INTERACTION OF A QUARK WITH THE PERTURBATIVE "MAGNETIC VACUUM"*

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Dedicated to Janusz Dąbrowski in honour of his 65th birthday

The angular distribution for a quark scattering from the modes of the magnetic vacuum is calculated in the first approximation. One finds different behaviors of the light and heavy quarks. Similarities to the fast moving electron in a ionic crystal (polaron) are pointed out.

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It is an honor and a pleasure to dedicate this paper to my old friend Janusz Dąbrowski. Since this note discusses an "optical potential" of a quark, it may be of interest to Janusz who has worked on the nuclear optical potentials.

1. Introduction

In order to understand both the qualitative and the quantitative features of the asymptotic freedom in an intuitive way it has been proposed to cast the QCD perturbative vacuum in the form of a magnetic medium [1]. Indeed, QCD vacuum is very similar to a paramagnetic substance and it can be identified with the following prototype system [2]: a charged, massless free field (boson or fermion), of charge g and spin S, interacting with a weak homogeneous external magnetic field pointed *e.g.* along the z-axis

$$\vec{H} = \vec{e}H \,. \tag{1.1}$$

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This system has a spectrum of modes whose energies are labelled by the three quantum numbers k_z, n, S_z (z-component of the momentum, the harmonic oscillator quantum no, z-component of the spin of the fields involved)

$$E(k_z, n, S_z) = [k_z{}^2 + 2gH(n + \tfrac{1}{2} - S_z)]^{1/2},$$

$$n = 0, 1, 2, ..., \tag{1.2}$$

$$S_z = \pm S.$$

Here g is the coupling constant.

To determine the Gell-Mann–Low function one computes first the vacuum energy per unit volume (by summing the energies of the modes (2)). This part of the energy which depends on H scales as H^2 , and its proportionality coefficient determines the susceptibility, from which the leading term of the GL function follows [2]. It agrees with the results obtained from more formal treatments of the QCD vacuum.

The purpose of this note is to test a little further this model of the perturbative vacuum as a paramagnetic substance on a somewhat academic system of one quark moving through such vacuum and polarizing it. A quark moving with its accompanying distortion of the vacuum is an analogue of a polaron (an electron moving in an ionic crystal) [3].

One is also tempted to speculate that the results of the calculations presented below are relevant for deep inelastic lepton-nucleon interactions. Indeed, quarks sitting inside the nucleon may be immersed in the magnetic vacuum. Our calculations imply that the behavior of the light (*e.g.* $m = 0.01$ GeV) and heavy (*e.g.* $m = 100.0$ GeV) quarks set in motion in the magnetic vacuum is quite different [4]: light quarks scatter from the $n = 0$ state of the gluonic field (compare (1.2)) in a well defined and very narrow forward cone, while the angular distribution of the heavy quarks does not look like a cone. This first case reminds one of the cone exhibited by a fast electron moving through an ionic crystal [3], and the hadronic analogue of the Cherenkov cone conjectured long time ago [5]. Note that although we are discussing the angular distribution of a quark, the emerging gluons must form a similar cone due to the momentum conservation.

2. Transition rate for a quark moving through magnetic vacuum

We do the calculations employing the time dependent perturbation theory. First, we write the operator of the unperturbed Hamiltonian

$$\mathcal{H}_0 = (p^2 + m^2)^{1/2}$$

$$+ \sum_{k_y, k_z, n, S_z} \left[k_z{}^2 + 2gH(n + \tfrac{1}{2} - S_z)\right]^{1/2} a^+(k_y, k_z, n, S_z) a(k_y, k_z, n, S_z), \tag{2.1}$$

where a^+, a are the creation and annihilation operators of the modes (1.2), and $L^3 = V$ is the normalization volume. $\int(Ldk_y/2\pi)$ reduces to the degeneracy factor $gH/2\pi$ (compare *e.g.* [1]) which takes care of the positions of the centers of classical orbits which could be anywhere in the plane normal to $\vec{H}$. We keep explicitly the k_y dependence because we shall need it to discuss the energy-momentum conservation (see below).

The time dependent interaction Hamiltonian density operator is

$$\mathcal{H}'(x,t) = g\overline{\psi}_{\vec{p}'}(\vec{x},t)\gamma_\mu W^\mu(\vec{x},t)\psi_{\vec{p}}(\vec{x},t), \tag{2.2}$$

where $\psi_{\vec{p}}(\vec{x},t) = \sqrt{\frac{m}{EV}}u(\vec{p})\exp[-i(p_0 t - \vec{p}\cdot\vec{x})]$ is the four component spinor and the gluon field operator $W^\mu(\vec{x},t)$ is expanded into the modes (2) as follows [1]:

$$W_\mu(\vec{x},t) = e_\mu \int \frac{Ldk_y}{2\pi} \int \frac{Ldk_z}{2\pi} \sum_{n,S_z} \frac{e^{ik_y y}}{\sqrt{L}} \frac{e^{ik_z z}}{\sqrt{L}} \Phi_n(x - \frac{k_y}{gH})$$

$$\times \frac{e^{-iE(k_z,n,S_z)t}}{\sqrt{2E(k_z,n,S_z)}} a(k_y,k_z,n,S_z) + \text{c.c.}, \tag{2.3}$$

where Φ_n is the harmonic oscillator wave function of the n-th level. Our formulation is not relativistic invariant. There exist a distinctive frame of reference in which we may specify the vacuum field $W_\mu(\vec{x},t)$. We hypothesize that this is, *e.g.* for a lepton deep inelastic scattering from a quark in a nucleon, the rest frame of the nucleon.

The transition matrix element for the quark to change its energy by Δp_0 and momentum by $(\Delta p_x, \Delta p_y, \Delta p_z)$ and excite the mode k_y, k_z, n, S_z of the vacuum is

$$\mathcal{H}'_{\text{fi}} = \int d^3x dt \mathcal{H}'(\vec{x},t) = g \int \frac{Ldk_y}{2\pi}\frac{Ldk_z}{2\pi} \sum_{n,S_z} \frac{1}{L}\sqrt{\frac{m^2}{V^2 p'_0 p_0}}\overline{u}(\vec{p}')e^*_\mu \gamma^\mu u(\vec{p})$$

$$\times(2\pi)^3\delta(\Delta p_0 - E(k_z,n,S_z))\delta(\Delta p_y - k_y)\delta(\Delta p_z - k_z)e^{-i\left(\frac{\Delta p_x k_y}{gH}\right)}\tilde{\Phi}_n(\Delta p_x), \tag{2.4}$$

where

$$\tilde{\Phi}_n(\Delta p_x) = \int\limits_{-\infty}^{+\infty} \mathrm{e}^{-ix\Delta p_x} \Phi_n(x) dx.$$

From Eq. (2.4) we compute the transition rate for the quark to go from $\vec{p}$ to $\vec{p}\,'$ and the vacuum to get excited to the state given by $k_z = \Delta p_z, n, S_z$ (we set $\hbar = c = 1$)

$$\Gamma_{\mathrm{fi}} = \frac{2\pi}{\hbar} \langle |\mathcal{H}'_{\mathrm{fi}}|^2 \rangle \delta(\varepsilon_{\mathrm{f}} - \varepsilon_{\mathrm{i}}), \tag{2.5}$$

where ε_{f}, ε_{i} are the final and initial energies of the system, and $\langle \ldots \rangle$ denotes the averaging over the initial and summing over the final quark spin states. With the z-axis taken as the quantization axis we obtain the rate for the quark of momentum p to scatter through the angle θ and excite the ($k_z = \Delta p_z, n, S_z = \pm 1$) mode of the vacuum:

$$\Gamma'_{\mathrm{fi}} = \frac{g^2}{p'_0 p_0} (p'_0 p_0 + p' p \cos\theta - m^2)$$

$$\times \frac{(2\pi)^3}{L^4} \frac{|\tilde{\Phi}_n(p' \sin\theta \cos\phi)|^2}{2E(\Delta p_z, n, S_z)} \delta(\Delta p_0 - E(\Delta p_z, n, S_z)). \tag{2.6}$$

We write

$$\delta(\Delta p_0 - E(\Delta p_z, n, S_z)) = \frac{1}{|F'(p_{\mathrm{f}})|} \delta(p_{\mathrm{f}} - p'),$$

where F' is the derivative of

$$F(p') = \Delta p_0 - E(\Delta p_z, n, S_z)$$

$$= \sqrt{p^2 + m^2} - \sqrt{p'^2 + m^2} - \sqrt{(p - p' \cos\theta)^2 + 2gH(n + \tfrac{1}{2} - S_z)},$$

and p_{f} is the solution of $F(p_{\mathrm{f}}) = 0$. Note that for a given $0 < |\cos\theta| < 1$, $S_z = +1$, and $n = 0$, there is only one solution for p_{f} (in all relevant cases, that is — see below).

We compute the rate for the initial quark to loose the energy $E(p - p_{\mathrm{f}} \cos\theta, n, S_z)$ to the vacuum, and to scatter into the angle $(\theta, \theta + d\theta)$:

$$\frac{d\Gamma'_{\mathrm{fi}}(\theta)}{d\theta} = \frac{L^3}{(2\pi)^3} \int \sin\theta d\phi p'^2 dp' \Gamma'_{\mathrm{fi}}(p')$$

$$= g^2 \sin\theta \big(1 + v_{\mathrm{f}} v \cos\theta - \frac{m^2}{p_{0\mathrm{f}} p_0}\big) \frac{p_{\mathrm{f}}^2}{|F'(p_{\mathrm{f}})|} \frac{\int_0^{2\pi} d\phi |\tilde{\Phi}_n(p_{\mathrm{f}} \sin\theta \cos\phi)|^2}{2E(\Delta p_z, n, S_z) L}, \tag{2.7}$$

where v and v_f are the initial and final velocities of the quark.

Observe that in this process only one finite size object from the vacuum is being excited. Thus no wonder $d\Gamma'_{fi}/d\theta \to 0$ when $L \to \infty$. Its wave function in one dimension (along the x-axis) is given by the oscillator wave function $\Phi_n(x)$ whose spatial extension is

$$\Delta x \approx \sqrt{\frac{(n+\frac{1}{2})}{2gH}}. \tag{2.8}$$

Therefore in our normalization volume we have about

$$\frac{L}{\Delta x} \approx L\sqrt{\frac{2gH}{(n+\frac{1}{2})}}$$

such objects available, and our final formula for the rate is

$$\frac{d\Gamma_{fi}(\theta)}{d\theta} = L\sqrt{\frac{2gH}{(n+\frac{1}{2})}}\frac{d\Gamma'_{fi}(\theta)}{d\theta}$$

$$= g^2\sqrt{\frac{2gH}{(n+\frac{1}{2})}}\sin\theta\big(1+v_f v\cos\theta-\frac{m^2}{p_{0f}p_0}\big)\frac{p_f^2}{|F'(p_f)|}\frac{\int_0^{2\pi}d\phi|\tilde{\Phi}_n|^2}{2E}. \tag{2.9}$$

Let us consider the energy-momentum conservation given by (2.4). Let θ be the scattering angle of the quark when it excites one mode of the magnetic vacuum. We get

$$\cos\theta = \frac{p}{p_f} - \sqrt{\frac{(\Delta p_0)^2 - 2gH(n+\frac{1}{2}-S_z)}{p_f^2}}, \tag{2.10a}$$

$$\Delta p_0 = \sqrt{p^2+m^2} - \sqrt{p_f^2+m^2},$$

$$E(k_z,n,S_z) = \sqrt{(p-p_f\cos\theta)^2 + 2gH(n+\tfrac{1}{2}-S_z)}, \tag{2.10b}$$

with the following conditions imposed: $|\cos\theta| < 1$ and E being positive. From (2.4) we can see that $\Delta p_z = p - p_f\cos\theta = k_z, \Delta p_y = k_y$, and that Δp_x is determined, approximately, by the size of Φ_n. One can show that $|\cos\theta| < 1$ only when $H \neq 0$, $n = 0$ and $S_z = 1$. Also, we have to have $\Delta p_z \geq gH$ to avoid imaginary E.

So, the interaction of the quark with only one mode of the magnetic vacuum, $E(\Delta p_z \neq 0,\ n = 0,\ S_z = 1)$, makes possible elastic or inelastic scattering of the quark. Therefore, we specialize Eq. (2.9) for this particular mode. For heavy quarks, when $p_\perp \geq gH$, we give the following estimate

$$\int_0^{2\pi} d\phi |\tilde{\Phi}_0(p_f \sin\theta\cos\phi)|^2 = \frac{2}{\sqrt{\pi gH}} \int_{-1}^{+1} \frac{dz}{\sqrt{1-z^2}} e^{-\frac{p_\perp^2 z^2}{gH}} \cong \frac{2\pi}{p_f \sin\theta}, \quad (2.11)$$

Inserting (2.11) into (2.9) we obtain for $p_f \sin\theta \gg \sqrt{gH}$ the rate of interactions of one quark with the "magnetic" vacuum:

$$\frac{d\Gamma_{fi}^{(n=0)}}{d\theta} = g^2 \pi p_f \sqrt{2gH} \frac{\left(1 + v_f v \cos\theta - \frac{m^2}{p_{f0} p_0}\right)}{|v_f E - (p - p_f \cos\theta)\cos\theta|}, \quad (2.12)$$

with $E = \sqrt{(p - p_f\cos\theta)^2 - gH}$ being the excitation energy of the vacuum. Once we fix the parameter $\sqrt{gH}$, which gives a structure to the vacuum, and the mass of the quark, m, the two parameters p and p_f determine the scattering (compare (2.10)). It turns out that, in some cases, only limited region of the p, p_f plane gives $|\cos\theta| < 1$ and E positive (see the numerics below and [6]).

3. Discussion and conclusions

Let us observe that the formula (2.9) exhibits the following "universal" property for the light, relativistic quarks ("universality" means here independence on H, the quark mass m, and the incident momentum p). We introduce the ratio of the final to the initial momenta, $p_f = xp$, and work out the denominator of (2.12) for p, $xp \gg m$, $\sqrt{gH}$ employing (2.10):

$$v_f E - (p - p_f\cos\theta)\cos\theta = EF'(p_f), \quad (3.1)$$

where $F'(p_f)$ turns out to be

$$F'(p_f) = -1 + \frac{1}{x}\left(x + \frac{gH}{p^2}\frac{2x-1}{2(1-x)^2}\right). \quad (3.2)$$

So, $F'(p_f) = 0$ for $x = 0.5$ and the rate (2.12) blows up because $E \neq 0$ for $x = 0.5$. This fact does not depend (for small m and $\sqrt{gH}$) on the values of m, $\sqrt{gH}$ and p, uniquely determines a "universal" $x = 0.5$, and is also borne out well by numerical calculations (see below).

We comment this result as follows. First, in dealing with this model which suffers from many maladies right from the beginning, one should not treat an infinity as a nonsensical results, it merely marks out something very large: Indeed, our system (a quark in a vacuum) lives presumably a very short time, hence the integration over time from $-\infty$ to $+\infty$ (compare (2.4)) which leads to a sharp energy dependence, $\delta(\varepsilon_f - \varepsilon_i)$, is certainly a very crude approximation, and a more realistic, smoother energy dependence will convert infinities to merely large effects.

Second, **the pole at $p_f \approx 0.5p$ tells us that there exist a well defined cone in the angular distribution of the scattered quark.** Eq. (2.9) is the angular distribution of the scattering rate, that is to say we should replace p_f by $\cos\theta$ through (2.10) and the pole gives us $\cos\theta_c$ at which angle occur virtually all scatterings of the quarks; very much as in the case of a fast polaron [3] (see also [5]). θ_c is very close to — but different from zero. On the other hand when one computes $d\Gamma_{if}/dp_f$ (see [4]) one finds that there are no singularities in p_f. One can understand its relation to $d\Gamma_{if}/d\theta$ realizing that the function $\cos\theta = c(p_f)$ given by (2.10) is very close to a constant and, at the singularity, $d\cos\theta/dp_f = 0$. This relation between $d\Gamma_{fi}/d\theta$ and $d\Gamma_{fi}/dp_f$ is similar to the relation between the angular distribution and the frequency distribution of the Cherenkov radiation.

Does this make sense? One may argue that the phenomenon of a quark moving through a magnetic vacuum is approximately realized when *e.g.* a lepton hits a quark which resides inside a nucleon which nucleon contains, presumably, a perturbative vacuum. When this interaction is deep inelastic the process is very local, and the surface effects are not important.

One should stress, however, that the comparison of our results with experiments is **very speculative.** The quark interacts with the vacuum through excitation of its $n = 0$ mode, whose Fourier transformed wavefunction is a gaussian (compare (2.11)) which becomes Dirac δ-function when $gH \to 0$. But all this was obtained for a spatially infinite system whereas the lump of perturbative vacuum in the nucleon is finite. Nevertheless our integrations (in *e.g.* (2.4)) extend over the infinite space-time. Also, exciting the $n = 0$ mode is a collective, nonperturbative effect which may become very distorted in a small piece of the perturbative vacuum contained inside the nucleon.

The simple (qualitative) results of our calculations is the formation of a jet: a light, fast quark moving through magnetic vacuum forms a jet at a very small, but finite, opening angle θ_c. Such jets do emerge from a nucleon hit by a lepton. (They also emerge from e^+e^- annihilation into hadrons but there it is not clear whether the magnetic vacuum is the right milieu for description of this reaction.)

A brief description of some of the numerical results obtained with equation (2.9): One observes a completely different behavior of (a) the light, fast quarks, and (b) of the heavy, slow ones.

(a) For $m = 0.01$ GeV, $p = 10, 50, 100, 150, 200$ GeV and $\sqrt{gH} = 1.0$ and 0.01 GeV, $d\Gamma/d\theta$ is dominated by the singularity at $x = 0.5 \pm 0.01$, hence by a well defined cone.
   — Although the whole range of $x (0 \leq x \leq 1)$ is allowed by (2.10), the singularity (a jet?) dominates the picture.

(b) For $m = 100.0$ GeV, $p = 5, 10, 50$ GeV and $\sqrt{gH} = 1$ and 0.01 GeV, only the region $0.99 \leq x \leq 1$ is allowed by (2.10). Hence heavy quarks dissipate only small fractions of their energies following $d\Gamma/d\theta$ which is rather flat.

The "magnetic" vacuum can take both energy and momentum from the quark but, also, only the momentum ($x = 1$, elastic scattering, the quark is only deflected) in both cases (a) and (b). The "magnetic" vacuum acts as an infinitely heavy medium. Also: The above numerical results suggest that there might exist a threshold for the formation of a well defined cone (see [4] for more details).

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