QUARK CONFINEMENT THROUGH HIDDEN BREAKING OF COLOUR SYMMETRY*

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The aim of this paper is to study the possibility of a non-linear mechanism of quark confinement. The sets of coupled equations for Dirac fields carrying colours and flavours are discussed. They contain non-linear self-interaction and mutual interaction terms of the same fractional form that was studied before for single Dirac fields (*Phys.Lett.*71**B**,357(1977); *Phys.Lett.*76**B**,391(1980); *Acta Phys.Pol.*B12,601(1981)). It turns out that the only way of preventing creation of isolated coloured objects consists in breaking global colour symmetry. An explicit form of the symmetry breaking term is proposed (different from that used in *Acta Phys.Pol.*B19,203(1988)), which implies that only white currents are conserved and the three colours are truly inseparable. Moreover, the new equations have the advantage of having strictly colour symmetric (white) solutions that correspond to an absolute minimum of the symmetry breaking term of energy.

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1. Introduction

There were many attempts to understand the physical mechanism of quark confinement. However, none of them is completely convincing and satisfactory. Even the most ambitious attempts based on QCD provide as yet rather vague explanations of the mechanism of confinement. The problem was earlier attacked also with the help of several less ambitious but more plausible models like naive strings, bags, suitable potentials *etc.* Therefore, it seems to be justified to investigate whether confinement can be obtained if one describes hadrons as systems of quark solitons with suitable non-linear interactions.

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The concept of confinement contains two equally important ingredients: localization and inseparability of the respective coloured components, *i.e.* quarks or quarks and gluons. Localization means that the respective basic fields can be different from zero only within small regions of the order of fm occupied by individual hadrons. Inseparability of colours means that no coloured object can be isolated and all the observed hadrons are "white".

With respect to the problem of localization of the quark fields we shall recall very briefly the properties of the soliton-like solutions of the single Dirac field equations with a fractional non-linear self-interaction (NLDE) implied by the following class of Lagrangians [1, 2]:

$$L = -i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi - b\kappa^d\bar{\psi}\psi\,, \tag{1}$$

with

$$\kappa = (\bar{\psi}\psi)^2, \quad b > 0, \quad d = (a-1)/2, \quad 1 > a > 0\,. \tag{2}$$

The corresponding NLDE contain an effective scalar potential that tends to $+\infty$ whenever $\psi(x)$ approaches the vacuum value $\psi = 0$. This produces strong forces which prevent spreading and produce non-dispersive lumps of field matter. In the limit $a \to 0$ one obtains what is called the MIT bag model for one particle [4].

The respective NLDE have been studied by the author in [1, 2]. They have the following peculiar properties:

(A) There exist stationary solutions that are eigen-functions of the angular momentum (in the quantum mechanical sense) corresponding to $j = 1/2$ which can be identified with the spin of the object. Such solutions and their Lorentz transforms behave as shape preserving, stable, isolated solitons.

(B) Explicit stationary solutions found in [1] describing such solitons at rest are different from zero within a sphere of definite radius determined by the constants a, b, m. In other words: these solutions are very well localized and have compact supports.

(C) For these explicit solutions the scalar field invariant $\bar{\psi}\psi$ is non-negative.

Numerical methods applied by other authors [4] support the conjecture that the properties (A, B, C) are rather generally valid for this type of non-linear self-interaction.

2. Colour symmetric equations for coupled Dirac fields

Consider now the systems of coupled Dirac fields

$$\psi_{cf}(x) \text{ with } c = 1, 2, 3 \text{ and } f = 1, 2, \ldots, n\,, \tag{3}$$

supposed to describe quarks carrying both colour and flavour quantum numbers. Let us start from the following tentative form of the Lagrangian:

$$L_0 = \sum_c \sum_f [-i\bar{\psi}_{cf}\partial\!\!\!/\psi_{cf} - \bar{\psi}_{cf}\psi_{cf}(m_f + b\mu_f^d + b'\lambda^{d'})], \tag{4}$$

where

$$\mu_f = \Big[\sum_c \bar{\psi}_{cf}\psi_{cf}\Big]^2, \quad \lambda = \Big[\sum_c \sum_f \bar{\psi}_{cf}\psi_{cf}\Big]^2. \tag{5}$$

The constants a, b, d as well as a', b', d' satisfy the same relations as those given in (2). It follows from the definitions of μ and λ that the Lagrangian (4) is invariant under the global colour symmetry group $SU(3)_c$ and the basic flavour symmetries $U(f)$. We shall ignore in this paper the higher, approximate flavour symmetries like isospin etc. In spite of containing summation over c the first non-linear term in (4) describes self-interaction of quarks with definite quantum numbers f. The last non-linear term in (4) describes mutual interaction between all quarks appearing in the system. The forms of both terms are only tentative. We have taken them in the same fractional form as in (1) expecting that the nice properties (A,B,C) survive at least in a slightly relaxed form expressed by the following conjecture:
The solutions of the corresponding set of the NLDE:

$$D_f\psi_{cf} = 0, \tag{6}$$

with

$$D_f = i\partial\!\!\!/ + m_f + ab\mu_f^d + a'b'\lambda^{d'} \tag{7}$$

for any value of time differ from zero only in some finite regions with compact support in which the value of each field scalar $\bar{\psi}_{cf}\psi_{cf}$ is non-negative.

The inherent symmetries imply that there are as many as 9n conservation laws for each of the currents:

$$j^{\alpha}_{cc'f} = \bar{\psi}_{cf}\gamma^{\alpha}\psi_{c'f}. \tag{8}$$

Any model in which not only the white currents but also the full colour octets of currents are conserved must be rejected as it would imply the possibility of creation of isolated coloured objects in collisions between white hadrons (*e.g.* colour octets of $\bar{q}_{cf}q_{c'f}$ pairs).

3. Inseparability of colours achieved by "hidden breaking" of colour symmetry

In order to remove this essential difficulty let us add to the former Lagrangian the following, tentative, colour symmetry breaking term:

$$L = L_0 - L', \quad \text{where } L' = gW^2/2, \quad \text{with } g > 0\,, \tag{9}$$

with

$$W = W^\dagger = \sum_f \sum_{cc'} \tfrac{1}{2}(\bar{\psi}_{cf} - \bar{\psi}_{c'f})(\psi_{cf} - \psi_{c'f}) + h \sum_{cc'c''} i\epsilon_{cc'c''}\bar{\psi}_{cf}\psi_{c'f}\,. \tag{10}$$

Obviously L' is non-negative and its absolute minimum is equal to zero.

The resulting equations of motion assume now the form:

$$D_f\psi_{1f} = gW[2\psi_{1f} - \psi_{2f}(1-i) - \psi_{3f}(1+i)]\,. \tag{11}$$

The remaining two equations can be obtained by cyclic permutations of the colour indices.

It can easily be checked that the coloured octets of currents are now nót conserved but the white currents are conserved as required.

We shall now show that this way of colour symmetry breaking implies inseparability of colours and absolute minimum of the additional energy obtained for certain physically interesting colour symmetric solutions.

The three colours are regarded as separable if there exists some compact region S of space-time (e.g. the interior of a sphere and some finite time interval Δt) in which, for at least some values of f, only one or two coloured quark fields are different from zero, while the remaining coloured fields vanish in S. This happens if, for example, either:

Case I:

$$\begin{aligned} &\psi_{1i}(x) \neq 0 \text{ for some } x \in S \text{ and for some values of } i\,,\\ &\psi_{2j}(x) = \psi_{3k}(x) = 0 \quad \text{for all } x \in S \text{ and all values of } j, k\,, \end{aligned} \tag{12}$$

or

Case II:

$$\begin{aligned} &\psi_{1i}(x) \neq 0,\ \psi_{2j}(x) \neq 0 \text{ for some } x \in S \text{ and some values of } i, j\,,\\ &\psi_{3k} = 0 \text{ for all } x \in S \text{ and for all values of } k\,. \end{aligned} \tag{13}$$

Let us investigate the forms of the new equations of motion in S in these two cases. We find for:

Case I:

$$D_i\psi_{1i} = 2gW_I\psi_{1i}, \quad 0 = -gW_I\psi_{1i}(1+i), \quad 0 = -gW_I\psi_{1i}(1-i)\,, \tag{14}$$

Case II:

$$\begin{aligned} D_f\psi_{1f} &= gW_{II}[2\psi_{1f} - \psi_{2f}(1-i)]\,, \\ D_f\psi_{2f} &= gW_{II}[2\psi_{2f} - \psi_{1f}(1+i)]\,, \\ 0 &= -gW_{II}[\psi_{1f}(1-i) + \psi_{2f}(1+i)]\,. \end{aligned} \tag{15}$$

In both cases W is non-vanishing by our assumptions (12) and (13). Therefore, it can easily be checked that the same separability assuptions lead in both cases to contradicting equations. In this way we have proved that the proposed way of breaking global colour symmetry ensures inseparability of colours and thus does not allow production of coloured objects.

Equations (11) have a set of colour symmetric solutions:

$$\psi_{1f}(x) = \psi_{2f}(x) = \psi_{3f}(x) = \psi_f(x) \quad \text{for all } f\,, \tag{16}$$

which display manifest inseparability of colours and coincide with the condition used in the naive quark model of hadrons. Eqs. (16) mean that, for each fixed value of f, the x-dependence of the wave function is for the three values of the colour index exactly the same. Moreover, since for these symmetric solutions W vanishes it follows that the symmetry breakig L' as well as the corresponding part in the expression for energy assume their absolute minimum equal to zero. Solutions breaking this symmetry are in principle possible but they would raise the value of energy of the system above the value zero valid for symmetric solutions.

These energetically favoured solutions satisfy the same colour symmetric equations of motion (6) and their relations to the symmetry breaking equations (11) become "hidden". These properties justify the name of "hidden breaking of colour symmetry" used in the title of this paper.

4. Concluding remarks

The author does not pretend that the model presented in this paper can be used for perfoming definite calculations concerning the structure of hadrons and their interactions. Its main aim is to draw attention to the very plausible possibility of understanding the properties of hadrons in terms of quarks regarded as Dirac solitons satisfying suitable non-linear field equations. The proposed forms of the non-linear terms preserving the colour symmetry, as well as those breaking it, should be regarded only as tentative and open to further investigations and improvements. Nevertheless it has been shown that both basic ingredients of confinement, *i.e.* localization and inseparability of coloured quarks, can be achieved with the help of suitable non-linearities.

Like in the first, but nevertheles quite successful, quark models of hadrons which have been mentioned in the introduction — no gluons appear in this paper explicitly. All the interactions are constructed with the help of white or coloured Dirac field scalars that can be understood in terms of particle concepts as suitable quark-antiquark pairs. It is quite possible that a description in terms of vector currents instead of scalars would be more suitable and closer to the gluon concept. At any rate, one can expect that at least in some approximation it should be possible to eliminate gluons and to obtain some effective Lagrangian for low energy region that is expressed in terms of quarks alone. In this region the exchange of virtual gluons may be hard to distinguish from the exchange of virtual pairs of light quarks.

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