Lipkin-Nogami Method for Rotating Nuclei

Piotr Magierski

Institute of Physics, Warsaw University of Technology,
ul. Koszykowa 75, PL-00-662 Warsaw.

The superconductivity approximation provides us with a reasonably simple treatment of the pairing interaction in nuclei. Nevertheless this method has a drawback in that it introduces an uncertainty of the nucleon number. The accuracy of this approximation is satisfactory if the interaction is very strong or the number of particles is very large. These conditions are often not satisfied in problems dealing with actual nuclei. It has been shown that the effect of the nucleon number fluctuation can be suppressed by using a model Hamiltonian $\hat{H} - \lambda_1 \hat{N} - \lambda_2 \hat{N}^2$ instead of the conventional $\hat{H} - \lambda \hat{N}$, where $\hat{H}$ is the original pairing Hamiltonian and $\hat{N}$ is the nucleon number operator. This method was first suggested by Lipkin [1], and developed by Nogami and his collaborators for nonrotating systems [2,3,4,5]. The important feature of the LN method is that there always exist a nontrivial (superfluid) solution regardless of the strength of pairing force.

Therefore it seems to be interesting to extend the method to the case of rotating nuclei, where the short-range attraction acting between the nucleons (pairing force) seems also to play a significant role for the coupling scheme. The pairing force tends to couple nucleons in pairs with zero angular momentum counteracting the Coriolis force and leads to the formation of superfluid correlations in the nucleus. The superfluid correlations manifest themselves by the existence of an energy gap separating the nuclear ground state in the even nucleus from the excited states of the single particle nature. In addition, the superfluid correlations tend to reduce considerably the nuclear moment of inertia as compared to its standard estimate following from the assumption of a rigid-body rotation. On the other hand it is well known that at the very high spin nuclei behave as macroscopic rotors, i.e. their moments of inertia are constant and close to the rigid body values. In this connection further studies of the Mottelson-Valatin and Stephens-Simon effects which lead to the phase transition from superfluid to normal state in rotating nuclei seem to be of great importance [6].

In the case of two-dimensional rotation the result of Coriolis and centrifugal forces are provided by the term $-\omega \hat{J}_x$. Thus in LN method we have to consider the following operator: $\hat{K} = \hat{H} - \lambda_1 \hat{N} - \lambda_2 \hat{N}^2 - \omega \hat{J}_x$. From the condition $\delta\langle \hat{K} \rangle = 0$ one can obtain the following equations:

$$\begin{aligned} \sum_l \{(\nu^{\omega}_{kl} - \lambda\delta_{kl})A_{il} + \Delta_{kl}B_{il}\} &= \mathcal{E}^{\omega}_i A_{ik} \\ \sum_l \{(\nu^{\omega *}_{\bar{k}\bar{l}} - \lambda\delta_{kl})B_{il} + \Delta^{*}_{kl}A_{il}\} &= -\mathcal{E}^{\omega}_i B_{ik}, \end{aligned} \tag{1}$$

where

$$\nu^{\omega}_{kl} = \epsilon_{kl} - \omega(j_x)_{kl}, \tag{2}$$

$$\epsilon_{kl} = e_k\delta_{kl} - Gsgn(k)sgn(l)\rho_{\bar{k}\bar{l}} + 4\lambda_2\rho^{*}_{kl}, \tag{3}$$

$$\lambda = \lambda_1 + 2\lambda_2(N+1), \tag{4}$$

$$\Delta_{kl} = -G\delta_{k\bar{l}}sgn(k)\sum_{k>0}\chi_{k\bar{k}}. \tag{5}$$

As it is easly seen the above equations have the well known form of Hartree-Fock-Bogolyubow equations. The only difference originated from the appearance of parameter λ_2 which is a state dependent quantity and can be calculated from the formula:

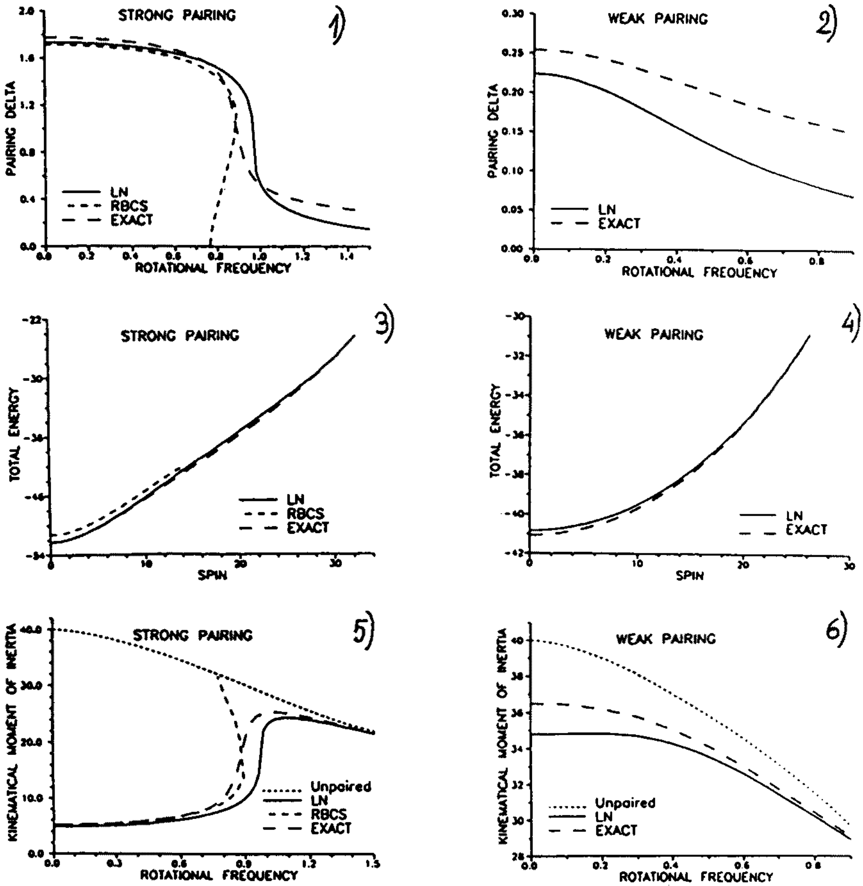
$$\lambda_2 = \frac{G}{4}\frac{\sum_{k,l>0}\{\chi^{*}_{k\bar{l}}(\rho^{*}_{\bar{k}\bar{l}} + \rho_{kl})\}\sum_{k,l>0}\{\chi_{k\bar{l}}(u_{\bar{k}\bar{l}} + u^{*}_{kl})\} - 2\sum_{k,l}(\chi\chi^{+})_{lk}(\chi\chi^{+})_{\bar{l}\bar{k}}}{(Tr(\chi\chi^{+}))^2 - 2Tr(\chi\chi^{+}\chi\chi^{+})}. \tag{6}$$

After solving the equations (1) one can find the ground state energy of the system:

$$E_{LN} = \sum_k e_k\rho_{kk} - \frac{\Delta^2}{G} - \frac{1}{2}G\sum_{k,l}\rho_{kl}\rho_{\bar{k}\bar{l}} - 2\lambda_2\sum_{k,l}\rho_{kl}u_{kl}. \tag{7}$$

Other quantities, like spin or kinematical moment of inertia are calculated from the standard HFB formulas. The comparison of LN method and RBCS[1] method with an exact solution was done for simple two-level model of Krumlinde-Szymanski [7] (see fig.(1),(2),...,(6)).

[1] BCS method for rotating systems.



References

[1] H.J. Lipkin, Ann. of Phys., **9**, 272(1960)

[2] Y. Nogami, Phys. Rev., **B313**, 134(1964)

[3] Y. Nogami, Phys. Lett., Vol. 15, 4(1965)

[4] J.F. Goodfellow, Y. Nogami, Can. J. Phys., **44**, 1321(1966)

[5] H.C. Pradhan, Y. Nogami, J. Law, Nucl. Phys., **A201**, 357(1973)

[6] W. Nazarewicz, J. Dudek, Z. Szymański, Nucl. Phys., **A436**, 139(1985)

[7] J. Krumlinde, Z. Szymański, Ann. Phys.(N.Y.), **79**, 201(1973)