THRESHOLD PION PHOTOPRODUCTION REVISITED

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Dedicated to Janusz Dąbrowski in honour of his 65th birthday

We show that the low energy expressions for the CGLN pion photoproduction invariant amplitudes when evaluated at threshold yield a 17% enhancement of the de Baenst $\gamma p \to \pi^0 p$ threshold electric dipole amplitude. One can recover the de Baenst result, to lowest two orders in m_π/m_N, by assuming that the Goldberger–Treiman (GT) relation $f_\pi g = m_N g_A$ is exact. However, accounting for the observed 5% GT discrepancy and the recent modifications of the Saclay and Mainz threshold data, and comparing the data to the enhanced de Baenst amplitude leads to a large explicit breaking of chiral symmetry. The magnitude of the explicit chiral symmetry breaking is not cleanly extracted from the threshold data because of the GT-like cancellations in the exact $\gamma p \to \pi^0 p$ threshold electric dipole amplitude.

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Recently there has been much concern that the standard low energy theorem for $\gamma p \to \pi^0 p$ photoproduction

$$E_{0+}(\pi^0 p) = -\frac{e}{4\pi}\frac{g}{2m_N}\left\{\frac{m_\pi}{m_N} - \frac{m_\pi^2}{2m_N^2}(3+\kappa_p) + O(m_\pi^3)\right\}$$
$$\approx -2.3\times 10^{-3} m_\pi^{-1}\,, \qquad (1)$$

may not be compatible with the threshold multipole amplitudes measured at Saclay [2] and at Mainz [3]. We believe there is no conflict here because of the existence of the photoproduction σ term. Four years ago Kamal [4], and more recently Holstein [5], claimed that there is no explicit chiral breaking (σ term-like) correction to the low energy theorem (1), counter to the original Furlan *et al.* [6] treatment using Breit frame methods and Mac Mullen and Scadron [7] using an analogue dispersion theory covariant analysis. The latter two studies concluded that the resulting chiral symmetry breaking equal-time σ commutator takes the quark model-dependent form (for nonstrange current quark mass $\hat{m}$)

$$\langle N(\vec{p})| \left[\int d^3x i\partial A^{(+)}(\vec{x}), \vec{V}_{em}(0) \right] |N(-\vec{p})\rangle = \hat{m}\langle N(\vec{p})|\vec{\sigma}|N(-\vec{p})\rangle \tag{2a}$$

in the Breit frame, and

$$\langle p'| \left[\int d^3x i\partial A^3(\vec{x}), \vec{V}_{v+s}(0) \right] |p\rangle = -\tfrac{14}{9}\hat{m}\bar{u}_{p'}\vec{\sigma}u_p\,, \tag{2b}$$

for proton matrix elements in covariant frames. Such photoproduction σ terms as Eqs (2) indeed can account for the discrepancy between the low energy theorem (1) and the data [2, 3] for threshold $\gamma p \to \pi^0 p$. This latter observation has been supported in refs [8, 9].

The (small) $\gamma p \to \pi^0 p$ electric dipole amplitude reflects a suppression of the Δ isobar $M1$ contribution which otherwise dominates the low-energy region and the chiral symmetric current algebra. A much more quantitative test of chiral symmetry in $\gamma N \to \pi N$ processes stems from the original soft-pion current algebra predictions of Fubini, Furlan and Rossetti (FFR) [10],

$$\bar{A}_1^{(+)} \to \frac{-g_A\kappa^v}{4m_N f_\pi} \approx -0.26 m_\pi^{-2} \tag{3a}$$

$$\bar{A}_1^{(0)} \to \frac{-g_A\kappa^s}{4m_N f_\pi} \approx 0.008 m_\pi^{-2}\,, \tag{3b}$$

where the former is Δ isobar dominated. Here $\bar{A}_1$ is the background (*i.e.,* the nucleon pole has been removed) first CGLN invariant amplitude [11], $\kappa^v = \kappa_p - \kappa_n \approx 3.70$ and $\kappa^s = \kappa_p + \kappa_n \approx -0.12$ are the respective isovector and isoscalar nucleon anomalous magnetic moments, $f_\pi \approx 93$ MeV is the pion decay constant and the latest PDG value for g_A is [12] 1.2573. Early confirmation of (3a) based on dispersion relations and phase shifts was given in Ref. [13]. Later studies of the specific $\bar{A}_1$ background amplitudes dominated by the nearby low-energy Δ resonance finds at the soft pion point

$q_\pi \to 0$ [14]

$$\bar{A}_1^{(+)} = -(0.27 \pm 0.02) m_\pi^{-2} \tag{4a}$$

$$\bar{A}_1^{(0)} = +(0.015 \pm 0.007) m_\pi^{-2} \ , \tag{4b}$$

both in excellent agreement with the FFR current algebra predictions (3). It was explicitly shown in Ref. [14] that the Δ isobar contributes almost all of the FFR soft amplitude (4a).

With that being said, we return to threshold $\gamma p \to \pi^0 p$ but formulated in the covariant form of Refs. [7]. Separating out the nucleon pole terms, σ term and background amplitudes, the relevant CGLN amplitudes are

$$A_1^{(+,0)}(\nu, t) = -\frac{k \cdot q g F_1^{v,s}}{s_m u_m} - \frac{g_A(m_\pi^2)\kappa^{v,s}}{4 m_N f_\pi} - \frac{\Sigma^{(+,0)}(t)}{f_\pi m_v^2} + B_1^{(+,0)}(\nu, t) \tag{5a}$$

$$A_3^{(+,0)}(\nu, t) = \frac{-\nu g \kappa^{v,s}}{m_N s_m u_m} + B_3^{(+,0)}(\nu, t) \tag{5b}$$

$$A_4^{(+,0)}(\nu, t) = \frac{k \cdot q g \kappa^{v,s}}{2 m_N s_m u_m} + B_4^{(+,0)}(\nu, t) \ , \tag{5c}$$

where $g \approx 13.4$ is the $\pi^0 pp$ pseudoscalar coupling constant [15], and m_v is the average vector meson mass ~ 800 MeV which arises in the covariant σ term in (5a) but not in the Breit frame σ commutator (2a). Also the invariants in (5) are $\nu = (s-u)/4$ and $s_m = s - m_N^2$, $u_m = u - m_N^2$. The form factors are evaluated for on-shell photons ($k^2 = 0$) and pions ($q^2 = m_\pi^2$); $F_1^v(0) = F_1^s(0) = 1$ and $g_A(m_\pi^2)$ will be discussed shortly. Note that Eqs (5) reduce to the FFR Eqs (3) and (4) in the soft-pion limit. In that limit the σ term and backgrounds B (proportional to the axial-vector divergence) vanish.

Now we return to the threshold limit, where the precise value of the threshold $\gamma p \to \pi^0 p$ electric dipole amplitude in the center of mass system is (recall $M(\pi^0 p) = M^+ + M^0$),

$$E_{0+}(\pi^0 p) = -\frac{e m_\pi (2m_N + m_\pi)[m_N(m_N + m_\pi)]^{1/2}}{8\pi (m_N + m_\pi)^2} \left[A_1(\pi^0 p) + \frac{m_\pi (1 + m_\pi/2m_N) A_3(\pi^0 p)}{1 + m_\pi/m_N} + \frac{m_\pi^2 A_4(\pi^0 p)}{2 m_N (1 + m_\pi/m_N)} \right] . \tag{6}$$

In contrast to de Baenst [1], we have separated the purely kinematical terms into a prefactor in order to isolate the dynamical aspects of the CGLN amplitudes in (6). To proceed, we recall the threshold values, $s_m u_m =$

$-4m_N^2 m_\pi^2(1-x^2)$, $\nu = m_N m_\pi$, $t = -m_\pi^2$ and $k \cdot q = m_\pi^2(1+x)/(1+2x)$, where $x = m_\pi/2m_N$. Substituting Eqs (5) at threshold into (6), the leading terms in $x = m_\pi/2m_N$ are

$$E_{0+}(\pi^0 p) = -\frac{e}{4\pi} m_\pi (1-2x)\left\{\frac{g}{2m_N^2}(1-x) - \frac{g_A(m_\pi^2)\kappa_p}{2m_N f_\pi} + \frac{g\kappa_p}{2m_N^2}(1-x) + \cdots\right\}, \tag{7}$$

where the first three terms between the brackets in (7) are due to A_1 and the remaining two terms are due to A_3. If we set $g_A(m_\pi^2) \approx g_A$, then the third and fourth terms in (7) cancel because of the Goldberger–Treiman (GT) relation $f_\pi g = m_N g_A$. With this *approximation* (good to 5%), and multiplying in the $(1-2x)$ factor in (7), the resulting expression is *precisely* the low-energy "theorem," Eq. (1). We regard this consistency with de Baenst as simply a check that the *more exact* expression (6) displays the correct threshold behavior of the nucleon pole terms for the electric dipole $\gamma p \to \pi^0 p$ amplitude. It is the threshold behavior of (6) reflected in (7) which is then our starting point to test the explicit chiral symmetry *breaking* in the threshold $\gamma p \to \pi^0 p$ amplitude.

To study the most exact $\gamma p \to \pi^0 p$ threshold behavior, we return to (6) using the CGLN amplitudes (5) but evaluated at threshold, $s_m u_m = -4m_N^2 m_\pi^2(1-x^2)$, $\nu = m_N m_\pi$, and $k \cdot q = m_\pi^2(1+x)/(1+2x)$, where $x = m_\pi/2m_N$. Then (6) becomes, postponing the σ term and the background corrections for the moment,

$$E_{0+}(\pi^0 p) = -0.0210 m_\pi \left[0.1393 m_\pi^{-2} - 0.2588 m_\pi^{-2} + 0.2494 m_\pi^{-2} - 0.0012 m_\pi^{-2}\right] \tag{8a}$$

$+\sigma$ term $+$ background term, where $e = +0.30282$ in rationalized units. The reason why the second and third terms in (8a) do not cancel is due to our choice [16] of $g_A(m_\pi^2) \approx 1.29$, according to the observed t-dependent slope ($m_A \approx 1.3$ GeV) of the form factor obtained from electro-weak production measurements. This $g_A(m_\pi^2)$ is also compatible with the observed 5% GT discrepancy [16]. Then the combined sum of the first four numerical terms in (8a) gives a lead term (-0.00270 m_π^{-1}) which is a 17% increase in magnitude over the de Baenst threshold value ($-0.0023 m_\pi^{-1}$) in (1):

$$E_{0+}(\pi^0 p) = -0.00270 m_\pi^{-1} + 0.0015 \hat{m} m_\pi^{-2} + \text{background}. \tag{8b}$$

The second term in (8b) corresponds to the chiral-breaking σ term, $\Sigma^{\pi^0 p} = (-14/9)\hat{m}\bar{u}_{p'}\vec{\sigma}u_p$ in (2b), divided by $f_\pi m_v^2$ in (5a) due to vector meson resonance-dominating the unsubtracted dispersion relation form of the CGLN invariant amplitudes [7]. This (covariant) σ-term($-\Sigma/f_\pi m_v^2$) arises as an application of the original Bjorken-Johnson-Low [17] infinite momentum frame $q^2 \to -\infty$, *i.e.*, $q_0 \to i\infty$, $|\vec{q}|$ fixed limit [7]. We believe it significant that Schäfer and Weise in Ref. [9] obtain the same $(-14/9)\widehat{m}\vec{\sigma}$ form for the leading order σ term as we do in (2b), only they work in the Breit frame as do Furlan *et al.* [6], with $f_\pi m_v^2$ replaced by $f_\pi m_\pi^2$ in (5a) [18]. In either case, one uses the standard PCAC relation applied to photoproduction [6-9],

$$i\int d^4x e^{iqx}\langle N'|T(\partial A^i(x)V_\nu^{em}(0)|N\rangle = \frac{f_\pi m_\pi^2}{q^2-m_\pi^2}T_\nu^i \,. \tag{9}$$

Here $T_\nu\epsilon^\nu$ is the invariant photoproduction amplitude containing the CGLN terms $A_1\frac{1}{2}[\not{k}\,,\not{\epsilon}\,]\gamma_5+\cdots$. In the covariant version, one replaces q^2 in (9) by m_v^2.

Finally, the Δ isobar-dominated threshold electric dipole background contribution to E_{0+} for $\gamma p \to \pi^0 p$ in (8a) is very small. The latter fact is because the $\gamma p \to \Delta^+$ transition is primarily of the magnetic dipole M_{1+} type, but the smallness of the background terms can also be verified explicitly from the covariant background amplitudes listed in Table I in the second reference of [7]. Neglecting this very small background in the threshold $\gamma p \to \pi^0 p$ multipole amplitude (8b), we can set a scale for the average nonstrange current quark mass $\widehat{m}$ in (8b) once we determine the LHS amplitude $E_{0+}(\pi^0 p)$ from data.

Although the original measurements respectively suggested the very small $\gamma p \to \pi^0 p$ threshold multipole amplitudes [2, 3]

$$E_{0+}(\pi^0 p) = (0.5\pm0.3)\times10^{-3}m_\pi^{-1}, \quad E_{0+}(\pi^0 p) = (0.7\pm0.3)\times10^{-3}m_\pi^{-1}, \tag{10a}$$

a recent study points to a strong energy dependence near threshold [19]. This is due to the difference between photon lab momentum $k = 144.7$ MeV where the $\gamma p \to \pi^0 p$ channel opens up and the rescattering channel $\gamma p \to \pi^+ n \to \pi^0 p$ with $k = 151.5$ MeV. Such a momentum difference in turn recalibrates the original data in (10a) to the new respective threshold values [5, 19]

$$E_{0+}(\pi^0 p) = -(1.5\pm0.3)\times10^{-3}m_\pi^{-1}, \quad E_{0+}(\pi^0 p) = -(2.0\pm0.3)\times10^{-3}m_\pi^{-1}. \tag{10b}$$

A similar conclusion was also obtained in Ref. [20]. While Eqs (10b) are more in line with the de Baenst prediction in (1), the 17% shift to the exact threshold value of size $-2.70\times10^{-3}m_\pi^{-1}$ means that the revised data in (10b) of Refs [2, 3] when substituted into the revised low energy expansion

(8b) with background→ 0 implies respectively

$$2.70 - (1.5 \pm 0.3) = 1.5\frac{\hat{m}}{m_\pi} \quad \text{or} \quad \hat{m} = (112 \pm 28)\ \text{MeV} \tag{11a}$$

$$2.70 - (2.0 \pm 0.3) = 1.5\frac{\hat{m}}{m_\pi} \quad \text{or} \quad \hat{m} = (65 \pm 28)\ \text{MeV}. \tag{11b}$$

While these nonstrange current quark mass values in (11) are rather large, the latter in (11b) is within one standard deviation of $\hat{m}$ determined from many other independent constraints on explicit chiral symmetry breaking [21]. Regardless of these latter results, however, we believe as do the authors of Refs [6] and [7] that threshold pion production does not offer a clean extraction of explicit chiral symmetry breaking effects in the $\gamma p \to \pi^0 p$ channel because then effects of the dominant Δ isobar are suppressed, exposing the delicate cancellations of (7) and (8a) [22]. Rather, the original soft-pion theorem of FFR [11] as tested in Eq. (3) and in (4) [13, 14] offers the clearest confirmation of chiral symmetry in low energy pion photoproduction for the $M^{(+)}$ and $M^{(0)}$ isotopic modes separately analyzed, but then with the Δ isobar playing a significant role. On the other hand, we believe that the present day experimental studies of threshold photoproduction carried out to high precision will ultimately prove valuable to studies of explicit chiral symmetry breaking. In this regard, we re-emphasize the suggestion made in Ref. [7] that the new data be analyzed in terms of the invariant amplitudes *below* threshold, as is done for πN scattering, rather than in terms of multipoles at threshold or slightly above threshold.

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[18] D. Drechsel and L. Tiator have claimed in *J. Phys. G* **18**, 449 (1992) that "there could (and probably will) be so-called Schwinger terms cancelling [the sigma term of [6] (and presumably of [8] and [9])]". That this is not so can be appreciated by recalling that the Bjorken–Johnson–Low limit is invoked in the covariant analysis of Ref. [7] for the purpose of *determining* those Schwinger terms which arise in solving the vector and axial-vector Ward identities for pion photo- and electroproduction.

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[22] For example, discussions at the Blacksburg Workshop, August, 1992 have suggested a provisional value for the $\pi^0 pp$ pseudoscalar coupling constant $g \approx 13.15$; πN *Newsletter* #7, Eds R. Cutkosky, G. Höhler, W. Kluge and B. Nefkens, (1992). Substituting this value into (6) yields a lead term in (8b) of -0.00255 m_π^{-1} and a concomitant $\widehat{m} = (98 \pm 28)$ MeV or $\widehat{m} = (51 \pm 28)$ MeV.

That is, a 2% reduction of g from the standard value of 13.4 is amplified by the GT type cancellation in (7) and (8a) to a 10-20% reduction of $\widehat{m}$.