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SUPERSYMMETRY SCHEME FOR NUCLEI $32 \leq A < 40$*

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Dedicated to Janusz Dąbrowski in honour of his 65th birthday

A new version of the supersymmetry scheme for nuclei $32 \leq A < 40$ has been proposed. The IBM bosons (pairs of nucleons approximately) have been taken with spin and isospin degrees of freedom (IBM4) while nucleons (nucleon) are bounded to the $j = {}^3\!/_2$ level only. The assumed supersymmetry group is then the unitary-unitary supergroup U(36/8). Theoretical energy levels and $E2$ transition probabilities have been compared with experimental data yielding quite a good agreement.

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1. Introduction

We have already considered the supersymmetry of light nuclei from *sd* shells. For several multiplets we have obtained fairly good agreement as well in energy levels [1] as in $E2$ transitions [2]. However, in the region $32 \leq A < 40$ the supersymmetry predictions were not so good as for lighter nuclei from *sd* shell. The possible reason might be the $L - S$ coupling in which the supersymmetry was assumed. For nuclei $32 \leq A < 40$ both protons and neutrons obey rather $j - j$ coupling because valence nucleons could be placed most probably on the $j = {}^3\!/_2$ level. Hence, the unitary transformation in the nucleon space including isospin degrees of freedom is of 8 dimensions, *i.e.* U(8). The boson space is based on s, d bosons of the standard interacting boson model (IBM) [3] enlarged by the spin-isospin

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formalism [4] which is followed as in [1] bythe symmetry group U(36). The assumed supersymmetry is then the unitary-unitary supergroup U(36/8).

2. Groups and generators

Realisation of generators of superunitary transformations is usually done in nuclear theory by quadratic products od creation and annihilation operators $a^\dagger(a)$ for fermions and $b^\dagger(b)$ for bosons which read

$$b^\dagger_\beta b_{\beta'}\,; \quad a^\dagger_\alpha a_{\alpha'}\,; \quad b^\dagger_\beta a_\alpha\,; \quad a^\dagger_\alpha b_\beta\,, \tag{1}$$

where $\alpha(\alpha') = (j\,m_j\,{}^1\!/\!_2\,m_t)$ for fermions and $\beta(\beta') = (l\,m_l\,\sigma\,m_\sigma\,\tau\,m_\tau)$ for bosons, with σ, τ — the spin and isospin boson quantum numbers which take on values 1 or 0. The first two operators (1) from the so-called Bose--sector of generators with commutation relations and the last two operators (1) form the Fermi-sector with anticommutation relations. Commutation relations are also imposed on the mixed commutators of Bose-Fermi operators (1). The generators are then closed under the above (anti)commutation relations and define the basis of the Lie superalgebra of the supergroup U(36/8). The generators (1) while acting on a given boson-fermion state preserve the total number of particles *i.e.* the sum of fermions $N_{\rm f}$ (nucleons) and bosons $N_{\rm b}$ (approximately pairs of nucleons). Hence, a set of states for a fixed number $N = N_{\rm f} + N_{\rm b}$ form the basis for an irreducible representation of the supergroup U(36/8). Such set of states is also called the supermultiplet (this notion should not be mixed with a Wigner supermultiplet). Bosons and fermions are taken from the last magic shell either as particles in the first half of the shell or as holes otherwise. For nuclei $32 \leq A < 40$ considered here there will be holes only below double magic shell $A = 40$.

The assumed supersymmetry consideration enters, in our context, only through the very important definition of a supermultiplet. Under this assumption all of nuclei which belong to the same supermultiplet (even-even, even-odd and odd-odd) must behave similarly in the supersymmetry scheme. The further standard procedure includes:

i) splitting of the supersymmetry group into two unitary subgroups in fermion and boson space,

ii) construction and analysis of symmetry group chains in both spaces,

iii) construction of generators of the combined boson-fermion groups from sums of boson and fermion operators of the Bose sector of (1).

The last step is a crucial one because it introduces the possibility of fermion-boson interaction in the constructed Casimir operators which then form an assumed Hamiltonian of the system.

At first we take the unitary group product as a subgroup of the U(36/8)

$$\mathrm{U}(36/8) \supset \mathrm{U}^{\mathrm{B}}(36) \times \mathrm{U}^{\mathrm{F}}(8)\,. \qquad (2)$$

The generators of these subgroups are the first and second operators of (1) respectively. The group U(36) chain has been considered in [1] and we take the same chain

$$\mathrm{U}^{\mathrm{B}}(36) \supset \mathrm{U}^{\mathrm{B}}_{L}(6) \times \mathrm{U}^{\mathrm{B}}_{ST}(6) \supset \mathrm{SO}^{\mathrm{B}}_{L}(6) \times \mathrm{SU}^{\mathrm{B}}_{S}(2) \times \mathrm{SU}^{\mathrm{B}}_{T}(2)\,. \qquad (3)$$

In the Fermi space we adopt

$$\mathrm{U}^{\mathrm{F}}(8) \supset \mathrm{SU}^{\mathrm{F}}_{J'}(4) \times \mathrm{SU}^{\mathrm{F}}_{T}(2)\,. \qquad (4)$$

The chain (3) is, in its orbital part, the IBM version for γ-unstable nuclei for which the symmetry subgroup $\mathrm{SO}^{\mathrm{B}}_{L}(6)$ appeared. There are two other symmetry subgroups of the IBM, namely SU(5) and SU(3) which are proper for vibrational and rotational nuclei respectively. We have chosen the symmetry $\mathrm{SO}^{\mathrm{B}}_{L}(6)$, besides of physical reasons, to fulfill the point *(iii)* because in the fermion chain (4) the group $\mathrm{SU}^{\mathrm{F}}_{J'}(4)$ is accidentally locally isomorphic with the group $\mathrm{SO}^{\mathrm{B}}_{L}(6)$. Hence it is possible to form generators for the group $\mathrm{SU}^{\mathrm{BF}}_{J''}(4)$ constructed by proper sums of generators of these two isomorphic groups. Exploiting this isomorphism and also an isomorphism (mathematical identity) of the groups $\mathrm{SU}^{\mathrm{B}}(2)$ and $\mathrm{SU}^{\mathrm{F}}(2)$ we have adopted the following supersymmetry chain for nuclei $32 \le A < 40$.

$$\begin{aligned}\mathrm{U}(36/8) &\supset \mathrm{U}^{\mathrm{B}}(36) \times \mathrm{U}^{\mathrm{F}}(8)\\ &\supset \mathrm{U}^{\mathrm{B}}_{L}(6) \times \mathrm{U}^{\mathrm{B}}_{ST}(6) \times \mathrm{SU}^{\mathrm{F}}_{J'}(4) \times \mathrm{SU}^{\mathrm{F}}_{T}(2)\\ &\supset \mathrm{SO}^{\mathrm{B}}_{L}(6) \times \mathrm{SU}^{\mathrm{F}}_{J'}(4) \times \mathrm{SU}^{\mathrm{B}}_{S}(2) \times \mathrm{SU}^{\mathrm{B}}_{T}(2) \times \mathrm{SU}^{\mathrm{F}}_{T}(2)\\ &\supset \mathrm{SU}^{\mathrm{BF}}_{J''}(4) \times \mathrm{SU}^{\mathrm{B}}_{S}(2) \times \mathrm{SU}^{\mathrm{BF}}_{T}(2)\\ &\supset \mathrm{Sp}^{\mathrm{BF}}_{J''}(4) \times \mathrm{SU}^{\mathrm{B}}_{S}(2) \times \mathrm{SU}^{\mathrm{BF}}_{T}(2)\\ &\supset \mathrm{SU}^{\mathrm{BF}}_{J''}(2) \times \mathrm{SU}^{\mathrm{B}}_{S}(2) \times \mathrm{SU}^{\mathrm{BF}}_{T}(2)\\ &\supset \mathrm{SU}^{\mathrm{BF}}_{J}(2) \times \mathrm{SU}^{\mathrm{BF}}_{T}(2)\,, \qquad (5)\end{aligned}$$

where

$$\boldsymbol{J}'' = \boldsymbol{J}'_{\mathrm{F}} + \boldsymbol{L}_{\mathrm{B}} \quad \text{and} \quad \boldsymbol{J} = \boldsymbol{J}'' + \boldsymbol{S}_{\mathrm{B}}\,. \qquad (6)$$

Generators of these group chains have been constructed in the following way

$$
\begin{aligned}
\mathrm{U^B}(36) &: (b^\dagger_{l\sigma\tau}\tilde{b}_{l'\sigma'\tau'})^{LS_1T_1} \\
\mathrm{U^B_L}(6) &: B^L_M(l,l') \equiv \sqrt{3}\sum_{\sigma,\tau}(b^\dagger_{l\sigma\tau}\tilde{b}_{l'\sigma\tau})^{L00}_M \\
\mathrm{SU^B_{ST}}(6) &: B^{ST}_{M_SM_T}(\sigma\tau,\sigma'\tau') \equiv \sum_l \sqrt{2l+1}(b^\dagger_{l\sigma\tau}\tilde{b}_{l\sigma'\tau'})^{0ST}_{M_SM_T} \\
& \quad B^{00} \equiv \sqrt{3}\big(B^{00}(10,10) - B^{00}(01,01)\big) \\
\mathrm{SO^B_L}(6) &: B^1_M(2,2),\ B^3_M(2,2),\ B^2_M(0,2) + B^2_M(2,0) \\
\mathrm{SU^B_S}(2) &: S^B_M \equiv \sqrt{2}B^{10}_M(10,10) \\
\mathrm{SU^B_T}(2) &: T^B_M \equiv \sqrt{2}B^{01}_M(01,01) \\
\mathrm{U^F}(8) &: (a^\dagger_{\frac{3}{2}\frac{1}{2}}\tilde{a}_{\frac{3}{2}\frac{1}{2}})^{JT},\ J = 0,1,2,3;\ T = 0,1 \\
\mathrm{SU^F_{J'}}(4) &: F^J_M \equiv -\sqrt{2}(a^\dagger_{\frac{3}{2}\frac{1}{2}}\tilde{a}_{\frac{3}{2}\frac{1}{2}})^{J0}_M,\ J = 1,2,3 \\
\mathrm{SU^F_T}(2) &: T^F_M \equiv \sqrt{2}(a^\dagger_{\frac{3}{2}\frac{1}{2}}\tilde{a}_{\frac{3}{2}\frac{1}{2}})^{01}_M \\
\mathrm{SU^{BF}_{J''}}(4) &: G^1_M \equiv B^1_M(2,2) - \tfrac{1}{\sqrt{2}}F^1_M \\
& \quad G^2_M \equiv B^2_M(2,0) + B^2_M(0,2) + F^2_M \\
& \quad G^3_M \equiv B^3_M(2,2) + \tfrac{1}{\sqrt{2}}F^3_M \\
\mathrm{Sp^{BF}_{J''}}(4) &: G^1_M, G^3_M \\
\mathrm{SU^{BF}_{J''}}(2) &: J''_M = \sqrt{10}G^1_M \\
\mathrm{SU^{BF}_T}(2) &: T_M = T^B_M + T^F_M \\
\mathrm{SU^{BF}_J}(2) &: J_M = J''_M + S^B_M\,.
\end{aligned}
\tag{7}
$$

3. The Hamiltonian

The chain of groups (5) is not, strictly speaking, the symmetry chain. It only defines the so called dynamical symmetry in the sense that the Hamiltonian is constructed with the help of generators of the chain (5), or, in other words, by generators of the Lie superalgebra (1). Both physical and technical reasons impose on the construction several restrictions. In the extreme but physically important case the Hamiltonian H is built with the help of the second order Casimir invariants of the groups (5). Such a construction fulfills the very important conservation laws and, moreover, the constructed H can be diagonalized by the standard group theoretical procedure. The Hamiltonian comprises also two-body interactions as well between nucleons, between bosons, as between nucleons and bosons. Then we assume

$$H = H_0 + \sum_i c_i \widehat{C}_i \,, \tag{8}$$

where H_0 is a constant part of the H, $\widehat{C}_i$ are the second order Casimir invariants and c_i are phenomenological parameters. The H_0 is irrelevant in our calculations because we usually calculate $H - H_0$, *i.e.*, energies of excited nuclear levels relative to the ground state. Due to the construction (8) each state vector of a nucleus is an eigenstate of the H. Moreover, lower energy levels under consideration belong to the unique irreducible representations of several subgroups of the chain (5) and, hence, the Casimir invariants of these subgroups can be put into H_0. Then the relevant subgroups of the supersymmetry group U(36/8) are the following

$$\begin{array}{ccccccc}
\mathrm{U}^{\mathrm{B}}_{L}(6) & \times & \mathrm{SU}^{\mathrm{F}}_{J'}(4) & \times & \mathrm{SU}^{\mathrm{B}}_{S}(2) & \times & \mathrm{SU}^{\mathrm{BF}}_{T}(2) \\
\downarrow & & | & & | & & \\
\mathrm{SO}^{\mathrm{B}}_{L}(6) & & | & & | & & \\
\hookrightarrow & \mathrm{SU}^{\mathrm{BF}}_{J''}(4) & \leftarrow & & | & & \\
 & \downarrow & & & | & & \\
 & \mathrm{Sp}^{\mathrm{BF}}_{J''}(4) & & & | & & \\
 & \downarrow & & & | & & \\
 & \mathrm{SU}^{\mathrm{BF}}_{J''}(2) & & & | & & \\
 & \hookrightarrow & \mathrm{SU}^{\mathrm{BF}}_{J}(2) & \leftarrow & & &
\end{array} \tag{9}$$

Eigenvalues of Casimir invariants of the groups in (9) which form the Hamiltonian (8) are expressed in proper quantum numbers labelling irreducible representations (IR's) of the groups (9). Hence, we have to analyse these IR's as well in a direct product appearing in (9) as in subgroups splitting.

For the boson group $\mathrm{U}^{\mathrm{B}}_{L}(6)$ we consider only the completely symmetric representation $[f,\, 0^5] \equiv [f]$ and of mixed symmetry $[f,\, 1,\, 0^4] \equiv [f,\, 1]$. Then it follows the irreducible representations $\langle g,\, 0,\, 0\rangle$ and $\langle g,\, 1,\, 0\rangle$ of the group $\mathrm{SO}^{\mathrm{B}}_{L}(6)$. Basis states of further irreducible representations are beyond of the considered region of excited energy levels. The group $\mathrm{SU}^{\mathrm{F}}_{J'}(4)$ is relevant for odd nuclei for which we consider one unpaired nucleon with state vectors belonging to the representation $\langle 1/2, 1/2, 1/2\rangle$. Hence, for odd nuclei we get the following direct product splitting

$$\begin{aligned}
&\mathrm{SO}^{\mathrm{B}}_L(6) \times \mathrm{SU}^{\mathrm{F}}_{J'}(4) \supset \mathrm{SU}^{\mathrm{BF}}_{J''}(4): \\
&\langle g,0,0\rangle \times \langle \tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2}\rangle = \langle g+\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2}\rangle + \langle g-\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2}\rangle \\
&\langle g,1,0\rangle \times \langle \tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2}\rangle = \langle g+\tfrac{1}{2},\tfrac{3}{2},\tfrac{1}{2}\rangle + \langle g+\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2}\rangle \\
&\qquad + \langle g-\tfrac{1}{2},\tfrac{3}{2},-\tfrac{1}{2}\rangle + \langle g-\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2}\rangle .
\end{aligned} \tag{10}$$

The IR's of (10) split into the IR's of $\mathrm{Sp}^{\mathrm{BF}}_{J''}(4)$

$$\begin{aligned}
&\mathrm{SU}^{\mathrm{BF}}_{J''}(4) \supset \mathrm{Sp}^{\mathrm{BF}}_{J''}(4): \\
&\langle g,\tfrac{1}{2},\pm\tfrac{1}{2}\rangle \supset (g,\tfrac{1}{2}),\ (g-1,\tfrac{1}{2}),\cdots,(\tfrac{1}{2},\tfrac{1}{2}), \\
&\langle g,\tfrac{3}{2},\pm\tfrac{1}{2}\rangle \supset (g,\tfrac{1}{2}),\ (g-1,\tfrac{1}{2}),\cdots,(\tfrac{3}{2},\tfrac{1}{2}), \\
&\qquad (g,\tfrac{3}{2}),\ (g-1,\tfrac{3}{2}),\cdots,(\tfrac{3}{2},\tfrac{3}{2}).
\end{aligned} \tag{11}$$

Then the lowest IR's of $\mathrm{Sp}^{\mathrm{BF}}_{J''}(4)$ involve the following J'' sets

$$\begin{aligned}
&\mathrm{Sp}^{\mathrm{BF}}_{J''}(4) \supset \mathrm{SU}^{\mathrm{BF}}_{J''}(2): \\
&(\tfrac{1}{2},\tfrac{1}{2}) \supset \tfrac{3}{2} \\
&(\tfrac{3}{2},\tfrac{1}{2}) \supset \tfrac{1}{2},\tfrac{5}{2},\tfrac{7}{2} \\
&(\tfrac{5}{2},\tfrac{1}{2}) \supset \tfrac{3}{2},\tfrac{5}{2},\tfrac{7}{2},\tfrac{9}{2},\tfrac{11}{2} \\
&(\tfrac{3}{2},\tfrac{3}{2}) \supset \tfrac{3}{2},\tfrac{5}{2},\tfrac{9}{2}.
\end{aligned} \tag{12}$$

We stress that in the direct product (10) we have made an assumption that the fermion part of a system consists, for odd nuclei, of only one fermion — the rest of them are paired to form bosons. For even nuclei there are bosons only and hence there is a boson group $\mathrm{SO}^{\mathrm{B}}_L(6)$, the same as in [1] where we considered the full *sd* fermion shell. However, we should not expect exactly the same theoretical energy levels for even nuclei because the adjusted parameters of the present fermion model $j = {}^3\!/_2$ differ from those in the *sd* model. Decomposition of the IR's of the boson groups into their subgroups can be found in [1, 3, 5, 6].

In the formulas (10–12) we have introduced the following notation for the IR's: $[f_1, f_2]$ is for the IR's of U(6), $\langle g_1, g_2, 0\rangle$ $\{\langle \gamma_1,\gamma_2,\gamma_3\rangle\}$ denotes the IR's of the SO(6) or {SU(4)}, (h_1, h_2) is for SO(5) or Sp(4) and for SU(2) we have $J; L; S; T$ whatever is considered.

Following the phenomenological Hamiltonian (8) with restriction (9) we write

$$\begin{aligned}
H = H_0 + P\widehat{C}[\mathrm{U}^{\mathrm{B}}_L(6)] + W_1\widehat{C}[\mathrm{SO}^{\mathrm{B}}_L(6)] + W_2\widehat{C}[\mathrm{SU}^{\mathrm{BF}}_{J''}(4)] \\
+K\widehat{C}[\mathrm{Sp}^{\mathrm{BF}}_{J''}(4)] + D\widehat{C}[\mathrm{SU}^{\mathrm{BF}}_{J''}(2)] + F\widehat{C}[\mathrm{SU}^{\mathrm{BF}}_{J}(2)].
\end{aligned} \tag{13}$$

The Casimir operators commute with each other and we can diagonalise H with the help of eigenvalues of the Casimir invariants which gives

$$\begin{aligned} E =& E_0 + P[f_1(f_1+5)+f_2(f_2+3)] + W_1[g_1(g_1+4)+g_2(g_2+2)] \\ &+ W_2[\gamma_1(\gamma_1+4)+\gamma_2(\gamma_2+2)+\gamma_3^2] + K[h_1(h_1+3)+h_2(h_2+1)] \\ &+ DJ''(J''+1)+FJ(J+1)\,. \end{aligned} \tag{14}$$

In (13–14) there are not the spin-isospin dependent numbers coming at first from the group $SU^{B}_{ST}(4)$ and then from $SU^{B}_{S}(2)$ and $SU_T(2)$. We have followed arguments [4] that the lowest energy representations of the $SU^{B}_{TS}(4)$ are those with the smallest value of the Casimir operator and then this operator is absorbed by $H_0(E_0)$. This assumption leads to the boson spin $S=0$ with an exception for odd-odd nuclei with odd number of bosons (*i.e.* ^{34}Cl) for which (ST) can take on values (0,1) or (1,0); see also Appendix of [1]. Hence only in this case we need to add to expressions (13–14) the term AS^2+BT^2 with its eigenvalue $AS(S+1)+BT(T+1)$. For excited states (^{34}Cl) the difference $A-B$ is relevant and we have adopted the value $A-B=0.5$ MeV.

TABLE I

The lowest considered irreducible representations. Values of the total angular momentum J are given in the last column when total boson spin $S=1$ (see text). In other cases ($S=0$) J is equal to J''.

Nucleus	$U^B(6)$ $[f_1,f_2]$	$SU^{BF}(4)$ $\langle\gamma_1,\gamma_2,\gamma_3\rangle$	$Sp^{BF}_{J''}(4)$ (h_1,h_2)	$SU^{BF}_{J''}(2)$ J''	$SU^{BF}_{J}(2)$ J
1	2	3	4	5	6
^{32}S					
	[4,0]	$\langle 4,0\rangle$	(0,0)	0	
			(1,0)	2	
			(2,0)	2,4	
			(3,0)	0,3,4	
			(4,0)	2	
^{33}S					
	[3,0]	$\langle\frac{7}{2},\frac{1}{2},\frac{1}{2}\rangle$	$(\frac{1}{2},\frac{1}{2})$	$\frac{3}{2}$	
			$(\frac{3}{2},\frac{1}{2})$	$\frac{1}{2},\frac{5}{2},\frac{7}{2}$	
			$(\frac{5}{2},\frac{1}{2})$	$\frac{3}{2},\frac{5}{2},\frac{7}{2},\frac{9}{2}$	
	[2,1]	$\langle\frac{5}{2},\frac{3}{2},\frac{1}{2}\rangle$	$(\frac{3}{2},\frac{1}{2})$	$\frac{1}{2},\frac{5}{2}$	

TABLE I continued

1	2	3	4	5	6
^{32}P					
	[3, 1]	⟨3, 1⟩	(1, 0)	2	1, 2
			(2, 0)	2, 4	1, 2; 3
			(3, 0)	3, 4, 6	2; 3; 5
			(1, 1)	1, 3	0, 1; 2, 3
			(2, 1)	1, 2, 3, 4, 5	0, 1; 1, 2; 2; 3; 4
			(3, 1)	2, 3, 4	1; 2; 3
		⟨2, 0⟩	(1, 0)	2	1
^{34}S					
	[3, 0]	⟨3, 0⟩	(0, 0)	0	
			(1, 0)	2	
			(2, 0)	2, 4	
			(3, 0)	0, 3, 4,	
		⟨1, 0⟩	(0, 0)	0	
			(1, 0)	2	
	[2, 1]	⟨2, 1⟩	(1, 0)	2	
			(2, 0)	2, 4	
			(1, 1)	1, 3	
			(2, 1)	1, 2, 3	
		⟨1, 0⟩	(0, 0)	0	
			(1, 0)	2	
^{35}Cl					
	[2, 0]	$\langle\frac{5}{2},\frac{1}{2},\frac{1}{2}\rangle$	$(\frac{1}{2},\frac{1}{2})$	$\frac{3}{2}$	
			$(\frac{3}{2},\frac{1}{2})$	$\frac{1}{2},\frac{5}{2},\frac{7}{2}$	
			$(\frac{5}{2},\frac{1}{2})$	$\frac{3}{2},\frac{5}{2},\frac{7}{2},\frac{9}{2}$	
		$\langle\frac{3}{2},\frac{1}{2},\frac{1}{2}\rangle$	$(\frac{1}{2},\frac{1}{2})$	$\frac{3}{2}$	
			$(\frac{3}{2},\frac{1}{2})$	$\frac{1}{2}$	
^{34}Cl					
$T = 0$	[3, 0]	⟨3, 0⟩	(0, 0)	0	1
			(1, 0)	2	1, 2
			(2, 0)	2, 4	1, 2; 3
			(3, 0)	3, 4, 6	2; 3; 5
	[2, 1]	⟨2, 1⟩	(1, 0)	2	1
			(2, 0)	2, 4	1; 3
			(1, 1)	1, 3	0; 2
			(2, 1)	2, 3, 4, 5	1; 2; 3; 4
$T = 1$	[3, 0]	⟨3, 0⟩	(0, 0)	0	
			(1, 0)	2	
			(2, 0)	2	
			(3, 0)	0	
	[2, 1]	⟨2, 1⟩	(1, 0)	2	
			(1, 1)	1	

The energy (14) is expressed by the standard labels of the IR's of groups involved. The strict group-theory considerations provide the values of those labels proper for the lowest energy levels (up to 4–7 MeV). The full set of the lowest irreducible representations are given in Table I. The model parameters are fitted to the experimental data [7] separately for each supermultiplet (Table II).

4. Applications

We have considered two supermultiplets from the region $32 \leq A < 40$ namely those with the total number of holes $N = 4$ and $N = 3$. The first

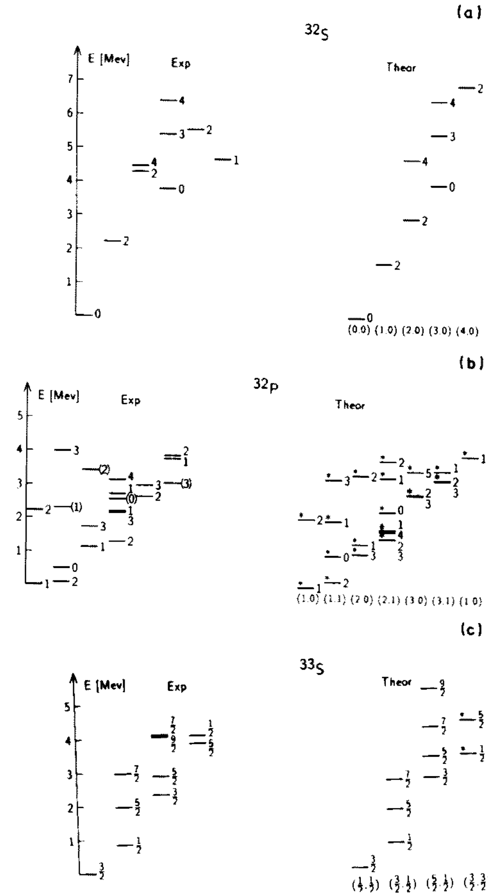

Fig. 1. Experimental positive parity [7] and theoretical energy levels of nuclei ^{32}S, ^{32}P and ^{33}S. Levels are grouped according to the IR's of the group Sp(4), (SO(5)), with IR labels placed below. Asterisks mark the levels belonging to the IR $[f, 1]$ of the group U(6).

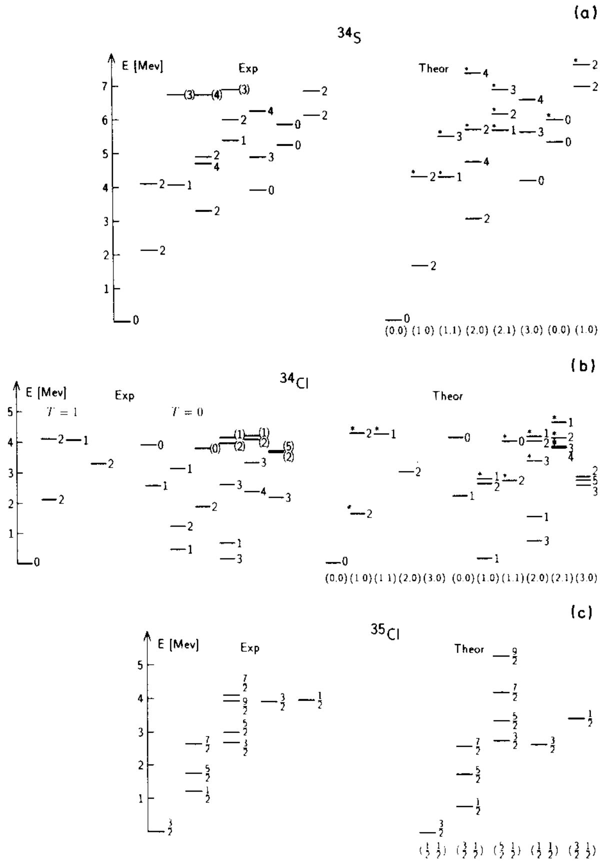

Fig. 2. Experimental [7] and theoretical energy levels of nuclei ^{34}S, ^{34}Cl and ^{35}Cl. See caption to Fig. 1.

supermultiplet contains nuclei $^{32}_{16}$S, $^{32}_{15}$P, $^{33}_{16}$S and $^{33}_{17}$Cl and the second one: $^{34}_{16}$S, $^{34}_{17}$Cl; $^{35}_{17}$Cl, $^{35}_{18}$Ar. We present the results only for one nucleus from mirror pair with $T = {}^1\!/_2$, namely $^{33}_{16}$S and $^{35}_{17}$Cl for which there exist wider experimental data. The model Hamiltonian depends practically on four (five for $N = 3$) free parameters (Table II). It has been stressed that each of the supermultiplets contains about 40 energy levels to be described in a supersymmetry scheme. All of the calculated as well as experimental angular momenta and energies of positive parity levels are taken into account in Fig. 1–2 up to considered energies. The levels are organized in multiplets of the group $\mathrm{Sp}^{\mathrm{BF}}_{J''}(4)$ labelled by numbers (h_1, h_2). All of the levels belong to two IR's of the group $\mathrm{U}^{\mathrm{B}}(6)$: the completely symmetric $[f]$ and of mixed symmetry $[f-1, 1]$, the last ones are marked on Fig. 1–2 by asterisks. The nuclei $^{32}_{16}$S; $^{33}_{16}$S; $^{34}_{16}$S; $^{35}_{17}$Cl have been considered in [1] but in $L-S$ coupling for the full *sd* shell. For nuclei $^{32}_{15}$P and $^{34}_{17}$Cl it is the first supersymmetry calculation.

TABLE II

Hamiltonian parameters in MeV

	P	$P+W_1+W_2$	K	D	F
$N=3$	-0.11	-0.44	0.23	-0.500	0.62
$N=4$		-0.44	0.22	-0.385	0.51

5. The electromagnetic $E2$ transitions

It is usually much easier to reproduce experimental energies as eigenvalues of a considered Hamiltonian of a system than to construct its eigenstates in a proper way. Hence the next to energy calculations there is a consideration of transition probabilities which strongly depend not only on a transition operator in the assumed model but also on constructed vector states. In what follows we will consider the $E2$ transition probabilities in our supersymmetry model.

To construct the electromagnetic transition operator we follow the procedure of [2] and write

$$T(E2) = \alpha T_B^{(JT)} + \alpha' T_F^{(JT)}, \tag{15}$$

where for $E2$ $J=2$ and $T=0$ as the isovector transitions ($T=1$) are of the order of magnitude lower. The $T(E2)$ operator should not change the boson (fermion) number and it is, by a simple assumption, of the second order in creation and annihilation operators. The next very strong assumption follows the SO(6) limit in the group chain (3) stating that the boson (fermion) part of (15) be built by means of generators of the group $\mathrm{SO}_L^{\mathrm{B}}(6)$ ($\mathrm{SU}_{J''}^{\mathrm{F}}(4)$). Then the total operator (15) is, by an assumption, the generator of the boson-fermion group $\mathrm{SU}_{J''}^{\mathrm{BF}}(4)$. These assumptions and approximations lead to formulas

$$T_B^{(2,0)} = \sum_{\sigma,\tau}\left(\left(b^\dagger_{0\sigma\tau}\tilde{b}_{2\sigma\tau}\right)^{200} + \left(b^\dagger_{2\sigma\tau}\tilde{b}_{0\sigma\tau}\right)^{200}\right),$$

$$T_F^{(2,0)} = \left(a^\dagger_{\frac{3}{2}\frac{1}{2}}\tilde{a}_{\frac{3}{2}\frac{1}{2}}\right)^{J=2;\,T=0}$$

and

$$\frac{\alpha}{\sqrt{3}} = -\frac{\alpha'}{\sqrt{2}} \equiv Q_{\mathrm{eff}}, \tag{16}$$

where Q_{eff} plays a role of an "effective charge". Then

$$T^{(2,0)} = Q_{\text{eff}} \left(\sqrt{3} T_B^{(2,0)} - \sqrt{2} T_F^{(2,0)} \right) . \tag{17}$$

The operator (17) has also the definitive tensor character in the transformation group $\mathrm{Sp}^{\mathrm{BF}}_{J''}(4)$ namely it is of the type $(h_1, h_2) = (1, 0)$. There are following selection rules coming from the construction (16–17):

(i) Nonzero matrix elements of (17) are only between states belonging to the same IR of $\mathrm{SU}^{\mathrm{BF}}_{J''}(4)$ for odd nuclei and to the same IR of $\mathrm{U}^{\mathrm{B}}_{L}(6)$ and $\mathrm{SO}^{\mathrm{B}}_{L}(6)$ in even nuclei.

(ii) For odd nuclei with initial and final states which belong to the IR's of $\mathrm{Sp}^{\mathrm{BF}}_{J''}(4)$: $(h_1, h_2)_{\mathrm{i}}$ and $(h'_1, h'_2)_{\mathrm{f}}$ respectively there must be

$$\Delta h_1 = 0; \pm 1 , \qquad \Delta h_2 = 0 .$$

(iii) For even nuclei the selection rule reads

a) for symmetric representations $[f, 0]$ of the group $\mathrm{U}^{\mathrm{B}}_{L}(6)$

$$\Delta h_1 = \pm 1 ; \qquad \Delta h_2 = 0 ,$$

b) for mixed symmetry representations $[f, 1]$

$$\Delta h_1 = \pm 1 , \quad \Delta h_2 = 0 \quad \text{or} \quad \Delta h_1 = 0 , \quad \Delta h_2 = \pm 1 .$$

The states of a given supermultiplet $N =$ const can be factorized by the quantum numbers of the irreducible representations of the groups (9):

$$|N[f_1, f_2]\langle \gamma_1, \gamma_2, \gamma_3 \rangle (h_1, h_2) J'', S, J, T\rangle \equiv |(\delta), J\rangle . \tag{18}$$

Then the $B(E2)$ probability reads

$$B(E2, \mathrm{i} \to \mathrm{f}) = \frac{1}{2J_{\mathrm{i}} + 1} |\langle (\delta)_{\mathrm{i}}, J_{\mathrm{i}} \parallel T^{(2,0)} \parallel (\delta)_{\mathrm{f}}, J_{\mathrm{f}} \rangle|^2 . \tag{19}$$

Labels of the IR's of the groups (9) describing initial and final state are listed in Table I. The reduced matrix elements (18) has been evaluated with the help of already performed calculations [3, 5].

For comparison with experimental data [8] we have taken the nuclei ^{34}S and ^{35}Cl from the supermultiplet $N = 3$ and ^{32}S, ^{33}S from $N = 4$. Then the "effective charge" Q_{eff} in one parameter formula (17) has been adjusted to the experimental data $E_{\mathrm{i}} \to E_{\mathrm{f}}$ (2.13 MeV$\to$ g.s.) of the nucleus ^{34}S and for $E_{\mathrm{i}} \to E_{\mathrm{f}}$ (2.23 MeV $\to$ g.s.) in ^{32}S. We have taken $Q^2_{\text{eff}} = 8.8\ \mathrm{e}^2\ \mathrm{fm}^4$ and $Q^2_{\text{eff}} = 9.4\,\mathrm{e}^2\ \mathrm{fm}^4$ respectively. The experimental [8] and theoretical values of $B(E2)$ are given in Table III.

TABLE III

$B(E2)$ transition probabilities (in e^2 fm^4)

E_i	E_f	J_i	J_f	Exp [8]	Theor
^{32}S					
2.23	0.00	2	0	59.9 ± 6.0	59.9
3.78	2.23	0	2	83.9 ± 12.0	0.0
4.28	0.00	2	0	8.4 ± 1.2	0.0
	2.23	2	2	55.7 ± 7.8	72.1
4.46	2.23	4	2	71.9 ± 12.0	72.1
5.41	2.23	3	2	16.2 ± 3.0	0.0
5.55	0.00	2	0	0.7 ± 0.2	0.0
6.41	2.23	4	2	18.0 ± 4.2	0.0
^{33}S					
0.84	0.00	$1/2$	$3/2$	23.7 ± 1.2	45.0
1.97	0.00	$5/2$	$3/2$	43.7 ± 6.9	45.0
2.31	0.00	$3/2$	$3/2$	25.6 ± 4.4	0.0
	0.84	$3/2$	$1/2$	53.7 ± 11.9	16.9
2.87	0.00	$5/2$	$3/2$	2.0 ± 0.9	0.0
2.97	0.00	$7/2$	$3/2$	35.0 ± 5.6	45.0
4.05	1.97	$9/2$	$5/2$	59.3 ± 10.0	33.7
	2.97	$9/2$	$7/2$	18.7 ± 5.0	9.4
4.09	0.00	$7/2$	$3/2$	0.7 ± 0.2	0.0
	1.97	$7/2$	$5/2$	23.1 ± 9.4	23.4
^{34}S					
2.13	0.00	2	0	37.0 ± 1.9	37.0
3.30	0.00	2	0	4.7 ± 0.3	0.0
	2.13	2	2	26.6 ± 6.5	40.6
3.91	2.13	2	2	27.9 ± 4.5	0.0
4.11	0.00	2	0	3.8 ± 0.6	0.0
	2.13	2	2	20.1 ± 7.8	0.0
4.69	2.13	4	2	56.5 ± 6.5	40.6
6.25	4.88	4	3	182.0 ± 39.0	0.0
^{35}Cl					
1.22	0.00	$1/2$	$3/2$	14.9 ± 2.7	24.7
1.76	0.00	$5/2$	$3/2$	74.3 ± 6.8	24.7
2.65	0.00	$7/2$	$3/2$	20.3 ± 2.7	24.7
	1.76	$7/2$	$5/2$	29.8 ± 12.2	7.9
2.69	0.00	$3/2$	$3/2$	8.1 ± 2.7	0.0
3.94	1.76	$9/2$	$5/2$	50.7 ± 8.8	14.1
4.11	0.00	$7/2$	$3/2$	5.0 ± 1.1	0.0
	1.76	$7/2$	$5/2$	1.9 ± 0.6	9.7

6. Conclusions

The present investigations have confirmed that light nuclei from the *sd* shell behave according to the assumed supersymmetry. Nobody has expected a perfect agreement between theoretical and experimental data or even as good as in a standard procedure of the shell model calculations. It is fully understood because in the supersymmetry scheme we have taken further approximations especially in the construction of the phenomenological Hamiltonian (13). The new feature of supersymmetry considerations comes from the treatment of even-even, even-odd and odd-odd nuclei on the same footing. In other words, the supersymmetry reveals the strong connections between these nuclei which had been treated before quite separately. From the microscopical point of view it means that for low energy behaviour of nuclei they can be treated as consisted of pairs of nucleons (bosons) and unpaired nucleons and these "super-particles" do not feel any differences while considered in the supersymmetry scheme. Deviations of theoretical data from experimental values point on approximate procedure in which an assumption of treating pairs of nucleons as bosons and the Casimir invariant structure of the Hamiltonian are, from the very beginning, evident approximation.

We want also to stress that the much simpler symmetry in the fermion space, namely $U^F(8)$ as compared to $U^F(24)$ considered before [1] does not spoil the theoretical results and even more: for odd nuclei for which the group $U^F(8)$ is relevant, we have obtained better results. Besides, we have also included in the supersymmetry multiplets two new nuclei ${}^{32}_{15}P$ and ${}^{34}_{17}Cl$ with a quite good accuracy of their calculated energy levels.

The energy calculations are indirectly confirmed by the $E(2)$ transition probability evaluations. In the same supersymmetry scheme the one parameter $B(E2)$ approximate formula (21) describes quite well the experimental values. Inspection of Table III shows up that the theoretical values follow experimental data with approximations accepted for far going assumptions of our model. There are only two distinct exceptions: $E_i \rightarrow E_f$ (3.78 MeV $\rightarrow$ 2.23 MeV) for ^{32}S and $E_i \rightarrow E_f$ (6.25 MeV $\rightarrow$ 4.88 MeV) for ^{34}S. In the spirit of our model we should say that the states with energy $E = 3.78$ MeV for ^{32}S and $E = 6.25$ MeV in ^{34}S are of the structure beyond of our model.

Summarizing the results we are able to state that for ground and low energy excited states of nuclei in the considered region there is an evident signature of the supersymmetry.

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