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THE MOMENTUM SUM RULES IN THE EMC EFFECT WITH MESON DEGREES OF FREEDOM*

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Dedicated to Janusz Dąbrowski in honour of his 65th birthday

The Momentum Sum Rule in deep inelastic scattering on nuclei is discussed and limitations of nuclear convolution model are pointed out. The pion and vector meson contributions to the nuclear structure function are derived from the composite picture of the nucleon in the nuclear medium and presented as correction to the nuclear convolution with nucleon degrees of freedom. Finite size effects of a nucleus are discussed.

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1. Introduction

In this paper, we want to examine the conventional nuclear models of deep inelastic electron-nucleus scattering, and show how mesonic degrees of freedom describe the effect of nuclear medium. The mean free path of nucleon in a nucleus is relatively long [1] (of the order of 1 fm) and therefore the nucleus can be treated in mean field approach as the sum of "bound" nucleons. The word "bound" refers to the natural approximation in this approach: the nucleons are off energy shell and their energies are shifted by the mean separation energies e_N given by the conventional nuclear physics.

In the kinematic region where the value of momentum transfer from the electron to parton (quark) is much bigger than the mass of the nucleon, $-q^2 >> M_N^2$, and transfer energy square $\nu^2 >> |q|^2$, the nucleon interaction with electron is much faster than the interaction with other nucleons and we expect that only Fermi motion [2] of free nucleons should be included as the medium correction. The experiment [3] revealed however that

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the ratio $R(x)$ of nuclear structure function to nucleon structure function as a function of Bjorken scaling variable x shows a strong departure from unity with a dip around $x \simeq 0.7$. This phenomenon (EMC effect) can be explained in conventional nuclear models, without explicit quark degrees of freedom (Convolution Models), by assuming a negative shift (around 30–40 MeV) of the nucleon longitudinal momentum $p_N^+ = p_N^0 + p_N^3$. On the other hand the Momentum Sum Rule is not satisfied [5] when the nucleons are off shell in the Convolution Model (CM). Consequently, the EMC effect has no simple explanation in terms of "bound" nucleons only. This is the reason to include the additional mesonic degrees of freedom in order to have a consistent description of the nuclear structure function.

The CM is an attempt to describe the EMC effect in terms of nucleon and meson degrees of freedom and is restricted to intermediate $x > 0.2$, because uncertainty of photon spatial localization (along transfer direction) $\sim 1/x$ can be very large for small x. For $x < 0.2$ these uncertainties are comparable with nucleon size and therefore this is the limit for CM.

The parton structure function $F_2^A(x)$ gives the distribution of the fraction of nuclear longitudinal momentum $x$carried by partons. Let us start from the following convolution formula [4] for the function $F_2^A(x)$:

$$\frac{F_2^A(x)}{A} = \int_x^\infty dy_N \ \rho^N(y_N) F_2^N\left(\frac{x}{y_N}\right) + \sum_m \langle n_m \rangle \int_x^\infty dy \ \rho^m(y) F_2^m\left(\frac{x}{y}\right). \quad (1)$$

This model includes conventional degrees of freedom: nucleons ($i = N$) and mesons ($i = m$), and is implemented as a convolution of the free hadron structure function $F_2^i(x)$ with the distribution $\rho^i(y_i)$ of longitudinal momentum $p_i^+ = y_i P^+$ for each type of hadron in the nucleus. The number of additional constituents per nucleon is $\langle n_m \rangle$ and the value of nucleus momentum $\langle P^+ \rangle$ is equal to the mass of nucleus in LAB.

Let us for a moment forget about meson degrees of freedom. Keeping only the nucleon term in the convolution (1), the nuclear structure function $F_2^A(x)$ has the form:

$$\frac{F_2^A(x)}{A} = \int_x^\infty dy \ \rho^N(y) F_2^N\left(\frac{x}{y}\right), \quad (1a)$$

and the nuclear distribution can be written as [4, 5]:

$$\rho^N(y_N) = \sum_\alpha \int d^3\vec{p} |\phi_\alpha(\vec{p})|^2 \delta^3 \left(y_N - \frac{E^A - E_\alpha^{A-1} - \vec{p}\cdot\vec{u}}{M_N + \varepsilon_N} \right) \quad (1b)$$

where $\phi_\alpha(\vec{p})$ is the single-nucleon overlap [7] between the A-particle ground state and the $(A-1)$-particle state α. The vector $\vec{u} = \vec{q}/\nu$ lies along the direction of the 3-component of momentum transfer and can be taken to be the unit vector $\vec{q}/|\vec{q}|$ in the Bjorken limit. The interpretation of $F_2^A(x)$ for small x, in terms of simple parton model is in fact impossible because here one has to take into account the interaction of partons from different nucleons. The usual Momentum Sum Rule can not be applied to such a strongly interacting system. There is an evidence [22] that in the nonperturbative parton wave function for nuclear matter, the momentum carried by charged partons is depleted by about 5%. This is connected with shadowing for small x. In our approach, however, $F_2^A(x)$ is given by CM (1) which assumes parton picture for all x, so we can assume in this approach that integration $\int F_2^A(x)dx$ should give 1. Neglecting mesonic degrees of freedom the Momentum Sum Rule in the nuclear medium is given by [5,8]

$$\int F_2^A(x)dx \simeq \int y\rho^N(y)dy = \langle y_N\rangle = \frac{M_n + e_N}{M_N + \varepsilon_N} \neq 1\,, \qquad (2)$$

where

$$e_N = \sum_\alpha \frac{E^A - E_\alpha^{A-1}}{A}$$

is the single particle energy of nucleon (average nucleon separation energy), where $\varepsilon_N = (AM_N - M_A)/A$. We see that Momentum Sum Rule (2) is violated by the difference $e_N - \varepsilon_N$. We know from the Hartree–Fock theory that this difference is related to the average NN potential V_{ij}, $2(e_N - \varepsilon_N) = \langle A|\sum_{ij} V_{ij}|A\rangle$. Since V_{ij} is produced by exchange of mesons, we expect to be able to satisfy Sum Rule (2) by including direct meson contributions to nuclear structure function.

The paper is organized as follows: in Section 2 we report the conventional nuclear model with pion (meson) degrees of freedom, in Section 3 we introduce our model with additional vector (omega) meson and in Section 4 we extend the pionic convolution model and we calculate nuclear structure function with a new vector meson contribution. The discussion and conclusions are given in the last Section.

2. Pions in the nuclear medium

In conventional nuclear physics meson degrees of freedom are eliminated in favor of a two body potential and remaining effects are described by exchange currents. However in deep inelastic scattering, especially for intermediate x we can start from the picture where the nucleon, a composite object, is surrounded by meson cloud and exchange interaction can change this cloud. In this treatment the shift of the average nucleon longitudinal momentum by $(e_N - \varepsilon_N)$ comes from the momentum balance with additional mesons in a nucleus. To estimate the pion excess number per nucleon we follow Ref. [10]. The nucleus is described in [10] by the Hamiltonian $H = H_0 + H'$, where the free Hamiltonian H_0 describes nucleons, pions and deltas, and H' describes the pion-nucleon interaction. The following relation between the pion excess operator per nucleon n_π and the two body pion exchange nucleon-nucleon interaction V_{ij}^π is derived in [10]:

$$n_\pi(k) = -\frac{V_{ij}^\pi(k)}{p_\pi^0}, \tag{3}$$

where the pion energy $p_\pi^0 = (m_\pi^2 + k^2)^{1/2}$.

The value of the pion excess number $\langle n_\pi \rangle = 0.12$ for $k_F = 220$ MeV was obtained in a variational calculation [10] with realistic Hamiltonian where the pion exchange diagrams with delta resonances were included. In nuclear matter with Fermi momentum $p_F = 220$ MeV, pion excess number $\langle n_\pi(p) \rangle$ is strongly peaked for $|p| = 400$ MeV, which allows us to determine $\langle y_\pi \rangle \cong A|\langle n_\pi \rangle|\langle p_\pi^0 \rangle / M_A = .05$ (for $\langle p_\pi^0 \rangle \cong 400$ MeV[4]).

The inclusion of the pion component in (1) changes the nucleon distribution for *intermediate* x and gives additional contribution for *small* x [9,14]. The region of small x can not be described by the convolution model, therefore the pions are working only as pilferers of nucleon momenta changing in this way the nucleon distribution. For heavy meson, however, we would expect also direct contribution for $x > 0.3$.

Now we present the Energy Sum Rule (considered previously in [5,12]) resulting from the Eq. (3) that relates the interaction energy operator and the meson excess operator. We sum up over the exchanged mesons and having $\langle y_m \rangle \cong A|\langle n_m \rangle|\langle p_m^0 \rangle / M_A$, we get:

$$\sum_m \frac{\langle n_m \rangle}{|\langle n_m \rangle|} \langle y_m \rangle (M_N + \varepsilon_N) = \sum_m \langle n_m p_m^0 \rangle = \left\langle A \middle| \sum_{m,i>j} V_{ij}^m \middle| A \right\rangle. \tag{4}$$

Thus we obtained the energy sum rule which states: *The average longitudinal momentum of the binding mesons in the nuclear medium is equal to the average nucleon interaction energy.* The interaction energy can be estimated from:

$$\left\langle A \middle| \sum_{m,i>j} V_{ij}^m \middle| A \right\rangle \cong \langle T_N \rangle - \varepsilon_N + e_c, \tag{4a}$$

where e_c is the Coulomb energy (per nucleon) which should be subtracted from ε_N because it is absent in the meson interaction energy. Note that when the numbers of protons and neutrons are equal, we have (from the Weizäcker formula) $\varepsilon_N - e_c \cong e_{\rm vol} + e_s$, where $e_{\rm vol}$ and e_s are the volume and surface energies.

In order to estimate $\langle y_m \rangle$ from (4a) we apply the local density approximation [13] and get for the average kinetic energy $T_M = 0.6\langle p_G^2 \rangle / 2M_N$, where $2p_F^3/3\pi^2 = \rho_0/\{1 + \exp[(r - R)]/.57\} = \rho(r)$ and R is the nuclear radius. The resulting spatial average gives for $A = 56$ $\langle T_N \rangle^{\rm Fe} \cong 15$ MeV and for $A = 192$ $\langle T_N \rangle^{\rm Au} \cong 18$ MeV, which is 1.5 MeV below the estimate $\langle T_N \rangle \cong 23$ MeV $(\rho/\rho_0)^{2/3}$ for constant ρ. In the pure pionic CM we have from (4):

$$\langle y_\pi \rangle = \frac{\langle T_N \rangle - \varepsilon_N + e_c}{M_N + \varepsilon_N}\,, \tag{4b}$$

and for $A = 56$ $\langle y_\pi \rangle^{\rm Fe} \cong 0.029$; Thus we see that $\langle y_\pi \rangle$ is determined by the nuclear saturation properties. For large A we obtain, by extrapolating (4b) with $e_c \cong 6$ MeV for $A > 200$, the limiting value of $\langle y_\pi \rangle_\infty \sim 0.036$.

Very good agreement with the SLAC data [11] for the EMC ratio (which are still actual for intermediate range of x) was obtained by Głazek *et al.* [14] in the convolution model with pions excess. They made calculations of EMC ratio $R(x)$ for a whole range of A, taking $\langle n_\pi \rangle$ and $\langle y_\pi \rangle$ from [10]. To describe the $A = 192$ data (see open circles in Fig. 1) the value $\langle y_\pi \rangle = 0.043$ was used. This value is inconsistent with our estimate $\langle y_\pi \rangle^{\rm Au} \cong 0.034$ coming from Eq. (4b). The values of $\langle y_\pi \rangle$ for heavy nuclei, calculated in [10], reach 0.05. However, the energy shift in nucleon distribution should be determined by the sum over all meson contribution, $\Delta = \sum_m \langle n_m y_m / |n_m| \rangle$, and not by the pions only. In fact $\langle y_\pi \rangle$ calculated in [10] is much greater than the our estimation from nuclear interaction energy (4a) but it shows that we can not restrict our model to pionic degrees of freedom only. In fact it is impossible to construct the realistic NN interaction with pion exchanges — the short range repulsion requires vector mesons [15]. The question — which mesons constitute the elementary interaction field between nucleons — is crucial in applying relation (4).

3. Vector mesons

The main features of the NN interaction are: an overall attraction at distances $r \gtrsim r_c \cong 0.5$ fm and a strong repulsion for $r \lesssim r_c$. In the relativistic Dirac phenomenology it corresponds to the scalar and vector part of the constant mean field. Here, we assume that in the nucleus we have pions which give one and two pion exchange contributions and omegas which provide the short range contribution to the NN interaction. In fact the contributions to the interaction energy from other mesons are much smaller [15] and are neglected in our simple considerations. Thus in our sum rule (4) we have two terms: a large negative one for omegas and a large positive one for pions. Similarly to (4a) we have:

$$(\langle T_N \rangle - \varepsilon_{\rm vol} + \varepsilon_c) = (M_N + \varepsilon)(\langle y_\pi \rangle + \langle y_\omega \rangle)\,. \tag{5}$$

As an estimate of $\langle y_\omega \rangle$ we insert the value $\langle y_\pi \rangle = 0.05$ into (5), obtained from the value $\langle n_\pi \rangle = 0.13$ calculated in [10] with a symmetrical maximum at $\langle p^0_\pi \rangle \cong 400$ MeV. This leads to the following, nonrelativistic estimate:

$$\langle y_\omega \rangle = .02\,,\ \langle n_\omega \rangle \cong -.026 \quad \text{and} \quad \langle y_\pi \rangle = 0.05\,,\ \langle n_\pi \rangle = 0.13\,, \tag{6a}$$

where for the average energy of ω meson we took $\langle p^0_\omega \rangle = 800$ MeV.

As another estimate of the vector meson contribution to the nuclear structure function $F_2^A(x)$, we use the value of the relativistic mean meson field in the nucleus taken from Dirac phenomenology. The ω contributions to the interaction energy for nuclear matter with $p_F = 220$ MeV for different OBE potentials vary around 100 MeV [16,17]. This, with the help of (4a), (4) and (5) leads to the following relativistic estimate:

$$\langle y_\omega \rangle = 0.10\,,\ \langle n_\omega \rangle \cong -0.12 \quad \text{and} \quad \langle y_\pi \rangle = 0.13\,,\ \langle n_\pi \rangle = 0.36\,. \tag{6b}$$

4. Calculations

We extend the simple phenomenological model of Ref. [14] to include vector meson contributions to the nuclear structure function according to formula (1). For the normalized distribution ρ^i we have the usual form with two parameters (α, β):

$$\rho^i(y) = \frac{\Gamma(\alpha_i + \beta_i)}{\Gamma(\alpha_i)\Gamma(\beta_i)} y^{(\alpha_i - 1)}(1 - y)^{(\beta_i - 1)}\,, \tag{7}$$

Momentum sum rule (2) for the meson part of nuclear structure function gives:

$$\langle n_m \rangle = \frac{\alpha_m + \beta_m}{\alpha_m} \langle y_m \rangle\,. \tag{8a}$$

In order to obtain the parameters of the meson distribution function we impose the constraints $(m = \pi, \omega)$:

$$\frac{M_m}{M_N} = \langle y_N \rangle \frac{\alpha_m + \beta_m - 2}{\alpha_m - 1}, \tag{8b}$$

which reflect the assumption that the ρ^N peaks at $M_N/\langle p^+ \rangle$ and ρ^m at $M_m/\langle P^+ \rangle$. This allows us to express the parameters $(\alpha_\omega, \beta_\omega)$, (α_π, β_π) of the vector and pion distribution functions by the more physical quantities (n_ω, y_ω) and (n_π, y_π).

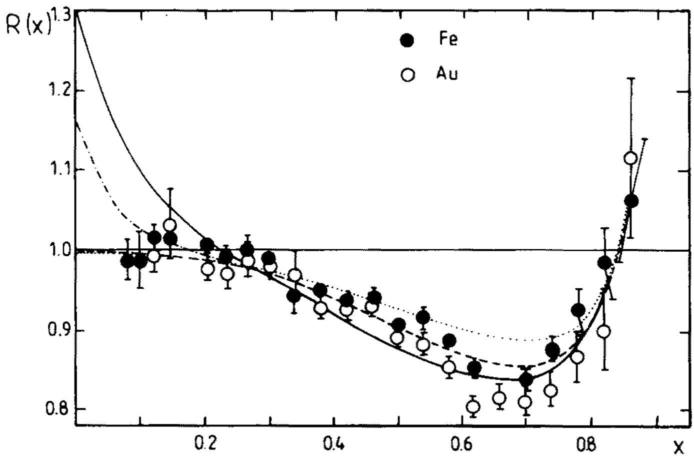

Fig. 1. The calculated ratio $R(x) = F_2^A(x)/F_2^D(x)$ in $A = 56$ (dotted line) and $A = 192$ (solid, dashed, and dashed-dotted lines). See text for explanation. The data are taken from Ref. [11]. We used the following parametrizations for the structure functions: for $F_2^N(x)$ from [19], for $F_2^D(x)$ from [20] and for $F_2^\pi(x) = F_2^\omega(x)$ from [21].

Our results for $A = 192$ with the choice (6b) (with Coulomb correction (5)) are presented in Fig. 1 as a solid line. The dashed line is obtained with the nucleon contribution only. The relatively large negative contribution from the vector meson is seen in the intermediate range of x. The nonrelativistic choice (6a) leads to a negligible vector meson correction for $x > 0.3$, which is connected with the smallness $\langle n_\omega \rangle = -\langle n_\pi \rangle/5$. For illustration, we show (as dotted line) the results for pure nucleon contribution in Iron $A = 56$ without Coulomb correction. The Coulomb correction would shift these results approximately to the middle between the dotted and dashed lines. Finally, we present, the result of the pure pionic model for $A = 192$ as the dotted-dashed line which coincides with the dashed line for $x > 0.3$.

5. Discussion and conclusions

We have shown that in the model of a nucleus with additional pions (positive excess number) and omegas (negative excess number) we have sum rule (5) which enables us to improve the Convolution Model for deep inelastic scattering. The nucleon momenta are diminished by the interaction energy which comes from the momentum balance between mesons and nucleons. The important new feature of this model is the additional contribution of the omega meson. It reduces the EMC ratio in the intermediate x region, up to 15% with the maximal reduction around $x = 0.5$. For small x the meson contributions are positive and increase rapidly but this happens outside the range of applicability of this model. We have replaced "bound" nucleons by free nucleons with "binding" mesons directly related to the NN interaction. This gives a more general picture of the deep inelastic scattering on nuclear targets.

Frankfurt *et al.* [8] used the nucleon energy shift Δ, which corresponds to $\sum_m \langle n_m p_m^0 \rangle$ in our model, of $\Delta \cong 24$ MeV estimated from $\Delta = \langle T_N \rangle - \varepsilon_n$. The commonly used value Δ, which gives the good agreement with experiment is $\Delta \cong 34$ MeV [5] (approximately equal to $\Delta = \langle T_N \rangle - e_{\rm vol}$). In this paper we have argued that in fact $\Delta = \langle T_N \rangle - \varepsilon_N + e_c$. This means that in order to estimate the meson interaction energy we *exclude* the nucleon Coulomb energy but *include* the finite size effect of the nucleus. These small changes (few MeV) in the Δ modify the EMC ratio, respectively by a few per cent with the maximum change at $x = 0.6$.

Another important question is how to explain the big difference between nonrelativistic (6a) and relativistic (6b) estimates of the pion and omega components in the meson interaction energy. The relativistic NN interaction between positive energy states (nucleons) can be considered, as an effective interaction which also takes into account the coupling to negative energy intermediate states. This can change significantly the magnitude of a particular meson contribution to the final NN interaction. For example, the mean scalar field in the relativistic models of nuclear matter [16] diminishes effectively the nucleon mass which enters the mean field Dirac equation by 200 MeV [16].

Finally, our single particle approximation can not describe with sufficient accuracy, even with the vector meson correction, the data for x around $x = 0.7$. This might hint at the importance of the NN correlations [18], neglected in our present approach.

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