MICROSCOPIC CALCULATIONS OF THE HYPERNUCLEUS ${}^{5}_{\Lambda}$He

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Dedicated to Janusz Dąbrowski in honour of his 65th birthday

Ground state results for the hypernucleus ${}^{5}_{\Lambda}$He are reported. They have been calculated with a variational Jastrow-like trial wave function and also within the Diffusion Monte Carlo method. Simple central potentials have been used to describe NN and ΛN interactions. The validity of the rigid core approximation is discussed.

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1. Introduction

When facing up the question of preparing a contribution dedicated to Professor Janusz Dąbrowski, we immediately realized the relevance of his contributions to the field of hypernuclear physics. Without aiming to be exhaustive, we may mention his studies on Λ-hypernuclear matter [1–4], light Λ hypernuclei [5], double Λ hypernuclei [6], $\Sigma - \Lambda$ conversion [7–9] and on Σ hypernuclear matter and hypernuclei [10–15]. Therefore, the subject of our research was already defined. Obviously, we had to apply our expertise in many body systems to a problem involving nuclei and strange particles. Probably, the simplest non trivial problem is the bound system of a Λ particle and the ^{4}He nucleus, *i.e.*, the ${}^{5}_{\Lambda}$He. This will be the subject of our work, with the objective of applying well established many body theories to the study of hypernuclei.

The ΛN interaction has been deduced from the existing Λp scattering data and the analysis of very light hypernuclei. The hypernucleus ${}^{5}_{\Lambda}$He constitutes a puzzling problem, since the separation energy of the Λ particle,

calculated using a variety of models and interactions, results to be considerably larger than the experimental value $B_\Lambda(^5_\Lambda\mathrm{He}) = 3.12 \pm 0.02$ MeV [16]. Several reasons have been argued to explain this discrepancy. The inclusion of tensor, spin-orbit and three body ΛNN terms in the ΛN interaction could reduce the Λ binding energy, but it results that these terms produce too small changes to appreciably modify the observed discrepancy. Bodmer and Usmani [17] reported a variational calculation of the hypernucleus $^5_\Lambda$He which includes two- and three-body spin independent correlation functions. They concluded that the experimental Λ binding energy can be well reproduced including phenomenological strongly repulsive ΛNN dispersive forces, whose origin should be associated with the suppression of two-pion exchange interactions arising from modifications of the intermediate Σ by the medium. These results suggest the convenience of carrying out a more precise calculation, without the incertitude of a variational approach.

The empirical Λ separation energy can be well reproduced within the α-Λ model, which takes advantage of the nuclear α-cluster model. However, there are no $\Lambda\alpha$ scattering experiments which could provide direct information on the $\Lambda\alpha$ potential, so that this interaction is obtained by convoluting the ΛN potential over the ^{4}He distribution. This approach is called rigid-core or adiabatic approximation [18] and it assumes that the ^{4}He-core is not very much perturbed by the presence of the Λ in the nucleus. But a readjustment of the original ΛN potential must be done to provide good agreement with the empirical binding energy. A detailed analysis of this approximation is given in Ref. [19].

In this paper we report on preliminary results of a Diffusion Monte Carlo (DMC) calculation of the system $^5_\Lambda$He. In this exploratory study we use two simple NN potentials: the Brink and Boeker B1 [20] and the Afnan and Tang S3 [21]. For the ΛN potential we use also effective interactions, whose parameters have been adjusted [19] to reproduce the empirical Λ binding energy in the rigid core approximation. All these NN and ΛN potentials are superpositions of Gaussians. Our purposes are to explore the validity of the rigid core approximation, to study the dependence of the Λ binding energy on the NN interaction, to determine the localization of the Λ particle around the ^{4}He core and, finally, to assess the quality of the used ΛN interactions.

The fact of dealing with a very light system requires a proper treatment of the center-of-mass motion. The best way to do it is to directly describe the system in terms of relative and translationally invariant coordinates. The set of coordinates used in this work is described in Section 2. This section also includes the description of the elementary ΛN interactions. In Section 3 are shown the results given by a variational calculation of the hypernucleus $^5_\Lambda$He with a Jastrow-like trial wave function. The obtained wave function per each NN and ΛN interactions is the input for the DMC

calculation, which is discussed in Section 4. Given that we are concerned with pure Wigner forces, the hypernucleus ${}^{5}_{\Lambda}$He may be considered as a system of bosons, and the DMC algorithm will provide the exact, within statistical fluctuations, description of the system. Finally, the conclusions are drawn in Section 5.

2. The intrinsic coordinates and the elementary interactions

To take proper care of the center-of-mass motion, we will use a set of intrinsic, *i.e.*, translationally invariant, coordinates. In the following the external coordinates of the four nucleons will be represented by $\boldsymbol{r}_1$ up to $\boldsymbol{r}_4$, and the external coordinate of the ${}^{5}_{\Lambda}$He particle by $\boldsymbol{r}_\Lambda$. The four internal coordinates will be denoted by ξ_1 up to ξ_4.

The first three coordinates are defined in the usual Jacobi form, namely

$$\xi_1 = \sqrt{\frac{1}{2}}(\boldsymbol{r}_2 - \boldsymbol{r}_1)\,, \tag{1}$$

$$\xi_2 = \sqrt{\frac{2}{3}}\left(\boldsymbol{r}_3 - \frac{\boldsymbol{r}_1 + \boldsymbol{r}_2}{2}\right)\,, \tag{2}$$

$$\xi_3 = \sqrt{\frac{3}{4}}\left(\boldsymbol{r}_4 - \frac{\boldsymbol{r}_1 + \boldsymbol{r}_2 + \boldsymbol{r}_3}{3}\right)\,. \tag{3}$$

The fourth coordinate is the distance of the Λ particle to the center-of-mass of the ^{4}He nucleus,

$$\xi_4 = \sqrt{\frac{4m_\Lambda}{4m + m_\Lambda}}\left(\boldsymbol{r}_\Lambda - \frac{\boldsymbol{r}_1 + \boldsymbol{r}_2 + \boldsymbol{r}_3 + \boldsymbol{r}_4}{4}\right)\,, \tag{4}$$

with a dimensionless scale factor, defined in terms of the masses of the elementary constituents, so that the intrinsic part of the kinetic energy operator becomes

$$\frac{\hbar^2}{2m}\sum_{k=1}^{4}\nabla_k^2 + \frac{\hbar^2}{2m_\Lambda}\nabla_\Lambda^2 \to \frac{\hbar^2}{2m}\sum_{k=1}^{4}\nabla_{\xi_k}^2\,, \tag{5}$$

as if it were the kinetic energy operator of equal mass particles. The definition of the last coordinate is not the usual one, and in fact the Jacobian of the transformation is not 1. However, this choice will simplify the application of the Diffusion Monte Carlo algorithm.

To describe the NN interaction we have considered two commonly used potentials, the B1 of Brink and Boeker [20] and the S3 of Afnan and Tang

[21]. The former is an effective interaction which produces a too large binding energy for ^{4}He, and the latter is a realistic interaction which was adjusted to the s-wave phase shifts in the singlet and triplet channels. The reason of using these two interactions is that, for an unknown reason, the ^{4}He one-body densities are almost indistinguishable for the two interactions. Consequently, both interactions will generate the same Λ-α effective interaction for a given ΛN interaction. The differences in the full microscopic calculation of $^5_\Lambda$He are to be assigned exclusively to many-body effects. Actually, the ^{4}He two-body distribution functions related to these two potentials are quite different, because of the short range repulsion of the S3 potential. In both cases, we will use the Wigner part of the interactions by taking advantage of the spin and isospin saturation of the core ^{4}He nucleus.

Concerning the ΛN interaction we have considered four of the forms adjusted by Daskaloyannis, Grypeos and Nassena [19] so as to reproduce the Λ-α binding energy in the rigid core approximation. The forms selected correspond to a convolution based on a single Gaussian (labelled ρ_{GI} in Ref. [19]), of the effective interactions of Dalitz and Downs [18], Bassichis and Gal [23], Gibson, Goldberg and Weiss [22] and Sotona (reported by Zofka in Ref. [24]). To refer to these four potentials we will use the symbols D, B, G and S respectively. All these interactions are written as a combination of Gaussians

$$V_{\Lambda N}(r) = \sum_i V_i \exp\left(-\frac{r^2}{a_i^2}\right). \tag{6}$$

Their parameters are displayed in Table I.

TABLE I

Parameters of ΛN potentials, taken from Ref. [19]

Potential	V_1(MeV)	a_1(fm)	V_2(MeV)	a_2(fm)
D [18]	−158.27	0.5907	–	–
B [23]	−44.98	1.031	27.6	0.5908
G [22]	−82.23	1.21	145.0	0.82
S [24]	−212.28	0.8	949.6	0.4

These ΛN interactions have quite different forms, as is shown in Fig. 3, ranging from the smooth Dalitz and Downs form which is purely attractive, to the realistic-like form of Sotona. The common characteristic of these four interactions, as we have already mentioned, is their fit to the Λ separation energy within the rigid core approximation for a given one-body distribution of the ^{4}He core.

3. Variational description

Prior to the DMC calculation we have obtained an approximation to the ground state wave function of the system by minimizing the expectation value of the intrinsic hamiltonian with respect to a Jastrow-like trial function. It is indeed convenient to use a good starting state for the DMC method serving as driving or importance sampling function. We have chosen the variational trial function as a product of two body correlation factors, both for the nucleon-nucleon as well as for the nucleon-lambda pairs

$$\Psi = \prod_{i<j=1}^{4} f^{NN}(|\boldsymbol{r}_i - \boldsymbol{r}_j|) \prod_{j=1}^{4} f^{N\Lambda}(|\boldsymbol{r}_\Lambda - \boldsymbol{r}_j|) . \tag{7}$$

The correlation functions are parameterized as

$$f^{NN}(r) = \exp(-a_1 r^2) + b \exp(-a_2 r^2) \tag{8}$$

and

$$f^{N\Lambda}(r) = \exp(-a_1' r^2) + b' \exp(-a_2' r^2) . \tag{9}$$

One expects a_1 and a_1' to be small, corresponding to a long range shell-model like wave function, and a_2 and a_2' to be rather large, corresponding to short range correlations. The parameters b and b' control the depth of the correlation at zero distance.

The trial wave function is thus characterized by six parameters, which are determined by minimizing the energy expectation value for each nucleon-nucleon and nucleon-lambda interaction. This minimization procedure is a rather lengthy job, but otherwise straightforward. The wave function, as well as the interactions, should be written in terms of the intrinsic coordinates. Moreover, the wave function Ψ is a linear combination of exponentials of quadratic forms of the intrinsic coordinates. For the kind of interactions we are dealing with, the expectation value of the potential energy requires the evaluation of integrals of Gaussian-like forms, which may be carried out analytically. Something similar happens for the kinetic energy. By using equation (5) one must carry out the second derivative of a Gaussian-like form, and subsequently integrate a polynomial times the same Gaussian form. This procedure must be repeated for each term coming both from the left-hand side and the right-hand side wave functions. We have used the algebraic manipulation program REDUCE to carry out all coordinate transformations for a generic Gaussian form, and generate afterwards an output acceptable as a FORTRAN source code. Apart from saving a lot of work, we have also avoided the unwanted presence of typing errors when writing these long formulae.

The results of the minimum energy search are shown in Table II, for the set of NN and NΛ interactions considered in this work. The table lists the parameters which minimize the expectation value of the hamiltonian, as well as the resulting variational upper bound for the energy of the hypernucleus. The first row of each block displays the variational results for the nuclear ^{4}He systems using an analogous trial wave function. The last column shows an estimate of the Λ binding energy obtained by subtracting the upperbound for the ^{4}He energy from the upperbound for the $^5_\Lambda$He energy. It is worth noting that this estimate is *not* variational.

TABLE II

Variational determination of the hypernucleus $^5_\Lambda$He. The table is divided in two blocks, one corresponding to the B1 interaction of Brink and Boeker, and the other corresponding to the S3 interaction of Afnan and Tang. The first line of each block corresponds to the minimum in the absence of the Λ particle. The remaining rows correspond to four types of NΛ interaction. The various columns identify the interactions and the parameters of the wave function. The last two columns correspond to the upper bound to the energy (column labelled E) and an estimate of the binding energy of the Λ particle (column labelled B).

N-N	N-Λ	$a_1(F^{-2})$	$a_2(F^{-2})$	b	$a_1'(F^{-2})$	$a_2'(F^{-2})$	b'	E (MeV)	B (MeV)
B1		0.08323	1.7033	−0.4875				−36.69	
	D [18]	0.07784	1.5832	−0.4914	0.04568	0.9723	0.6648	−43.81	7.12
	B [23]	0.07778	1.6538	−0.4910	0.01727	0.2438	0.6381	−40.76	4.07
	G [22]	0.07817	1.6457	−0.4863	0.03058	1.2019	−0.1185	−39.71	3.02
	S [24]	0.07670	1.7916	−0.4980	0.05411	4.3802	−0.4820	−42.60	5.91
S3		0.08590	2.1332	−0.7187				−24.35	
	D [18]	0.08096	2.0135	−0.7196	0.04446	0.8981	0.6847	−31.73	7.38
	B [23]	0.08136	2.0840	−0.7212	0.01784	0.2535	0.6376	−28.60	4.25
	G [22]	0.08057	2.1284	−0.7197	0.03063	1.6136	−0.1451	−27.48	3.13
	S [24]	0.08105	2.0854	−0.7207	0.05752	4.7229	−0.4940	−30.46	6.11

The most appealing result of this variational study is that the lambda separation energy is almost independent of the nucleon-nucleon interaction. In fact, the B1 and the S3 interactions are rather different, the latter having an important soft core at short distances, whereas the former is quite smooth. As can be seen in Table II,these interactions produce rather different values for the energies, as well as for the wave function parameters. As we have mentioned in Section 2, the one-body densities of ^{4}He obtained with these interactions are almost indistinguishable, but the two-body densities differ appreciably at short distances.

The Λ binding energies are very sensitive to the ΛN interaction. This is somehow surprising because the interactions used were adjusted to give the experimental value of the binding energy of the Λ particle in the ^{4}He nucleus, within the rigid core approximation and for a given nuclear density. The tentative conclusion is that many-body effects are important, and that the rigid core approximation fails. A glance to Table II illustrates this result. The parameters of the nucleon-nucleon correlation function are different for the ^{4}He nucleus and for the ^{4}He core in the ${}^{5}_{\Lambda}$He hypernucleus, particularly the parameter a_1 which is related to the long-range part of the correlation. The same kind of result is observed in the exact calculations which will be presented in the next section.

4. Diffusion Monte Carlo calculation

The DMC method is a procedure of solving the imaginary time A-body Schrödinger equation and obtaining stochastically the true ground state energy as well as the corresponding wave function. The method was originally proposed by Anderson [26] and improved with the addition of an importance-sampling driving function by Ceperley and Alder [27] and Reynolds and coworkers [28].

The procedure consists in leaving an initial wave function to evolve with time, by using an approximate Green function valid for short time steps, to reach the ground state wave function, by diminishing the amplitudes of all other components which may be present in the starting state. The wave function is represented by a collection of random vectors $\boldsymbol{R} \equiv \{\xi_1, \xi_2, \xi_3, \xi_4\}$ called *walkers*, each of them representing a snapshot of the system at a given time. The time evolution turns out to correspond, in the short time approximation, to a succession of drifting, isotropic diffusion and multiplication of each walker.

Instead of studying the evolution of the ground state wave function $\Psi(\boldsymbol{R}, t)$, it is better to consider its product by an importance or trial wave function $\Psi_T(\boldsymbol{R})$, so that the quantity which is evolving with the time is

$$f(\boldsymbol{R}, t) = \Psi_T(\boldsymbol{R})\Psi(\boldsymbol{R}, t)\,. \tag{10}$$

A step forward with time is obtained by means of the equation

$$f(\mathbf{R}', t+\tau) = \int G(\mathbf{R}', \boldsymbol{R}, \tau) f(\boldsymbol{R}, t) d\boldsymbol{R}\,, \tag{11}$$

where τ is the time-step, and G is the small-time approximate Green function. Among the various approximation to this Green function [29–31], we have used the $\mathcal{O}(\tau^2)$ form

$$G(\boldsymbol{R}', \boldsymbol{R}, \tau) = (4\pi D\tau)^{-3A/2} \exp\left\{ -\frac{\left(\boldsymbol{R}' - \boldsymbol{R} - D\tau \boldsymbol{F}(\boldsymbol{R})\right)^2}{4D\tau} \right\}$$

$$\times \exp\left\{ \left(E - \frac{E_L(\boldsymbol{R}) + E_L(\boldsymbol{R}')}{2} \right) \tau \right\} + \mathcal{O}(\tau^2), \quad (12)$$

where the *local* energy is

$$E_L(\boldsymbol{R}) = \frac{1}{\Psi_T(\boldsymbol{R})} H \Psi_T(\boldsymbol{R}), \quad (13)$$

i.e., corresponds to the action of the hamiltonian on the trial wave function divided by the trial wave function, and the diffusion constant is $D = \hbar^2/2m$, m being the nucleon mass. Note that the different mass of the lambda particle has been absorbed into the coordinate transformation described in Section 2.

Finally, the energy, as well as other observables, is determined by means of the mixed estimator

$$E \approx \lim_{t \to \infty} \frac{\langle \Psi_T | H | \Psi(t) \rangle}{\langle \Psi_T | \Psi(t) \rangle} = \frac{\int E_L(\boldsymbol{R}) f(\boldsymbol{R}, t) d\boldsymbol{R}}{\int f(\boldsymbol{R}, t) d\boldsymbol{R}}. \quad (14)$$

This equation has null variance in the limit in which the trial function is the exact ground state wave function. Furthermore, when dealing with observables other than the hamiltonian, a correction has to be applied to the above equation [32].

The stochastic DMC method has been carried out by using as importance sampling functions the variational wave functions determined in the previous section, for each set of elementary interactions. The initial set of walkers was determined from the squared trial function by means of the Metropolis algorithm [33], and the total number of walkers was maintained around 500, within the statistical fluctuation. A total of 40 blocks of 1000 time-steps per block was used to determine the energy value, the distributions and the radii of the distributions. This corresponds to 20 million samples per interaction. The time step was taken as 0.0001 MeV^{-1}, which is short enough to have an extrapolation error less than the statistical error of the result [25].

The main set of results is shown in Table III. The structure is analogous to that of Table II. The table is also divided into two blocks, one corresponding to the B1 interaction and the other to the S3 interaction. The first row of each block displays the high-precision determination of the ^{4}He binding

energy for the referred interactions from Ref. [25]. The third column lists the energies of the system ${}^{5}_{\Lambda}$He, and by subtracting the energy of the ^{4}He nucleus one gets the values of the binding energies of the lambda particle which appear in the sixth column. These values are in a rather wide range between 3 and 7 MeV, strongly depending on the $\Lambda - N$ interaction.

TABLE III

Diffusion Monte Carlo determination of the binding energy of the Λ particle in ^{4}He for several nucleon-nucleon and nucleon-lambda interactions. The table is divided into two blocks. The first line of each block contains the binding energy of ^{4}He [25] in absence of the lambda particle, as well as the point mean square radius. The other rows include the effect of the lambda particle in the total binding energy (third column), the point mean square radius of nucleons from the center-of-mass of the ^{4}He nucleus (fourth column) and the point Λ mean square radius again from the center-of-mass of the ^{4}He nucleus (fifth column). The sixth and seventh columns contain the value of the binding energy of the Λ-particle, the former as determined by our DMC calculation, and the later from the rigid core approximation.

N-N	N-Λ	E (MeV)	$r_{N-\alpha}$ (F)	$r_{\Lambda-\alpha}$ (F)	B_Λ (MeV) DMC	B_Λ (MeV) Rigid core
B1		-38.32 ± 0.01	1.395			
	D [18]	-45.61 ± 0.08	1.308	1.768	7.4 ± 0.1	3.43
	B [23]	-42.23 ± 0.05	1.352	2.423	4.0 ± 0.06	3.30
	G [22]	-41.8 ± 0.09	1.366	2.583	3.6 ± 0.1	3.12
	S [24]	-45.87 ± 0.08	1.329	1.935	7.6 ± 0.1	3.23
S3		-27.35 ± 0.02	1.398			
	D [18]	-34.57 ± 0.08	1.293	1.750	7.3 ± 0.1	3.43
	B [23]	-31.32 ± 0.12	1.339	2.417	4.0 ± 0.15	3.29
	G [22]	-30.97 ± 0.15	1.358	2.613	3.7 ± 0.2	3.11
	S [24]	-35.1 ± 0.3	1.315	1.918	7.7 ± 0.3	3.22

One can see that the Λ binding energy is quite insensitive to the NN interaction, as we have already noticed in Section 3 with the variational results. To understand this insensitivity let us examine the one-body distribution functions. We have found convenient to refer these distribution functions to the center-of-mass of the ^{4}He nucleus, instead of taking as origin the center-of-mass of the hypernucleus. The distribution functions are defined as

$$\rho_N(\boldsymbol{r}) = \int d\xi_1 \cdots d\xi_4 |\Psi(\xi_1, \cdots \xi_4)|^2 \delta\left(\boldsymbol{r} - \left(\boldsymbol{r}_1 - \frac{\boldsymbol{r}_1 + \boldsymbol{r}_2 + \boldsymbol{r}_3 + \boldsymbol{r}_4}{4}\right)\right), \tag{15}$$

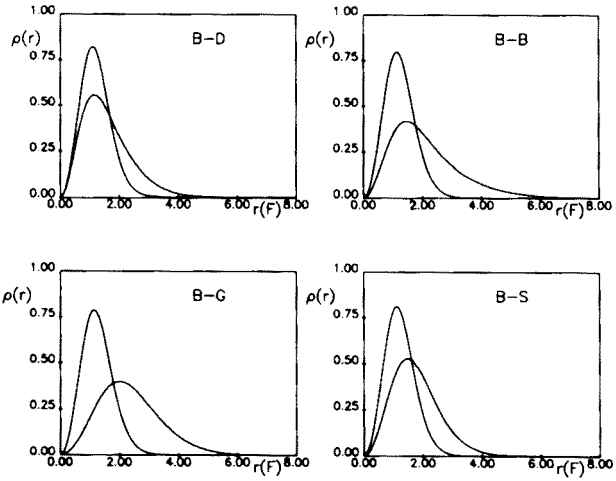

Fig. 1. The one-body distribution functions of a nucleon and the lambda particle referred to the center-of-mass of the ^{4}He core nucleus, obtained from the DMC calculation for the B1 nucleon-nucleon interaction and for the four ΛN interactions used in this work. The distribution corresponding to the nucleon has a strong peak around 1 F.

for the nucleons, and

$$\rho_\Lambda(\boldsymbol{r}) = \int d\xi_1 \cdots d\xi_4 |\Psi(\xi_1, \cdots \xi_4)|^2 \delta\left(\boldsymbol{r} - \left(\boldsymbol{r}_\Lambda - \frac{\boldsymbol{r}_1 + \boldsymbol{r}_2 + \boldsymbol{r}_3 + \boldsymbol{r}_4}{4}\right)\right), \tag{16}$$

for the lambda particle. In both cases, the wave function should be the exact wave function, but we have obviously used the mixed estimator to determine the distributions. These distribution functions are spherically symmetric, and in Fig. 1 we have plotted both functions for the various combinations of NN and ΛN interactions, normalized to unity (*i.e.*, $\int_0^\infty \rho(r)dr = 1$), so that these functions include the angular average and the r^2 factor of the three-dimensional integration. A parameter characterizing these distributions is the root mean square radius, which is defined in both cases as

$$r^2 = \int dr \rho(r) r^2 . \tag{17}$$

The values of the different radii are also displayed in Table III, columns fourth and fifth. The shape of the distributions and the values of the radii, clearly show that the Λ particle is quite *outside* of the ^{4}He core but not very far away. Actually, after adding the Λ particle, the ^{4}He core shrinks noticeably, the radius being almost a 10 % smaller than in the isolated nucleus. On the other hand, the Λ particle is at a distance of almost twice the nuclear radius from the center-of-mass of the ^{4}He core.

Another way of visualizing the shape of the nuclear core and the situation of the lambda particle in the hypernuclear system is to plot the instant positions of a given nucleon and of the lambda particle, along the Diffusion Monte Carlo stochastic evolution. This is shown in Fig. 2, for the B1 nucleon-nucleon interaction. At the left side, each point represents the projection of the position of a nucleon on the X-Y plane, and the same is shown at the right side for the lambda particle. The axes extend from -4 F to 4 F, and the total number of points plotted is 10000. The various plots from the top to the bottom correspond to the interactions D, B, G and S presented in Section 2. Particularly appealing is the plot corresponding to the Gibson interaction, where the position of lambda particle is spread in a wide region compared with the size of the ^{4}He core. This is also evident from the very large radius $r_{\Lambda-\alpha}$ quoted in Table III for this interaction.

It is now the moment of discussing a consistent rigid core approximation [18]. The basic idea of this approximation is that the interaction of the added lambda particle is weak enough so as not to disturb too much the ^{4}He core. In other words, a good approximation to the wave function of the hypernucleus is the product form

$$\Psi = \Psi_{\rm He}(\xi_1, \xi_2, \xi_3)\Psi_\Lambda(\xi_4)\,, \tag{18}$$

where $\Psi_{\rm He}$ is the exact ground state of the ^{4}He core in absence of the lambda particle. By using the variational principle, with a fixed wave function $\Psi_{\rm He}$ and an arbitrary wave function for the lambda, there results an one-body Schrödinger equation with the Hamiltonian

$$H_{\rm eff} = -\frac{\hbar^2}{2\mu}\nabla^2 + V_{\rm eff}\,, \tag{19}$$

where μ is the reduced Λ-^{4}He mass, and the effective interaction is given by

$$V_{\rm eff}(r) = \int d\xi_1 d\xi_2 d\xi_3 |\Psi_{\rm He}(\xi_1, \xi_2, \xi_3)|^2 \sum_{k=1}^{4} V_{\rm N-\Lambda}(r_\Lambda - r_k)\,. \tag{20}$$

The lowest eigenvalue of $H_{\rm eff}$ is an upper bound to the binding energy of the lambda particle, provided the wave function $\Psi_{\rm He}(\xi_1, \xi_2, \xi_3)$ is the *exact* wave function of the uncoupled He core. The evaluation of the effective interaction is quite simple, being the convolution of the free ΛN interaction with the one-body density defined in Eq. (15). In Fig. 3 we have plotted the elementary ΛN interactions (left part) and the effective $\Lambda\alpha$ interaction coming from the convolution with the one-body density of the isolated ^{4}He core. The strong differences among the four ΛN interactions are clearly

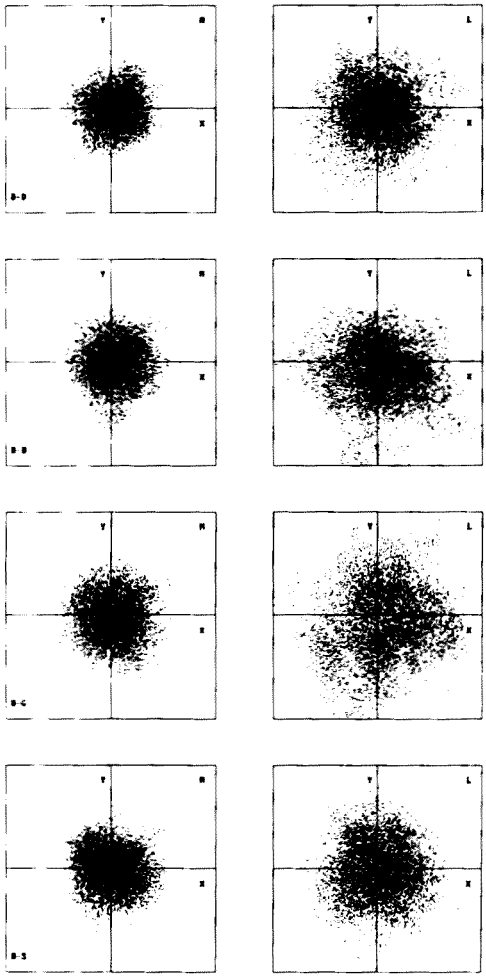

Fig. 2. A pictorial view of nucleons and lambda particle with respect to the center-of-mass of the ^{4}He core. The instantaneous position of a given nucleon (left plot) and of the lambda particle (right plot) has been recorded along 10000 steps of the DMC stochastic evolution, and projected on the XY plane. The NN interaction is the B1, and the ΛN interactions are, from top to bottom, D, B, G and S, as defined in the text.

reduced when obtaining the effective forms. In all cases the effective $\Lambda\alpha$ interaction is attractive, the range being inversely proportional to the depth of the potentials.

The upper bounds to the binding energy of the lambda particle in this approximation are shown in the last column of Table III. Certainly, the

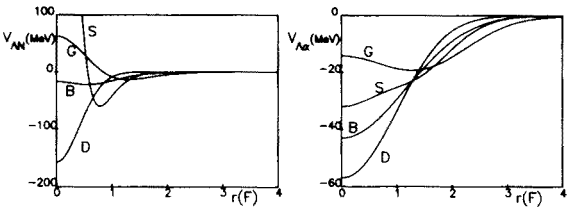

Fig. 3. Plot of the free ΛN interactions and the effective Λα interactions obtained by convoluting the free potentials with the one-body density of the isolated ^{4}He nucleus. The one-body density is evaluated by means of the DMC algorithm for the B1 nucleon-nucleon interaction.

upper bound condition is always respected. Moreover, the values obtained in this approximation are quite similar to the experimental values of the binding energy, but this is to be expected, the free interactions having been adjusted [19] to reproduce this quantity within the rigid core approximation. In the work of Ref. [19] a parameterized one-body distribution was used. This distribution is not too different from the distributions we are obtaining from the nucleon-nucleon interactions in the DMC algorithm, and so our values for the rigid core approximation do not differ appreciably from the adjusted value of Ref. [19]

However, the rigid core approximation to the values of the binding energy of the lambda particle differs noticeably from the exact, within statistical fluctuations, results of the DMC algorithm of the full hypernuclear system. The conclusion is obvious, namely that the rigid core approximation is not as good as expected, or, in other words, that many-body effects, in the form of polarization of the core, are very important.

5. Conclusions

The main contribution of this work is the determination of the binding energy of the ${}^{5}_{\Lambda}$He hypernucleus by means of a Jastrow variational approach and, especially, by solving exactly, within statistical fluctuations, the five-body Schrödinger equation with the help of the Diffusion Monte Carlo algorithm.

It would not make much sense to compare our results for the separation energy of the Λ particle to the experimental value since the elementary interactions used are not realistic. Actually we have obtained values ranging from 3.6 to 7.7 MeV, depending on the ΛN interaction used. These numbers are very different from the experimental measurement of 3.12 ± 0.02 MeV.

On the other hand we may state conclusions on the validity of the rigid core approximation. Due to the obvious difficulties in measuring precisely

the ΛN phase shifts, the usual method to determine the ΛN interaction is based on the analysis of hypernuclei which may be described as a system of α particles plus the strange baryon. The required $\Lambda\alpha$ interaction is constructed by folding the free ΛN interaction with the one-body density of the ^{4}He nucleus. This procedure will produce upper bounds to the Λ separation energy provided that the *exact* one body distribution of the nucleon in the ^{4}He nucleus is used. The comparison of the upper bounds determined in this form with the microscopic calculation shows that the rigid core approximation is not valid, and this means that many-body effects are important, and also that polarization of the core ^{4}He nucleus is not small.

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