DEUTERON MEAN SQUARE RADIUS AND THE SACLAY EXPERIMENT

D.W.L. SPRUNG AND HUA WU

Department of Physics and Astronomy, McMaster University
Hamilton, Ontario L8S 4M1, Canada

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Dedicated to Janusz Dąbrowski in honour of his 65th birthday

We discuss the current status of the deuteron matter radius. We compare the deuteron $A(q^2)$ structure function from the recent Saclay elastic electron-deuteron scattering experiment (S. Platchkov *et al.*, *Nucl. Phys.* **A510**, 740 (1990)) and that from earlier Mainz data (G.G. Simon *et al.*, *Nucl. Phys.* **A364**, 285 (1981)). The inconsistency of the two sets of data in the region of overlap is discussed in the light of a comparison with various deuteron potential models. The new experiment suggests a larger deuteron radius. We also analyse the Saclay fits to $G_{\rm En}$, and deduce an implied value for the deuteron matter radius.

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1. A Problem of consistency

Some time ago, Klarsfeld *et al.* [1] made a careful evaluation of the deuteron mean square radius based on the then existing electron-deuteron scattering data. They invented three new methods for this analysis, of which the "asymptotic method" yielded the most accurate value $r_{Ed} = 1.953(3)$ fm. From this one may subtract Kohno's value [2] for the relativistic and meson exchange current (denoted RMEC) contribution to the radius, leading to the matter radius $r_m = 1.950(3)$ fm, which is to be compared with the non-relativistic deuteron wave function. There exists an empirical linear relation [1] between r_m and a_t (the triplet scattering length) which is obeyed by most if not all phenomenological potential models. The experimental values of these quantities lie about three standard deviations off this empirical line. This discrepancy has been the subject of a good deal of work. On the one hand, Nogami [3] and van Dijk [4] have shown that the linear relation holds for other classes of potential models. Bhaduri *et al.* [5] and also Sprung *et al.* [6] have shown that the linear relation is a result of the weak binding of the deuteron, and the associated large scattering length. It has also been argued that the discrepancy is an indication of non-locality in the neutron-proton interaction [4, 6, 7].

Recently a new and very accurate elastic electron-deuteron scattering experiment has been carried out at Saclay, and used to determine the neutron electric form factor [8] in the range of momentum transfers $1 < q^2 < 20$ fm^{-2}. Clearly it is interesting to know whether this new data adds new information concerning the deuteron radius. Since the Mainz data [9] extend to lower momentum transfers, where only the mean square radius is important, they are still required. Traditionally, only data below $q^2 = 0.5$ fm^{-2} were used to determine the radius. In the analysis of Klarsfeld *et al.*, four points between $q^2 = 1$ and 4 were used mainly to stabilize the fitting of the function to the lower momentum transfer data. It was noted by Platchkov *et al.* that in the region of overlap of the two experiments, $1 < q^2 < 4$ fm^{-2} there is a clear discrepancy between the two sets of data. This is worrisome as it may strongly influence the fit to the Mainz data if the Saclay data were to replace these four points. It also makes it difficult to use the two sets in a combined analysis, as we further show below.

We will first illustrate the problem in two different but related ways. The slope of the $A(q^2)$ structure function at $q^2 = 0$ is directly related to the deuteron radius. In the absence of relativistic and meson current exchange contributions, one would write

$$A = G_{Cd}^2(q^2) + \frac{2}{3}\tau G_{Md}^2(q^2) + \frac{8}{9}\tau^2 G_{Qd}^2(q^2), \tag{1}$$

where

$$G_{Md}^2(q^2) = \frac{3}{4\tau(1+\tau)}B(q^2), \quad \tau = \frac{q^2}{4m_d^2}. \tag{2}$$

Since the deuteron monopole form factor is

$$G_{Cd} = \frac{G_{En}(q^2) + G_{Ep}(q^2)}{\sqrt{1 + q^2/(4m_p^2)}} C_E(q^2) \tag{3}$$

and

$$r_m^2 = -6 \left. \frac{dC_E(q^2)}{dq^2} \right|_{q^2=0} \tag{4}$$

we have

$$r_m^2 = -3 \left. \frac{dA(q^2)}{dq^2} \right|_{q^2=0} - r_p^2 - r_n^2 - \frac{3}{4m_p^2} + \frac{\mu_{Md}^2}{2m_d^2}. \tag{5}$$

In Eq. (5), $G_{Md}(q^2 = 0) = \mu_{Md}$ is the deuteron magnetic dipole moment. We have retained the Darwin–Foldy contribution in Eq. (3) because it is simply a matter of consistency in the definition of the nucleon form factors. One sees that an inconsistency between the two sets of data might lead to different radii, if they imply different slopes of $A(q^2)$.

In order to compare the two sets of data, we plot them together. To highlight differences on an expanded scale, we plot their deviation from the theoretical $A_{\text{Paris}}(q^2)$ values for the Paris potential. In our calculation, we have used the Mainz proton form factor parameterization [11], and for the neutron electric form factor we have used [12] the simple form

$$G_{En} = -\frac{r_n^2}{6} q^2 G_{Ep}(q^2) \tag{6}$$

which is satisfactory at small momentum transfers. It should be pointed out that any other choice of potential model or any other particular form of the nucleon form factors, would lead to a very similar figure. What would change if RMEC corrections were included, is the placement of the data points relative to the potential lines.

Fig. 1 includes $A - A_{\text{Paris}}$ for the two data sets as well as for various potential models (denoted as in [1]). Since we are interested mainly in the deuteron radius, only q^2-values up to 4 fm^{-2} are plotted. Almost all the Mainz data lie above Paris, which results in the radius deduced from it (1.950 fm) being less than that for Paris (1.9716 fm). On the other hand, the Saclay data beyond $q^2 = 1\text{fm}^{-2}$ lie below Paris, and this general trend suggests a radius larger than Paris. In fact, the GK3 potential appears to fit the Saclay data rather well in this region, and its radius is 1.984 fm. But note the caveat above concerning the effect of RMEC effects on the radius.

As discussed in Klarsfeld *et al.* [1], all the potential models give curves with similar shape in the low momentum transfer region, and the curves are ordered according to the deuteron radius. There is no obvious mechanism which can give a curve with a drastically different dependence on momentum transfer, which could lead to an oscillation in δA in this region. In order to fit both sets of data simultaneously, one would require a theory which is dramatically different in the region $q^2 < 2$ fm^{-2}. Until the Mainz results are confirmed, it is not clear whether a larger deuteron radius or a new theory is the solution.

The Mainz experiment measured the ratio of the e-d to e-p cross sections, from which they deduced the ratio of form factors

$$R(q^2) = \frac{G_{Cd}(q^2)}{G_{Ep}(q^2)} = \left[1 + \frac{G_{En}(q^2)}{G_{Ep}(q^2)}\right] \frac{C_E(q^2)}{\sqrt{1 + q^2/(4m_p^2)}}. \tag{7}$$

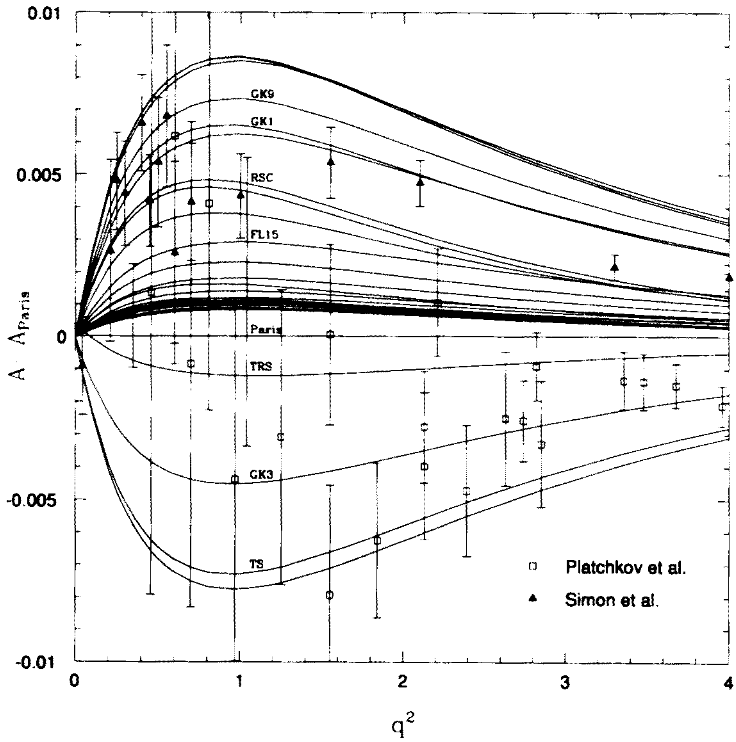

Fig. 1. The deuteron A structure function for the two sets of data and various potential models relative to that of the Paris potential model. Filled triangles: Mainz data; open squares: Saclay data. The rms radii for the labeled models in fm. are:

GK9	GK1	RSC	FL15	Paris	TRS	GK3	TS
1.9511	1.9528	1.9569	1.9636	1.9716	1.9753	1.9836	1.9915

The values of R for the Mainz experiment were given in Ref. [9] with error bars which are extremely small, namely a few parts per thousand. This was achieved by measuring the ratio of (e,d) to (e,p) scattering, so that many systematic errors cancel out in the ratio.

On the other hand, Platchkov *et al.* [8] did not quote their values for R. If one wishes to derive $R(q^2)$ from the Saclay $A(q^2)$, the uncertainties in G_{Qd} (a model average), $B(q^2)$ [10], and G_{Ep} [11] must be combined with the uncertainty of $A(q^2)$, and all of these become larger as the momentum transfer increases. As a result, $R_{\rm Sac}$ appears to have larger error bars than the older data. If one worked from the original cross-section ratios, just the opposite should be true. However, the mean values should be unaffected, so the plot is still useful. It would be interesting indeed to have values for R directly from the Saclay experiment. By adopting Eq. (6), the $R(q^2)$ value

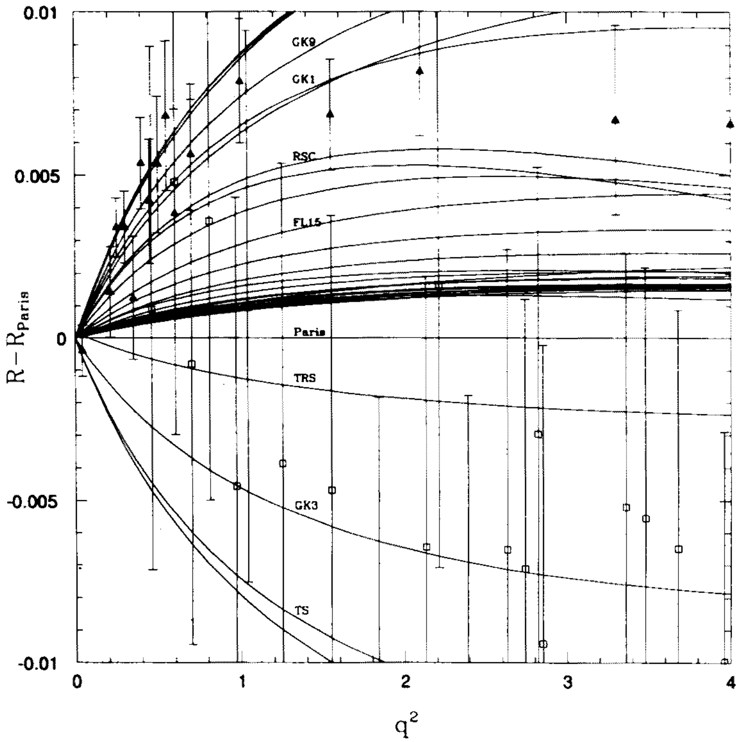

Fig. 2. $R - R_{\text{Paris}}$ for the two sets of data and various potential models.

for given potential model can be found easily. Taking a parallel approach to Fig. 1, we then plot $R - R_{\text{Paris}}$ in Fig. 2. As one can see, this reveals the same problem as seen in Fig. 1.

2. A simplified analysis

Here we briefly describe the analysis of Platchkov *et al.* and attempt to deduce a deuteron radius from it. Their main aim was to understand deuteron structure in the range of momentum transfers between 1 and 20 fm^{-2}. Calculations using the Paris potential plus the usual q^2/m^2 relativistic and MEC corrections by Mosconi and Ricci [13] indicated that these would play an important role in explaining the deviation of experiment from the impulse approximation. After considering various available calculations, they adopted the relativistic corrections according to Arnold, Carlson and Gross [14] and the $\rho\pi\gamma$ MEC effect as corrected by Mosconi and Ricci [15] in an erratum. These were subtracted from the experimental $A(q^2)$, as was the magnetic dipole contribution, according to Eq. (1).

This left them with a corrected $A'(q^2)$ containing only monopole and quadrupole contributions, to be understood on the basis of the non-relativistic impulse approximation, in which the three ingredients are the proton and neutron electric form factors $G_{Ep}(q^2), G_{En}(q^2)$ and the deuteron structure factors which can be computed from a given potential model. If two of these are given, the third can be deduced from experiment. In the view of the Saclay group, the least well established quantity was the neutron electric form factor, so this is what they deduced as their main result.

To this end, they parameterized the neutron electric form factor in the form

$$G_{En}(q^2) = \frac{-a\mu_n \tau}{(1+b\tau)} G_D(q^2)\,, \tag{8}$$

where $\tau = q^2/4m_d^2$ as before and G_D is the well known dipole form factor which provides a reasonable fit ($\pm 10\%$) to the proton form factor over the entire range of data. The above form with two adjustable parameters a, b has often been used. Then, for each of four recent potential models, they deduced the best fit values of a, b. Only for the Reid Soft Core and Paris models did they find $a \approx 1$ which would give agreement with the well established neutron electric radius as measured in neutron-electron scattering. Moreover, fixing a to give the neutron radius did not significantly impair the quality of fit in these two cases: see Tables II and III of Platchkov *et al.* [8]. There are still quite large differences between the G_{En}'s deduced for the various potential models, leaving a $\pm 25\%$ uncertainty in this quantity. Further improvement in our knowledge of G_{En} will come from experiments now under way at the new Mainz accelerator MAMI.

Now, in the low momentum transfer region, one may expand

$$A'(q^2) = 1 - \frac{q^2}{6}(r_m^2 + r_n^2 + r_p^2) + \cdots\,, \tag{9}$$

where the three mean square radii are those of the deuteron wave function, the proton and the neutron.

It is our hypothesis that in fitting the data, one has determined *inter alia* the slope of the form factor, and therefore the sum of these three radii squared. The proton and neutron radii are well established, so one can deduce an output value for r_m for each of the four fits, simply by computing the sum and then removing the known neutron and proton radii. All one needs to know is the theoretical or input value for r_m for each model, and the value of a determined by Platchkov *et al.* This calculation is displayed in Table I. What we found is that for three of the four models one obtains output values of r_m which are equal within the error bars. This shows internal consistency of our hypothesis with their fits. Averaging all four values gives $r_m = 1.961(7)$ fm. This is slightly higher than, but consistent with earlier values.

Table I

Deuteron matter radius from Platchkov *et al.* fits. Entries are in fm or fm^2. Standard deviations are in brackets, in units of the last digit.

Potential		RSC	Paris	V14	Nijmegen
r_m	(theory)	1.9569	1.9716	1.9814	1.9873
r_m^2	(in)	3.8295	3.8872	3.9259	3.9494
$-r_n^2$	(fit)	0.1242 (150)	0.1584 (162)	0.2167 (162)	0.1913 (120)
$r_m^2 + r_n^2$	(fit)	3.7053	3.7288	3.7092	3.7581
$-r_n^2$	(expt)	0.1192	0.1192	0.1192	0.1192
r_m^2	(out)	3.8245 (151)	3.8480 (163)	3.8284 (163)	3.8773 (130)
r_m	(out)	1.9556 (39)	1.9616 (41)	1.9567 (41)	1.9691 (31)
average		$r_m = 1.9608$ (67) fm			

3. Discussion and conclusion

Figs. 5 and 12 of Ref. [8] are quite interesting. The former shows the Saclay data plotted as a ratio to $A_{\rm Paris}$, showing that the new measurements lie below Paris with a minimum near $q^2 = 10$ fm^{-2}, but then rising to cross near $q^2 = 20$ fm^{-2}. In Fig. 12, this is explained as being due to the relativistic and pion-pair effects giving a steady decrease, while the $\rho\pi\gamma$ effect is essentially negligible up to 7 fm^{-2}, and then rises rapidly to be a +20% contribution near $q^2 = 20$ fm^{-2}. Reading the slope of the line denoted ACG from Fig. 12, one finds $A_{\rm rel} = A_{nr}(1 - \alpha q^2)$ with $\alpha = 0.012$ fm^{-2}. This represents a correction to the deuteron radius of $1.5\alpha/r_m = 0.009 \pm 2$ fm. The error arises from the difficulty of reading from a graph. This correction goes in the sense that when one analyses data and takes account of RMEC corrections, the model radius will be that much smaller than if one neglected them. This RMEC value is three times larger than that of Kohno [2] which we have been using.

If one reads the slope from Fig. 10 of Ref. [8], the corresponding $\alpha = 0.0168$, and leads to $\delta r_m = -0.013 \pm 2$ fm, this not including the $\rho\pi\gamma$ contribution. The agreement is satisfactory. This explains why in our Fig. 1, the Saclay data is lying close to the line for the GK3 potential, radius 1.984 fm, while after taking account of RMEC, one arrives at a radius smaller than that of the Paris potential, 1.972 fm. In Fig. 6 of Ref. [13], it appears that the calculation of Mosconi and Ricci was able to obtain good agreement with the Saclay data using the Paris potential and the Höhler [16] form factors. Unfortunately this was due to an error in their $\rho\pi\gamma$ correction, and when this was corrected [15], one sees that a much larger relativistic effect is still needed to obtain such agreement. These calculations were carried out, as were ours [17], by computing relativistic effects to order q^2/m^2 and MEC according to Gari and Hyuga, which would be compatible with Kohno's result. It is clear then, that in order to obtain a radius as small as Paris, or smaller, requires corrections of the size of ACG [14], *i.e.* several times larger.

Our analysis of the fits of Platchkov *et al.* [8] suggests that the radius is $r_m = 1.961(7)$ fm. This is not much larger than the Klarsfeld result [1], but it comes about because the data suggest a much larger radius, which is then reduced by much larger RMEC effects. This radius is nearly compatible with the semi-empirical $a_t - r_m$ relation and reduces the need for non-locality in a potential model.

It would be extremely useful to have values of $R(q^2)$ extracted from the Saclay cross-sections, and also to resolve the discrepancy between this and the Mainz experiment. One would then have much more confidence in the extracted deuteron radius.

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