NUCLEAR SYMMETRY ENERGY AND THE PROPERTIES OF NEUTRON STAR MATTER*

P. HAENSEL**

DARC - UPR 176 CNRS, Observatoire de Paris, Section de Meudon
F-92195 Meudon Cedex, France

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Dedicated to Janusz Dąbrowski in honour of his 65th birthday

Knowledge of the symmetry energy of nuclear matter is crucial for the determination of many properties of matter at ultra high density. Its value determines the neutron drip density in the crust of neutron star. The symmetry energy determines the proton fraction in the neutron star matter at supranuclear density, and therefore turns out to be of a paramount importance for the rate of cooling of neutron stars. It determines also the response of neutron star matter to the deviations from chemical equilibrium, and enters explicitly the formula for the bulk viscosity of the neutron star matter.

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1. Introduction

Due to Coulomb effects, atomic nuclei have an excess of neutrons over protons, characterized by the neutron excess parameter, $\alpha = (N - Z)/A$. In the idealized limit of infinite nuclear matter this corresponds to the case of isospin asymmetric nuclear matter, with nonequal number densities of neutrons, n_n, and protons, n_p. It is suitable to characterize asymmetric nuclear matter by the nucleon density, $n = n_\mathrm{n} + n_\mathrm{p}$, and by $\alpha = (n_\mathrm{n} - n_\mathrm{p})/n$.

In view of the isospin dependence of the relevant quantities, asymmetric nuclear matter is a significantly more complicated many-body system than the standard, symmetric nuclear matter. This can be clearly seen in

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** Permanent address: N. Copernicus Astronomical Center, Polish Academy of Sciences, Bartycka 18, PL-00-716 Warszawa, Poland.

the pioneering calculations of the properties of asymmetric nuclear matter, performed within the Brueckner many-body theory by Brueckner and Dabrowski [1] and by Brueckner, Coon and Dabrowski [2].

In view of the charge symmetry of nuclear forces, the energy per nucleon of an asymmetric nuclear matter, $E_{\rm N}$, is (at fixed n) an even function of α (by definition, Coulomb interaction is switched off). For $\alpha \ll 1$ we have thus

$$E_{\rm N}(n,\alpha) \approx E_0(n) + S(n)\alpha^2 \ . \tag{1.1}$$

Here, $E_0(n)$ is the energy per nucleon of symmetric nuclear matter of density n, and $S(n)$ is the symmetry energy.

The many-body calculations of asymmetric nuclear matter show, that Eq. (1.1) is valid, to a very good approximation, even for the values of α close to unity, so that the energy per nucleon of pure neutron matter, $\widetilde{E}(n)$, can be very well approximated by $E_0(n) + S(n)$ (see, e.g., [3-6]).

The neutron excess in experimentally studied atomic nuclei is rather small. Also, finite size and Coulomb effects in heavy nuclei imply, that the relation between the theoretically studied asymmetric nuclear matter, which is par excellence an infinite system, and real atomic nuclei, is not straightforward. However, genuine asymmetric nuclear matter, with a very large neutron excess, exists in the superdense cores of neutron stars (for a general review of the properties of neutron stars and their astrophysical incarnations, see the monograph of Shapiro and Teukolsky [7]).

Experimental determinations of the bulk symmetry energy at the saturation density $a_{\rm sym}$ yield $a_{\rm sym} = 27 - 36$ MeV [8]. The quantity $a_{\rm sym}$ should be reproduced by the many-body calculations of asymmetric nuclear matter. Ideally, we would like to get $S(n_0) = a_{\rm sym}$. However, the density dependence of the symmetry energy, and its value at, say, $n > 2n_0$ ($n_0 = 0.16$ fm^{-3} is the experimental saturation density of symmetric nuclear matter), are not available experimentally. Therefore, in the neutron star matter calculations, we have to rely on the theoretical determination of $S(n)$. Detailed discussion of the relevance of the density dependence of S for the neutron star structure can be found in Refs. [9,10].

In the present paper I review theoretical studies of some properties of matter in the interior of neutron stars, for which the nuclear symmetry energy is a crucial input quantity. In Section 2 I show how its value determines, to a large extent, the neutron drip density in the crust of neutron star. The symmetry energy determines the proton fraction in the neutron star matter at supranuclear density, and therefore turns out to be of a paramount importance for the rate of cooling of neutron stars. This is reviewed in Section 3. As I discuss in Section 4, nuclear symmetry energy determines also the response of neutron star matter to the deviations from chemical equilibrium, and enters explicitly the formula for the bulk viscosity of the neutron star matter. Finally, Section 5 contains discussion and conclusion.

2. Neutron drip in the neutron star crust

The outer envelope of neutron star consists of matter of the density ρ increasing with depth d below the stellar surface. Let us consider the problem of the dependence of the nuclear composition of matter on ρ, assuming that the matter is in a complete thermodynamic equilibrium. Thermal effects are negligible for $\log_{10}\rho > 6$ and $\log_{10}T < 9$ (conditions prevailing for $d > 1$ m in neutron stars of age > 1 year), and full thermodynamic equilibrium corresponds then to the ground state of matter. At $\log_{10}\rho > 6$ atoms are so squeezed that they loose their individuality, and matter forms a plasma composed of nuclei (mass number A, proton number Z), embedded in an ultrarelativistic electron gas. Pressure, P, increases monotonically with d. At given value of P, the values of A, Z, electron density n_e, and mass density of matter, ρ, are obtained from the minimization of the Gibbs potential per nucleon, g, which (neglecting small solid state effects) reads

$$g(A, Z; P) = \tfrac{1}{A}W(A, Z) + \tfrac{Z}{A}\mu_e\,, \tag{2.1}$$

where $W(A, Z)$ is the energy of the nucleus (which includes rest energy of nucleons), and the electron chemical potential (equal to the Fermi energy of the electron gas) is

$$\mu_e = 1.0\left(\rho_6 \tfrac{Z}{A}\right)^{1/3}\ \text{MeV}\,, \tag{2.2}$$

and $\rho_6 = \rho/10^6$ g cm^{-3}. The pressure of matter is provided by the ultrarelativistic electron gas, $P \propto \mu_e^4$, so that the electron contribution to g increases as $\mu_e \propto P^{1/4}$. With increasing P (and ρ), the nuclide corresponding to the minimum of g shifts gradually to lower and lower value of the proton fraction, $x = Z/A$ (larger and larger value of the neutron excess, $\alpha = 1 - 2x$). Let us approximate the value of energy per nucleon in the atomic nucleus, with rest energy excluded and neglecting finite size and Coulomb effects, W'/A, by the formula $W'/A = -a_{\rm vol} + a_{\rm sym}\alpha^2$. The chemical potentials of neutrons and protons in nuclei, with rest energy subtracted, are then given by

$$\begin{aligned}\mu'_n &= -\,a_{\rm vol} + a_{\rm sym}\,(\ 2\alpha - \alpha^2)\\ \mu'_p &= -\,a_{\rm vol} + a_{\rm sym}\,(-2\alpha - \alpha^2)\,.\end{aligned} \tag{2.3}$$

As the equilibrium value of x decreases with increasing ρ, the value of $\mu'_{\rm n}$ increases, too, and vanishes at $\rho = \rho_{\rm ND}$. For $\rho > \rho_{\rm ND}$ some neutrons are not bound in nuclei, and form a gas. So, for $\rho > \rho_{\rm ND}$, neutron star crust is composed of nuclei, embedded in electron and neutron gases. Using our simple model, we may estimate the limiting value of proton fraction, corresponding to "neutron drip", $x_{\rm ND}$. Using Eq.(2.3), we get

$$x_{\rm ND} = \frac{1}{2}\sqrt{1 - \frac{a_{\rm vol}}{a_{\rm sym}}}\,. \tag{2.4}$$

Putting $a_{\rm vol} = 16$ MeV and $a_{\rm sym} = 30$ MeV we get $x_{\rm ND} = 0.34$.

Matter in the neutron star crust is in equilibrium with respect to weak interactions (beta equilibrium), which results in the relation between the chemical potentials: $\mu_{\rm e} = \mu_{\rm n} - \mu_{\rm p}$ ($\mu_{\rm i} = \mu'_{\rm i} + m_{\rm i}c^2$, i=n, p). In our approximation, this relation reads, neglecting the rest mass difference between neutron and proton:

$$(\mu_{\rm e})_{\rm ND} = 4(1-2x)a_{\rm sym}\,. \tag{2.5}$$

Combining this result with the formula for $\mu_{\rm e}$, we get an approximate expression for the neutron drip density,

$$\rho_{\rm ND} = 6.4 \times 10^7\ (1 - 2x_{\rm ND})^3 \frac{(a_{\rm sym})^3}{x_{\rm ND}}\ {\rm g\ cm^{-3}}\,, \tag{2.6}$$

where $a_{\rm sym}$ is expressed in MeV. Putting $a_{\rm sym} = 30$ MeV and using our previous estimate $x_{\rm ND} = 0.34$, we get $\rho_{\rm ND} \simeq 2 \times 10^{11}$ g cm^{-3}.

Detailed calculations of the composition of the ground state of dense cold matter give somewhat higher values of the neutron drip density. For example, the calculation in which masses of neutron rich nuclei were calculated using the Hartree–Fock–Bogoliubov scheme with Skyrme interaction yielded $\rho_{\rm ND} = 4.4 \times 10^{11}$ g cm^{-3} [12]. The calculation based on the extrapolation of the semiphenomenological droplet model mass formula gives a slightly higher value of $\rho_{\rm ND} = 5.0 \times 10^{11}$ g cm^{-3} [12].

Typical case in which matter in the neutron star crust *is not* in the ground state, is that of a mass accreting neutron star. Strong deviations from the ground state result from the relatively low temperatures, at which the crust is formed via compression of accreted matter. For accretion rate $\sim 10^{-10}\ M_\odot$/year typical temperature in the neutron star interior is a few times 10^8 K, and thermonuclear reactions are effectively blocked by the Coulomb barriers. The composition of the crust of accreting neutron star has been studied in [13]. While the mass number of the nuclei — present in matter at the neutron drip density — turns out to be less than half of these in the ground state of dense matter, the value of the neutron drip density does not change much compared to the ground state case; for a nuclear model considered in [13] one gets $\rho_{\rm ND} = 6 \times 10^{11}$ g cm^{-3}.

The neutron drip density in the neutron star crust turns out to be rather insensitive to the models used to describe the neutron rich nuclei in superdense matter. It is also quite independent of the scenario of formation of the neutron star crust. The value of $a_{\rm sym}$ determines a simple estimate of $\rho_{\rm ND}$, with typical result $\log_{10}(\rho_{\rm ND}) \sim 11$.

3. Cooling of neutron stars via Urca reactions

Neutron stars are formed as very hot objects, with interior temperatures exceeding 10^{11} K ($kT > 10$ MeV). During the first $10^5 - 10^6$ years, solitary neutron stars cool via neutrino emission from their interior. However, although a short (~ 10) s neutrino burst from a neutron star formed in the SN 1987A has been detected, neutrinos emitted even from a nearby neutron star are expected to be detectable only during a few minutes after the neutron star formation, due to a very rapid decline of neutrino luminosity with time. Young neutron stars could (hopefully) be detected via photon emission from their surface. Their photon luminosity decreases in time at the rate resulting from the neutrino cooling of the stellar interior.

The simplest model of the hot, liquid neutron star interior (baryon density $n > n_0$), is an electrically neutral mixture of neutrons, protons, and electrons, of number densities $n_{\rm n}$, $n_{\rm p}$, $n_{\rm e}$, respectively. The chemical composition of the npe matter is suitably characterized by the proton fraction x, with $n_{\rm p} = n_{\rm e} = xn$, $n_{\rm n} = (1-x)n$. The proton fraction is related to the neutron excess parameter by $\alpha = 1 - 2x$. Under prevailing conditions, each of the components is a strongly degenerate Fermi liquid with Fermi energy $E_{\rm Fi} >> kT$ (i = n, p, e). The energy per nucleon is a sum of the nucleon component, $E_N(n,x)$, and the contribution of the free, ultrarelativistic electron gas, $E_{\rm e}$. As the timescale of cooling is much longer than that of the weak interactions in the hot, dense plasma, the npe matter is in the equilibrium with respect to beta reactions

$$n \longrightarrow p + e^- + \bar{\nu}_{\rm e} \tag{3.1a}$$

$$p + e^- \longrightarrow n + \nu_{\rm e}\ . \tag{3.1b}$$

Neutron decay (electron capture on a proton) is accompanied by emission of $\bar{\nu}_{\rm e}$ ($\nu_{\rm e}$), which leaves the star, taking away some of the available thermal energy. Both processes (3.1) are a specific example of the Urca process, introduced by Gamow and Schoenberg as very efficient sinks of thermal energy of highly evolved (hot and dense) stars [14].

The condition of the beta equilibrium results in the relation between the chemical potentials, $\mu_{\rm n} = \mu_{\rm p} + \mu_{\rm e}$, and strong degeneracy of all components restricts severely the momentum space available for the particles participating in processes (3.1). As the prevailing temperatures are well below typical Fermi temperatures, $T \ll T_{\rm Fi} \equiv E_{\rm Fi}/k \sim 10^{12}$ K, all fermions participating in the processes (3.1) must have momenta close to the corresponding Fermi momenta, $p_{\rm Fi}$. Since neutrino and antineutrino momenta are at most $\sim kT/c \ll p_{\rm Fi}$, the condition of momentum conservation in (3.1) is the triangle condition $p_{\rm Fp} + p_{\rm Fe} > p_{\rm Fn}$. For simple *npe* model of the neutron star matter the triangle condition becomes $2p_{\rm Fp} > p_{\rm Fn}$, or $x > x_{\rm c} = \frac{1}{9}$.

For more than 26 years since the classical paper of Chiu and Salpeter [15], who were first to consider the Urca process as an efficient mechanism for neutron star cooling, it was believed, that at the baryon densities, n, prevailing in the interiors of neutron stars, the equilibrium proton fraction, $x = n_{\rm p}/n$, is so low that the simple "direct Urca processes", Eq. (3.1), cannot proceed. This standard assumption was corroborated by the simplest free Fermi gas model of the *npe* plasma, for which $x = 5 \times 10^{-3} n/n_0$ ($n_0 = 0.16$ fm^{-3} is normal nuclear density). The paper of Boguta [16], who pointed out that for some models of the *npe* matter proton fraction could be quite large, was unnoticed by the neutron star theorists. It view of this, the standard assumption was, that the beta equilibrium is maintained through the *modified* Urca processes,

$$\begin{aligned} n + n &\longrightarrow n + p + e^- + \bar{\nu}_{\rm e} \\ n + p + e^- &\longrightarrow n + n + \nu_{\rm e} \ . \end{aligned} \tag{3.2}$$

The participation of an additional "active spectator" nucleon in the neutron decay or electron capture reactions is necessary to allow for the conservation of energy and momentum in the degenerate neutron star matter, in which neutron, proton and electron Fermi momenta are assumed to violate the inequality $p_{\rm Fn} < p_{\rm Fp} + p_{\rm Fe}$ [15].

Recently, it has been shown that for numerous models of dense nucleon matter the momentum condition $p_{\rm Fn} < p_{\rm Fp} + p_{\rm Fe}$ *is* actually satisfied at a sufficiently high n, allowing thus for the *direct* Urca processes in the neutron star matter: $n \longrightarrow p + e^- + \bar{\nu}_{\rm e}$, $p + e^- \longrightarrow n + \nu_{\rm e}$ [17]. This would dramatically increase (by many orders of magnitude) the neutrino emissivity of the neutron star interior [17], implying therefore a very rapid cooling of young neutron stars.

For the *npe* model of the neutron star matter, the energy per baryon , E, is the sum of the nucleon, $E_{\rm N}$, and electron, $E_{\rm e}$, contributions: $E = E_{\rm N} + E_{\rm e}$. The beta equilibrium condition at a given n, $(\partial E/\partial x)_n = 0$, determines then the equilibrium proton fraction $x_{\rm eq}$ as a solution of equation

$$\frac{x^{\frac{1}{3}}}{1-2x} = \frac{4S(n)}{\hbar c (3\pi^2)^{\frac{1}{3}} n^{\frac{1}{3}}} \ . \tag{3.3}$$

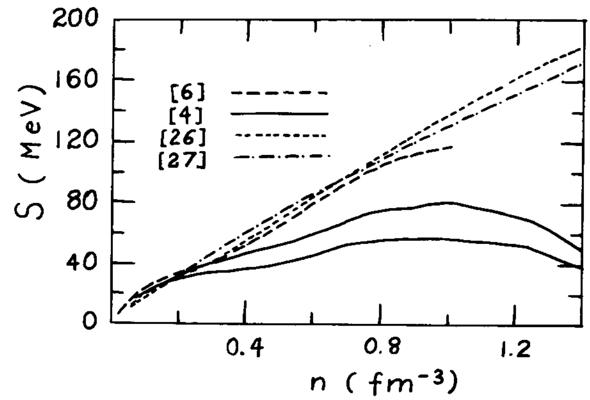

Fig. 1. Symmetry energy versus baryon density, n, for several models of nuclear matter. Number in square brackets gives the reference number corresponding to a given model.

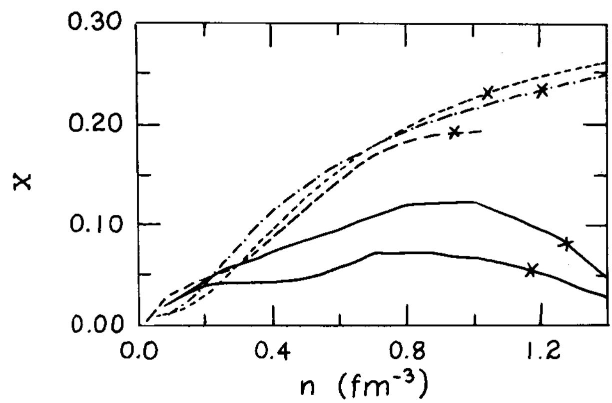

Fig. 2. Equilibrium proton fraction in the npe matter versus baryon density. Notation for curves as in Fig. 1. Cross corresponds to the maximum density in the neutron star models calculated using the specific model of dense matter.

For the npe matter, the value of x_{eq} is thus determined solely by the nuclear symmetry energy, S. A sufficiently rapid increase of S with density implies increase of the proton fraction, and for the densities such, that $x_{eq} > x_{crit} = \frac{1}{9}$ (corresponding to $p_{Fp} + p_{Fe} = 2p_{Fp} > p_{Fn}$), direct Urca reactions can proceed [17]. Density dependence of symmetry energy and of the corresponding equilibrium proton fraction in the npe matter, calculated for several models of dense matter, is shown in Figs 1, 2. For some of them, $x > x_{crit}$ at sufficiently high n, but still below the maximum density for neutron star models. For such models of dense matter, direct Urca process is operative in the central core of sufficiently massive neutron stars.

The neutrino emissivity (energy emitted from 1 cm^3 in 1 s), Q, from the direct Urca processes, Eq.(3.1), was calculated in Ref. [17], using Standard Model of weak interactions. It was assumed, that neutrons and protons form normal Fermi liquids. The phase space integrations could then be done analytically, assuming strong degeneracy of all constituents of the *npe* matter. The final formula for the total emissivity from direct Urca processes (3.1) reads

$$Q(dir.Urca) = 4.0 \times 10^{27} \frac{m_{\rm n}^*}{m_{\rm n}} \frac{m_{\rm p}^*}{m_{\rm p}} \left(x \frac{n}{n_0}\right)^{1/3} \times (T_9)^6 \ \Theta(p_{\rm Fe} + p_{\rm Fp} - p_{\rm Fn}) \ \frac{\rm erg}{\rm cm^3 \ s}, \qquad (3.4)$$

where $T_9 = T/10^9$ K, $m_{\rm i}^*$ (i=n, p) are effective nucleon masses at the corresponding Fermi surface, and the effect of strong interactions in dense medium on the weak interaction parameters has been neglected. The threshold factor $\Theta(y)$ is 1 for $y > 0$ and zero otherwise.

At typical temperatures, prevailing in the interior of neutron star, the quantity $Q(dir.Urca)$ is by many orders of magnitude larger, than that resulting from the modified Urca processes, Eq. (3.2). The neutrino emissivity from the modified Urca processes, as calculated in [18], is

$$Q(mod.Urca) \approx 10^{22} \left(\frac{m_{\rm n}^*}{m_{\rm n}}\right)^3 \frac{m_{\rm p}^*}{m_{\rm p}} \left(x \frac{n}{n_0}\right)^{1/3} (T_9)^8 \ \frac{\rm erg}{\rm cm^3 \ s}. \qquad (3.5)$$

One finds thus $Q(dir.Urca)/Q(mod.Urca) \sim 10^6 \ T_9^{-2}$. Roughly speaking, the obtained ratio reflects the fact that the bystander neutrons in the initial and the final state of the modified Urca process each lead to an additional phase space factor $T/T_{\rm Fn}$, where $T_{\rm Fn} \sim 10^{12}$ K.

The presence of muons in neutron star matter introduces minor modifications. Muons will be present when $\mu_{\rm e} > m_\mu c^2 = 105.7$ MeV. In the presence of muons $p_{\rm Fe} < p_{\rm Fp}$. Their presence will therefore slightly increase the value of the critical proton fraction, above which $p_{\rm Fe} + p_{\rm Fp} > p_{\rm Fn}$, but by a small amount, to not more than 15% [17]. On the other hand, at fixed n, the proton fraction $x = x_{\rm e} + x_\mu$, is larger then for the *npe* matter. This actually *decreases*, for a given $S(n)$, the critical value of n, above which the direct Urca process is operative. One notices, that for $p_{\rm Fp} + p_{F\mu} > p_{\rm Fn}$, which occurs somewhat above the threshold density for the appearance of muons [17], direct Urca processes with muons will proceed, $n \longrightarrow p + \mu^- + \bar{\nu}_\mu$, $p + \mu^- \longrightarrow n + \nu_\mu$, increasing further the neutrino emissivity of neutron star matter.

Dense neutron star matter ($n > 2n_0$) may contain hyperons, beginning with Σ^-, Λ^0 and Ξ^-. Hyperons provide additional sources of neutrino emissivity via direct hyperon Urca processes [19], which is generally less than that from the direct nucleon Urca processes, because of reduced matrix elements for the corresponding weak interaction processes.

Summarizing, nuclear symmetry energy, which increases sufficiently rapidly with baryon density, leads to significant proton fraction in the neutron star cores. This opens possibility of direct Urca processes, which lead to neutrino emissivity many orders of magnitude larger that from the "standard" modified Urca processes. Direct Urca processes in the central core result in a very rapid cooling of young neutron stars. Specific models of neutron star cooling by the direct Urca processes, including the possible effects of neutron superfluidity and/or proton superconductivity, were studied in [20].

4. Bulk viscosity of hot neutron star matter from non-equilibrium Urca reactions

The viscosity of neutron star matter determines damping timescales of radial vibrations of neutron stars [21]. Such vibrations can be excited in the process of formation of neutron star, or can result from the neutron star quakes. The viscosity enters in criteria for the gravitational wave instabilities in rapidly rotating neutron stars [22], which are essential for the determination of the maximum rate of rotation of neutron stars.

Recently, the problem of the actual viscosity of the hot neutron star matter has been reexamined [23, 24]. It has been demonstrated that at temperatures higher than $\sim 10^9$ K, the bulk viscosity exceeds significantly the shear viscosity. This could have important consequences of for hot, pulsating and/or rapidly rotating neutron stars. The source of the bulk viscosity of the neutron star matter is the deviation from beta equilibrium, and the ensuing non-equilibrium reactions, implied by compression and rarefaction of the matter in the pulsating neutron star.

As in Section 3, neutrons and protons are assumed to be normal Fermi liquids (possible effects of neutron superfluidity and proton superconductivity will be discussed in Section 5). It is also assumed, that matter is transparent to neutrinos, so that neutrino absorption can be neglected. The non-equilibrium reactions in compressed and decompressed *npe* matter are driven by the nonzero value of $\Delta = \mu_{\rm n} - \mu_{\rm p} - \mu_{\rm e}$, where $\mu_{\rm n}$, $\mu_{\rm p}$, and $\mu_{\rm e}$ are instantaneous neutron, proton and electron chemical potentials, respectively. Assuming $\Delta << kT$, one can calculate the linear response of the rate (in 1 cm^3, per 1 s) of the reaction $p + e^- \longrightarrow n + \nu_{\rm e}$ (denoted by Γ_ν), and the inverse one, $n \longrightarrow p + e^- + \bar{\nu}_{\rm e}$ (denoted by $\Gamma_{\bar{\nu}}$), to the instantaneous nonzero value of Δ. This response is determined by the coefficient λ, defined as [23]

$$\lambda = 2\left(\frac{\partial \Gamma_\nu}{\partial \Delta}\right)_{\Delta=0}, \tag{4.1}$$

so that in the linear approximation $\Gamma_\nu(\Delta) - \Gamma_{\bar{\nu}}(\Delta) = \lambda\Delta$. The value of λ determines the rate of the nonequlibrium Urca reactions in the *npe* matter. In the limiting case of a strongly degenerate *npe* matter, the calculation of λ can be done analytically. In the case when $x_{\rm eq} > x_{\rm crit}$, direct Urca processes are allowed, yielding [24]

$$\lambda(dir.Urca) = -3.5 \times 10^{40}\,\frac{m_n^*}{m_n}\,\frac{m_p^*}{m_p}\left(Y_e\frac{n}{n_0}\right)^{\frac{1}{3}} T_9{}^4\ {\rm cm}^{-5}\ {\rm g}^{-1}\ {\rm s}\,, \tag{4.2}$$

where $T_9 = T/10^9$ K.

In the case when $x_{\rm eq} < x_{\rm crit}$, only modified Urca reactions can proceed. The corresponding value of λ is then typically many orders of magnitude lower, $\lambda(mod.Urca) \sim 10^{-8}T_9^2\ \lambda(dir.Urca)$, which reflects the difference in the linear response of the rates of the direct Urca and modified Urca reactions in the strongly degenerate neutron star matter to the perturbation of matter density [24].

The instantaneous value of Δ during neutron star vibrations depends on the value of the neutron excess parameter, α, and on the baryon density, n, which oscillate around their equilibrium values, $\alpha_{\rm eq}$ and $n_{\rm eq}$. The linear response of Δ to the deviation of α and n from the equilibrium values is determined by the parameters B and C, defined as

$$B = \left(\frac{\partial \Delta}{\partial \alpha}\right)_n, \tag{4.3}$$

$$C = n\left(\frac{\partial \Delta}{\partial n}\right)_\alpha, \tag{4.4}$$

where all the derivatives should be calculated at equilibrium values of variables. Using the explicit expression for the chemical potentials of protons, neutrons and electrons, one can express the parameters B and C in terms of the symmetry energy and its derivative with respect to n,

$$B = \frac{8}{3}S\left(1 + \frac{1}{4x}\right), \tag{4.5}$$

$$C = 4(1-2x)\left(nS' - \frac{1}{3}S\right). \tag{4.6}$$

All quantities are evaluated at the equilibrium (*i.e.*, at $\alpha = \alpha_{\rm eq}$, $n = n_{\rm eq}$) and $S' = dS/dn$. The fourth coefficient, relevant for the bulk viscosity of the neutron star matter, measures the linear response of pressure, at fixed n, to the changes in α,

$$D = \left(\frac{\partial P}{\partial \alpha}\right)_n , \tag{4.7}$$

and has to be evaluated at $n = n_{\rm eq}$, $\alpha = \alpha_{\rm eq}$. Using thermodynamic relations, valid also off chemical equilibrium, one may show that $D = \frac{n}{2}C$ [24].

Neutron star vibrations of angular frequency ω induce perturbation of n and α around their equilibrium values, $n_{\rm eq}$ and $\alpha_{\rm eq}$. Following the considerations of Sawyer [22], one can calculate the time lag of $\alpha - \alpha_{\rm eq}$ as compared to $n - n_{\rm eq}$, and resulting time dependence of the perturbation of the local pressure, $P - P_{\rm eq}$. This enables one to derive expression for the time average of the energy dissipation rate (in cm^3, per 1 s), $Q_{\rm diss}$, due to non-equilibrium Urca reactions,

$$\langle Q_{\rm diss}\rangle = -\frac{\omega^2}{2}\left(\frac{\delta n}{n_{\rm eq}}\right)^2 \frac{\lambda C^2}{\omega^2 + 4\lambda^2 B^2/n_{\rm eq}^2} . \tag{4.8}$$

Identifying this dissipation rate with that due to the macroscopic bulk viscosity coefficient, ζ, given in our case by the expression $\frac{1}{2}\zeta\omega^2\left(\delta n/n_{\rm eq}\right)^2$, one gets final formula for the bulk viscosity coefficient of the *npe* matter [23],

$$\zeta = -\frac{\lambda C^2}{\omega^2 + 4\lambda^2 B^2/n^2} , \tag{4.9}$$

where all the quantities are to be calculated at $x = x_{\rm eq}(n)$.

Within the approximations used, $\zeta(Urca)$ depends, in an essential manner, on the nuclear symmetry energy, $S(n)$, and its derivative, $S'(n)$. As an example of the calculation of $\zeta(Urca)$, one can use a simple model of the density dependence of S, proposed by Prakash et al. [9], in order to simulate the results of many-body calculations of dense nucleon matter and parametrized as

$$S(n) = 13\ {\rm MeV}\left[u^{\frac{2}{3}} - u\right] + S_0 u , \tag{4.10}$$

where $u = n/n_0$. Such a model has an asymptotic behavior $S \propto u$ for $u >> 1$, characteristic of relativistic mean field theory models of dense baryonic matter [5,6,9]. Results for $x_{\rm eq}(n)$, B, and C, obtained for a rather conservative choice of $S_0 = 30$ MeV, are shown in Figs 3, 4. For such a model

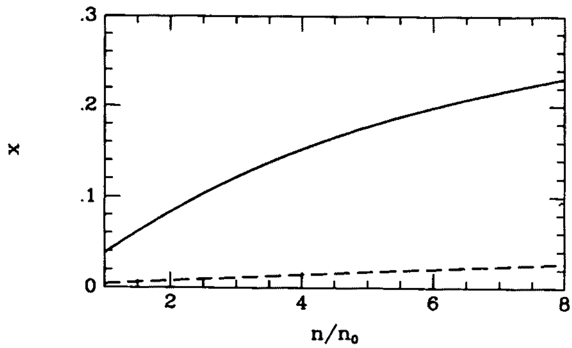

Fig. 3. The equilibrium proton fraction as a function of nucleon density of the *npe* matter, for the model of symmetry energy, given by. Eq. (4.10) (solid line). Dashed line corresponds to the free Fermi gas model.

of $S(n)$, direct Urca processes are operative in the *npe* matter at $n > 2.7n_0$, and, therefore, can be expected to be relevant for massive neutron stars.

As one can see from Figs. 4 a, b, the values of the linear response parameters, B and C, are actually determined by the interaction contribution to S.

Let us consider the denominator of expression (4.9) for $\zeta(dir.Urca)$. The second term in the denominator is strongly temperature dependent ($\propto T^8$). Using Fig. 4b we estimate it, at $n = 3n_0$, as $\sim 10^{-3}{T_9}^8$ s^{-2}. In view of the fact, that the angular frequency of the fundamental mode of radial pulsations $\omega \sim 10^4$ s^{-1} (and those of the higher modes are even higher), we see that at, say, $T_9 < 10$, the second term in the denominator is negligibly small compared to ω^2. Notice, that these arguments are even stronger in the case of the modified Urca process, for which the value of λ is much lower. A suitable expression for $\zeta(dir.Urca)$ can be casted in a simple "high frequency" formula [24]

$$\zeta(dir.Urca) = 8.9 \times 10^{24} \, \frac{m_{\rm n}^*}{m_{\rm n}} \frac{m_{\rm p}^*}{m_{\rm p}} \left(x \frac{n}{n_0}\right)^{\frac{1}{3}} \left(\frac{C}{100 \text{ MeV}}\right)^2 \frac{{T_9}^4}{\omega_4^2} \frac{\text{g}}{\text{cm s}}, \tag{4.11}$$

where $\omega_4 = \omega/10^4$ s^{-1}.

In the case of $x < x_{\rm crit}$, Sawyer [24] gets, at $n = 4n_0$,

$$\zeta(mod.Urca) \simeq 7 \times 10^{15} \frac{{T_9}^6}{\omega_4^2} \frac{\text{g}}{\text{cm s}}, \tag{4.12}$$

so that at such density one gets an order-of-magnitude estimate $\frac{\zeta(dir.Urca)}{\zeta(mod.Urca)}$ $\sim 10^9 \, T_9^{-2}$. Actually, this ratio can be a few times smaller, because of the

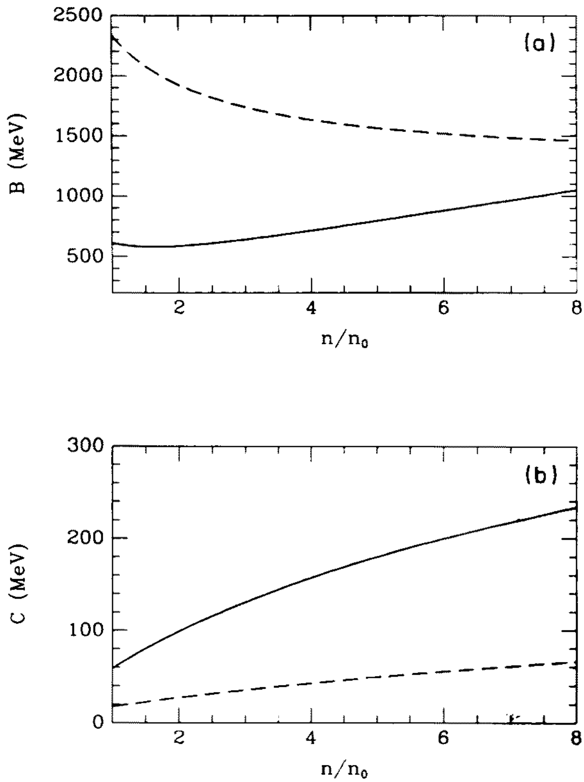

Fig. 4. Linear response parameters B and C, for the model of symmetry energy, given by Eq. (4.10) (solid line). Dashed line: free Fermi gas model.

Sawyer's use of the Fermi gas model for neutrons and protons. In the central region of a massive neutron star, where the condition $p_{\mathrm{Fe}} + p_{\mathrm{Fp}} > p_{\mathrm{Fn}}$ is satisfied, modified Urca processes yield a negligibly small correction to the dissipation via the direct Urca reactions.

By comparing results for the bulk viscosity of the *npe* matter with existing estimates for the shear viscosity [25] one concludes, that bulk viscosity dominates over the shear one for $T > 10^9$ K, when only modified Urca processes can proceed [23], and for $T > 10^8$ K when the direct Urca processes are allowed [24].

The presence of muons in neutron star matter introduces minor modifications. For $p_{\mathrm{Fp}} + p_{\mathrm{F}\mu} > p_{\mathrm{Fn}}$, which occurs somewhat above the threshold density for the appearance of muons [17], direct Urca processes with muons will proceed, $n \longrightarrow p + \mu^- + \bar{\nu}_\mu$, $p + \mu^- \longrightarrow n + \nu_\mu$, increasing further bulk viscosity of neutron star matter. At lower densities, muons contribute via the modified Urca processes, analogous to the electron ones, Eq. (3.2). Hyperons, which are likely to be present in neutron star at a density exceed a few times n_0, will also contribute to the bulk viscosity of neutron s matter. However, as long as they are kinematically allowed, direct U processes represent a strongly dominating source of dissipation, dwarf contributions coming from other weak interaction processes.

5. Discussion and conclusion

Numerical results, reviewed in Sections 3, 4 have been obtained assu ing that both neutrons and protons form normal Fermi liquids. We exp that at least at some densities, neutrons may be superfluid and/or prot superconducting. Calculated gaps in the single-particle spectra, are v uncertain but are typically of the order of a few hundred keV, which co sponds to critical temperature, T_c, of the order of a few 10^9 K. For $T \ll$ the Urca reaction rates and consequently, also the values of $Q(dir.Urca)$ a $\zeta(dir.Urca)$, will be significantly reduced as compared to those obtained normal neutrons and protons. The corresponding reduction of $Q(dir.Ur$ and $\zeta(mod.Urca)$ will then be even stronger.

Neutron stars contain strongly asymmetric and highly compressed clear matter. Cooling of neutron stars and their bulk viscosity stron depend on the composition of the nucleon component of matter. Obs vations of neutron stars represent therefore, at least at principle, a uni opportunity to learn about the behavior of nuclear symmetry energy at densities significantly higher, than the normal nuclear density.

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