CONVECTION-DRIVEN GROWTH IN FLUCTUATING VELOCITY FIELD* **

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The growth process of an initially ideal sphere in a convective fluctuating velocity field is considered. The influence of fluctuations of the velocity field on the growth process is examined.

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1. Introduction

The growth processes attract a great scientific and technological efforts in the last few years [1]. The most explored topics are connected with a random and/or fractal aggregates produced by different fields [2]. The famous example here is the diffusion-limited aggregation process [3] considered from the analytical (Partial Differential Equation (PDE)-based) [4] or numerical (computer simulation-based) [5] point of view. Also, the growth process driven by a diffusion field has been examined with respect to its basic features (structural stability, formation of dendritic patterns, *etc.*) and results are reported elsewhere [6, 7]. There are, however, some theoretical [8] as

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well as a few experimental evidences [9] that the growth process could also be driven by convection fields.

In this work we focus ourselves on the sphere growth driven by a convective fluctuating field. The practical realization of such a process could be observed in, *e.g.*, polymer physics where the growth of spherulites (polycrystalline aggregates having a sphere-like shape) from melts or solutions is often of interest [8].

The aim of this work is to examine how the fluctuations of the velocity field influence the growth velocity of the sphere, *i.e.*, how the mean value $\langle R(t)\rangle$ of the radius $R(t)$ of the sphere is changing with time and how it fluctuates.

2. Model and results

An equation describing evolution of a growing object can be obtained using the mass conservation law [6]. For objects with an ideal or perturbed spherical symmetry, it has the form (*cf.* Eq. (2.16) in Ref. [10])

$$[\mathbf{C} - c(\tilde{r}, \vartheta, \phi)]\frac{d\tilde{r}}{dt} = -\vec{J}[c(\tilde{r}, \vartheta, \phi)] \cdot \vec{n}_0 \,, \tag{1}$$

where

$$\tilde{r} \equiv \tilde{r}(\vartheta, \phi; t) \,, \tag{2}$$

is a surface equation (in the spherical coordinate system $(\tilde{r}, \theta, \phi)$ of the object of interest at instant t. $\mathbf{C}$ is its density (constant), $c(r, \vartheta, \phi)$ stands for a concentration of the external medium at point (r, ϑ, ϕ), $\vec{J}[c(r, \vartheta, \phi)]$ is a flux of particles of the external surroundings and $\vec{n}_0$ is the inward normal [10] to the surface (2),

$$\vec{n}_0 = \vec{e}_r - \frac{1}{\tilde{r}}\frac{\partial \tilde{r}}{\partial \vartheta}\vec{e}_\vartheta - \frac{1}{\tilde{r}\sin\vartheta}\frac{\partial \tilde{r}}{\partial \phi}\vec{e}_\phi \,. \tag{3}$$

Its length $||\vec{n}_0||$ is given by

$$||\vec{n}_0||^2 = 1 + \left(\frac{\partial \tilde{r}}{\partial \vartheta}\right)^2 + \left(\frac{\partial \tilde{r}}{\partial \phi}\right)^2 . \tag{4}$$

For an external convective field

$$\vec{J}[c(r, \vartheta, \phi)] = c(r, \vartheta, \phi)\vec{v}(r, \vartheta, \phi) \,, \tag{5}$$

where $\vec{v}(r, \vartheta, \phi)$ is a velocity of convective particles at the point (r, ϑ, ϕ). For a local thermodynamic boundary condition, $c(\tilde{r}, \vartheta, \phi)$ is given by the Gibbs-Thomson relation

$$c(\tilde{r}, \vartheta, \phi) = c_0[1 + \Gamma K(\tilde{r}, \vartheta, \phi)] \,, \tag{6}$$

where Γ is the capillary constant (*cf.* [6] or [10]), c_0 is the concentration field at a flat interface: growing object-environment and K is twice the mean curvature of the object surface.

Here, we wish to study the simplified model of an ideal sphere of radius R immersed in a radial convective field

$$\vec{v}(r, \vartheta, \phi) = -v(r, t)\hat{e}_r , \tag{7}$$

with a given velocity field $v(r, t)$ and $\hat{e}_r$ stands for a radial unit vector. In this case $\tilde{r}(\vartheta, \phi; t) = R(t)$, $K = 2/R$ and Eq. (1) reduces to the form

$$\frac{dR}{dt} = A(R)v(R, t) , \quad A(R) = \frac{c_0(R + 2\Gamma)}{(\mathbb{C} - c_0)R - 2\Gamma c_0} . \tag{8}$$

In the case when $v(r, t)$ is a deterministic function, a solution of Eq. (8) can easily be found. A more realistic situation is when the velocity field $v(r, t)$ fluctuates about its mean value $\langle v(r, t)\rangle$, *i.e.*, $v(r, t)$ consists of two parts: deterministic and fluctuating. Let us analyze the following case

$$v(r, t) = v_0 + V(t) , \tag{9}$$

where $v_0 = \langle v(r, t)\rangle$ is a positive constant and a fluctuating part $V(t)$ is assumed to be a Gaussian white noise (non-correlated fluctuations) of the strength $D > 0$,

$$\langle V(t)\rangle = 0 , \quad \langle V(t)V(s)\rangle = 2D\delta(t - s) . \tag{10}$$

Under these assumptions, the process $R(t)$ described by Eqs (8)–(10) is a stochastic Markovian process of a diffusional type. A Stratonovich interpretation [11] is adopted and, therefore, a single-event probability distribution $P(R, t)$ obeys the Fokker–Planck–Kolmogorov equation in the form [11]

$$\frac{\partial P(R, t)}{\partial t} = -v_0 \frac{\partial}{\partial R}A(R)P(R, t) + D\frac{\partial}{\partial R}A(R)\frac{\partial}{\partial R}A(R)P(R, t) , \tag{11}$$

with suitable initial and boundary conditions. As an initial condition, let us assume that at $t = 0$, the object is an ideal sphere of radius R_0 and in consequence

$$P(R, 0) = \delta(R - R_0) . \tag{12}$$

As boundary conditions, we should take into account reflecting boundaries:

$$J(R_0, t) = 0 , \quad J(\infty, t) = 0 , \tag{13}$$

where

$$J(R, t) = v_0 A(R)P(R, t) - DA(R)\frac{\partial}{\partial R}A(R)P(R, t) \tag{14}$$

is a probability current and $R \geq R_0$, $t \geq 0$. We have investigated the case of the absorbing boundary conditions, $P(R_0, t) = 0$, $P(\infty, t) = 0$. It leads to a trivial solution $P(R, t) \equiv 0$ and should be rejected in the case in question.

The solution of the problem (11)–(13) is

$$P(R, t) = \frac{\mathbf{C} - c_0}{c_0} \frac{R - R^*}{R + 2\Gamma} \left\{ (\pi D t)^{-1/2} \exp\left[-\frac{(x(R) - x(R_0) - v_0 t)^2}{4Dt} \right] \right.$$
$$\left. - \frac{v_0}{2D} \exp\left[\frac{v_0}{D}(x(R) - x(R_0)) \right] \operatorname{erfc}\left[\frac{x(R) - x(R_0) + v_0 t}{(4Dt)^{1/2}} \right] \right\}, \tag{15}$$

where

$$x(R) = \frac{\mathbf{C} - c_0}{c_0}[R - (R^* + 2\Gamma)\ln(R + 2\Gamma)], \quad R^* = \frac{2\Gamma c_0}{\mathbf{C} - c_0} < R_0. \tag{16}$$

and erfc(z) is a complementary error function. The deterministic growth ($D = 0$) is governed by the equation

$$R - R_0 - (R^* + 2\Gamma)\ln\left[\frac{R + 2\Gamma}{R_0 + 2\Gamma} \right] = \frac{v_0 c_0}{\mathbf{C} - c_0} t. \tag{17}$$

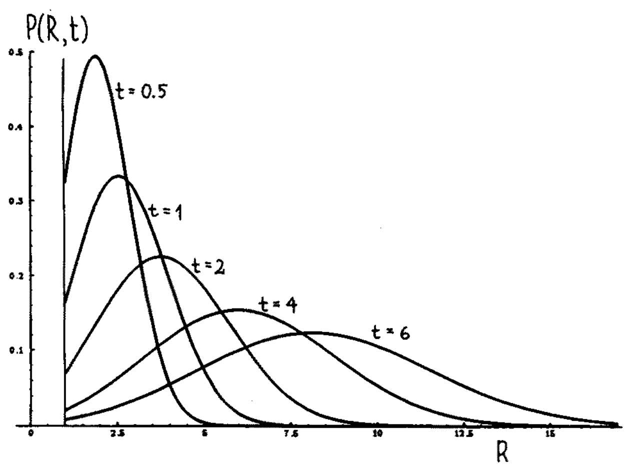

Fig. 1. The distribution function for different instants increasing from left to right $D = v_0 = \frac{c_0}{\mathbf{C} - c_0} = R_0 = 1$, $R^* = 0.1$).

The evolution of the probability distribution is sketched in Fig. 1. It is worth to mention that the most probable value of the sphere radius $R(t)$, which is determined by a maximum of $P(R, t)$, does not evolve according to (17).

3. Concluding remarks

The key idea of our preliminary investigations is to study some growth process driven by a convective field. Our motivation comes from the fact that an enormous effort is devoted to describe the growth processes by diffusional or diffusion-reaction mechanisms [1] but, in fact, almost none considers the non-diffusional ones. Since in [12] we have studied and compared two processes, the growth driven by a diffusion field and by a deterministic convective field, in this paper we consider a stochastic description of a convection-controlled growth. To be more precise, we study the convection-driven growth in a non-correlated fluctuating velocity field. Our investigations are mostly oriented on the evolution rules for a spherical object. Then, we examine the influence of the fluctuations of the external field on the growth velocity of the sphere. To be more specific, we look for the changes in time of the mean value $\langle R(t)\rangle$ of the sphere radius and we also examine its fluctuations.

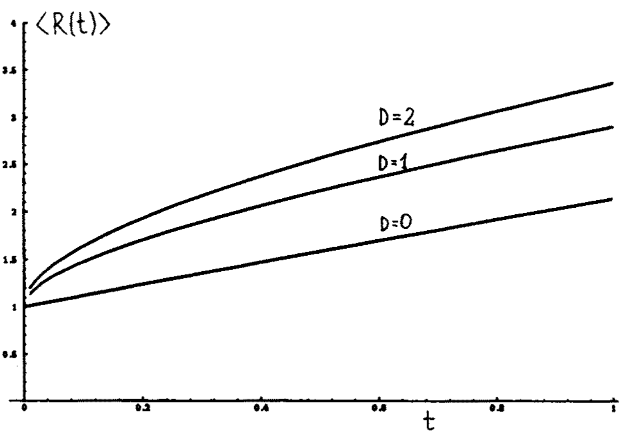

Fig. 2. The $\langle R(t)\rangle$ *versus* t for several values of the fluctuation strength D: the effect of increasing of growth velocity of a sphere by changing of D. The case $D = 0$ corresponds to the deterministic growth.

Our results show also that when the field fluctuations grow then the growth velocity increases (see Fig. 2). They also show that at the early stage of evolution the field fluctuations cause a very fast growth of the sphere in a nonlinear manner. For the long times, in turn, the growth is observed to be linear in time.

It results also from our findings that the surface fluctuations manifest a rather opposite behaviour (see Fig. 3), *i.e.*, at the early stage of evolution they grow linearly with time, but in the long time limit the nonlinear effect

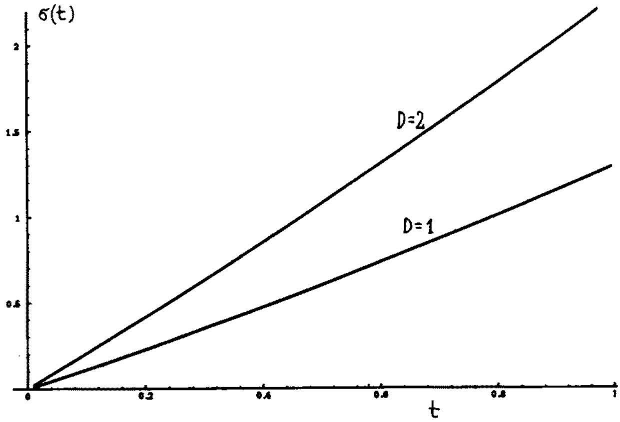

Fig. 3. The fluctuations $\sigma(t) = \langle R^2(t)\rangle - \langle R(t)\rangle^2$ of the sphere surface with respect to time t and for two different values of D.

is easy to notice (*cf.* Fig. 2 and Fig. 3 for comparison). We also observe that the fluctuations, growing in time, are stronger for greater values of the fluctuations strength D.

We also would like to mention, at least, a few possibilities of extension (or generalization) of our treatment. *E.g.*, it could be done by:

— incorporating the chemical reaction into our equations, at first, into Eq. (5) (in the same way as it was done in [13] — making sure that the Curie-rule has not been violated);

— involving a kinetic term in Eq. (6) (*cf.* Goldenfeld and Goldenfeld and Liu in Ref. [8]), which represents the physical situation that the interface is not static, but moves with certain velocity (note that Eq. (6) is a statement of thermal equilibrium at the interface [8]);

— studying the system driven by correlated noises (field fluctuations) with non-zero correlation time [14, 15].

Finally, we wish to emphasize a more practical aspect of our work. Namely, we are of the opinion that, in general, our approach could be proposed as another way to describe the evolution of the spherulites in polymer surroundings (better known is the way due to Goldenfeld and Goldenfeld and Liu [8], but it describes a diffusion-driven growth).

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REFERENCES

[1] I. Sunagawa (ed.), *Morphology of Crystals*, Terra Scientific Publishing Company, Tokyo 1987.

[2] F. Family, D.P. Landau (eds.), *Kinetics of Aggregation and Gelation*, North Holland, Amsterdam 1984.

[3] T.A. Witten, L.M. Sander, *Phys. Rev. Lett.* **47**, 1400 (1981).

[4] R. Ball, M. Nauenberg, T.A. Witten, *Phys. Rev.* **A29**, 2017 (1984).

[5] P. Meakin, *Phys. Rev.* **A33**, 3371 (1986).

[6] J. Łuczka, A. Gadomski, Z.J. Grzywna, IMA Preprint Series no 869, University of Minnesota 1991.

[7] V. Pines, M. Zlatkovsky, A. Chait, *Phys. Rev.* **A42**, 6129 (1990).

[8] N. Goldenfeld, *J. Crystal Growth* **84**, 601 (1987); F. Liu, N. Goldenfeld, *Phys. Rev.* **A42**, 895 (1990).

[9] H.D. Keith, F.J. Padden, Jr., *J. Appl. Phys.* **34**, 2409 (1963).

[10] J. Łuczka, A. Gadomski, Z.J. Grzywna, *Czech. J. Phys.* **42**, 577 (1992).

[11] H. Risken, *The Fokker-Planck Equation*, Springer-Verlag, Berlin 1989.

[12] A. Gadomski, Z.J. Grzywna, J. Łuczka, *Chem. Engng. Sci.*, to be published.

[13] D. Woermann, *J. Membrane Sci.* **7**, 127 (1980).

[14] S. Isogami, M. Matsushita, *J. Phys. Soc. Jpn.* **61**, 1445 (1992).

[15] J. Łuczka, *Physica* **A153**, 619 (1988).