). Of course, $j^{0(0)} = c\rho^{(0)}$ is the same as given through Eq. (29). The first-order state vector $\Psi^{(1)}(t_0)$ fulfils Eq. (31) with $\Gamma^{(1)}(t_0)$ determined by Eq. (32), and has the form given in Eqs. (34) and (35).

APPENDIX B

Time field created by the excitation of a heavy target

Let us consider an extension of our pedagogical example which now is intended to simulate (on the level of the average matter current (A.5)) the excitation of a fixed spherical target, centred at the point $\vec{r} = 0$, in an act of inelastic scattering of a particle at the moment $t_0 = 0$. To this end, we put in the zeroth perturbative order

$$\rho^{(0)}(\vec{r}, t_0) = \Theta(-t_0)\rho^{(0)}_{\rm inc}(\vec{r}, t_0) + \Theta(t_0)\rho^{(0)}_{\rm sc}(\vec{r}, t_0) \quad \text{(B.1)}$$

and similarly for $\vec{j}^{(0)}(\vec{r}, t_0)$, where

$$\rho^{(0)}_{\rm inc}(\vec{r}, t_0) = \delta^3(\vec{r} - \vec{v}\,t_0)\,, \quad \vec{j}^{(0)}_{\rm inc}(\vec{r}, t_0) = \vec{v}\delta^3(\vec{r} - \vec{v}\,t_0) \quad \text{(B.2)}$$

and

$$\rho^{(0)}_{\rm sc}(\vec{r}, t_0) = AF(r^2)\exp(-\gamma\, t_0) + B\delta^3(\vec{r} - \vec{v}\,'t_0)\,, \; \vec{j}^{(0)}_{\rm sc}(\vec{r}, t_0) = B\vec{v}\,'\delta^3(\vec{r} - \vec{v}\,'t_0) \quad \text{(B.3)}$$

with some dimensionless constants $A > 0$ and $B > 0$. Here, $\Theta(t_0) = 1$ or 0 for $t_0 > 0$ or $t_0 < 0$, respectively. The particle momenta and energies are

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - (v/c)^2}}\,,\; \vec{p}\,' = \frac{m'\vec{v}\,'}{\sqrt{1 - (v'/c)^2}} \quad \text{(B.4)}$$

and

$$E = c\sqrt{p^2 + (mc)^2}\,,\; E' = c\sqrt{p'^2 + (m'c)^2}, \quad \text{(B.5)}$$

satisfying the balance $E = E' + (M^* - M)c^2$ up to the width $\hbar\gamma$ of the target excited level $M^* c^2$.

In order to apply the perturbative chronodynamic equation (A.20), we calculate the four-divergences:

$$\partial_{t_0} \cdot j^{(0)}_{\rm inc} \equiv \frac{\partial \rho^{(0)}_{\rm inc}}{\partial t_0} + {\rm div} j^{(0)}_{\rm inc} = 0 \quad \text{(B.6)}$$

and

$$\partial_{t_0} \cdot j^{(0)}_{\rm sc} \equiv \frac{\partial \rho^{(0)}_{\rm sc}}{\partial t_0} + {\rm div} j^{(0)}_{\rm sc} = -\gamma AF(r^2)\exp(-\gamma\, t_0)\,. \quad \text{(B.7)}$$

Thus, from Eq. (A.20) we obtain in the first order

$$\phi^{(1)}(\vec{r}, t_0) = \Theta(-t_0)\phi^{(1)}_{\rm inc}(\vec{r}, t_0) + \Theta(t_0)\phi^{(1)}_{\rm sc}(\vec{r}, t_0) \quad \text{(B.8)}$$

(for $t_0 \neq 0$), where $\phi^{(1)}_{\rm inc}(\vec{r}, t_0) \equiv 0$ and

$$\phi^{(1)}_{\rm sc}(\vec{r}, t_0) = -4\pi g_\Gamma \lambda_\Gamma \gamma A \left(\Delta - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t_0}\right)^{-1} F(r^2)\exp(-\gamma_\Gamma\, t_0) \quad \text{(B.9)}$$

with $\gamma_\Gamma = \gamma + c/2\lambda_\Gamma$. Inserting

$$F(r^2) = \frac{1}{(2\pi)^3}\int d^3\vec{k}\, f(k^2)\exp(i\vec{k}\cdot\vec{r}) = \frac{1}{(2\pi)^2}\frac{1}{ir}\int_{-\infty}^{\infty} k\, dk\, f(k^2)\exp(ikr) \tag{B.10}$$

and choosing for the Green function in Eq. (B.9) the Feynman convention $k^2 \to k^2 - i\varepsilon$ where $\varepsilon \to +0$, we find taking the real part:

$$\phi^{(1)}_{\rm sc}(\vec{r}, t_0) = g_\Gamma\lambda_\Gamma\gamma A\, f(\gamma_\Gamma/\lambda_\Gamma c)\,\frac{1}{r}\cos(\sqrt{\gamma_\Gamma/\lambda_\Gamma c}\; r)\exp(-\gamma_\Gamma\, t_0)\,. \tag{B.11}$$

Note that $f(0) = 1$, when the space integral of $F(r^2)$ is normalized to 1.

Making use of Eqs. (B.3) and (B.9) we can calculate from Eq. (32) the first-order energy width

$$\begin{aligned}\Gamma^{(1)}(t_0) &= \Theta(t_0)g_\Gamma\hbar\int d^3\vec{r}\,\rho^{(0)}_{\rm sc}(\vec{r}, t_0)\phi^{(1)}_{\rm sc}(\vec{r}, t_0)\\ &= \Theta(t_0)\, g^2_\Gamma\lambda_\Gamma\hbar\gamma\, f\left(\frac{\gamma_\Gamma}{\lambda_\Gamma c}\right)\, {\rm e}^{-\gamma_\Gamma\, t_0}\\ &\times\left\{\left[A^2\int_0^\infty r\,dr F(r^2)\cos(\sqrt{\frac{\gamma_\Gamma}{\lambda_\Gamma c}}r)\right]{\rm e}^{-\gamma t_0} + AB\frac{1}{v' t_0}\cos(\sqrt{\frac{\gamma_\Gamma}{\lambda_\Gamma c}}v' t_0)\right\},\end{aligned} \tag{B.12}$$

where g^2_Γ is very small.

The result (B.12), when inserted into Eq. (31), determines the state equation for $\Psi^{(1)}(t_0)$, whose solution has the form given in Eqs. (34) and (35).

The formula (B.12) shows that $\Gamma^{(1)}(t_0) \to \infty$ at $t_0 \to +0$ (though $\Gamma^{(1)}(t_0) = 0$ for $t_0 < 0$), and then $\Gamma^{(1)}(t_0) =\to 0$ with $t_0 \to \infty$. This means that at the moment of excitation of the matter system (in our case, in the act of inelastic collision) a time field is created, causing an amount of energy width (in our case, infinite) to be transferred to the system from the physical spacetime represented by the time field, perturbing thereby the temporal equilibrium. Afterwards, in the process of its deexcitation, the matter system loses its energy width to the physical spacetime still represented by the time field, tending to the temporal equilibrium.

Though in our case, $\Gamma^{(1)}(t_0) = 0(g^2_\Gamma)$ and $\Gamma^{(1)}(t_0) =\to 0$ with $t_0 \to \infty$, the time integral of $\Gamma^{(1)}(t_0)$ from $t_0 = 0$ to any $t_0 > 0$, instead of being very small, is divergent to ∞ due to the singularity $\Theta(t_0)/t_0$ of $\Gamma^{(1)}(t_0)$ at $t_0 = 0$. This requires a kind of renormalization or other reformulation of our pedagogical example. Note that such a singularity is caused by the interaction of the time field of the excited target with the scattered point-like particle.

It is also seen from Eq. (B.12) that $\Gamma^{(1)}(t_0) \to 0$ for $\hbar\gamma \to 0$, what relates the temporal equilibrium of the system to its quantum-mechanical stability.

As it was already emphasized, in the mixed theory described in this note the physical spacetime is represented by the parameter-valued time field (or inverse-time field) having the status of a thermodynamic quantity, called chronodynamic in this case.

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