PARTICLES WITH ONE HEAVY QUARK IN THE MIT BAG MODEL*

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We propose a model based on the MIT Bag Model for particles containing one heavy quark. We eliminate the colour electric and the colour magnetic energy terms in the mass formula, by replacing them by a renormalization of the constants of the model.

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1. Introduction

The Isgur–Wise symmetry supports the model of the particles containing one heavy quark, as "hydrogen-like" systems. The heavy quark is at the centre of this system and the light antiquark or diquark circulates around it. However, before the Isgur–Wise symmetry was discovered there had been several papers in which authors used this "hydrogen-like" picture. In particular, it was applied in the MIT Bag Model [1]. The standard way of evaluating the masses of particles in the MIT Bag Model [1], [2] breaks down for a system containing one heavy quark. In this old picture the heavy quark appears as a particle moving in the whole bag in the same manner as the light one(s). There was disagreement between that picture and experimental data [3], [4]. Paper [5] was one of the first trying to reconcile the MIT Bag Model with experimental data. Although the author of this paper didn't apply the analogy between the particle with the heavy quark and "hydrogen-like" atoms, the changes he introduced, improved the agreement with data. Further work [6], [7] used the "hydrogen-like" picture. The results they got, confirmed the correctness of the new view on particles containing one heavy quark.

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In this paper we try to create an effective theory, which can be useful in practical computations. In the particle mass formula there are three fundamental terms: the mass of the heavy quark, which is independent of R-the bag radius, the zero point energy proportional to $1/R$ and the bag volume energy proportional to R^3. In first order of α_s, (strong coupling constant of QCD) we have additional contributions to the mass of the particle from colour electric and magnetic fields. Both contributions are very complicated functions of R. We propose to approximate these functions in the simple form of the three fundamental terms already present in the mass formula. This means that the coefficients of our new expression renormalize appropriate constants changing them into new effective ones. This gives a simple and useful expression describing the mass of the particle.

2. Mass of the particle

The mass of the particle containing one heavy quark can be expressed in the following standard form [6], [7]:

$$M = m_Q + E_k + \tfrac{4}{3}\pi B R^3 + \frac{Z}{R} + E_E + E_M \,, \tag{1}$$

where m_Q is the heavy quark mass, E_k is the light antiquark or diquark kinetic energy, the third term is the standard bag volume energy, Z/R is the zero-point energy and two last terms represents colour electric and magnetic field energy.

We start from the model described in papers [6], [7]. The heavy quark occupies the centre of the bag and is surrounded by light quarks. The position of the heavy quark defines the centre of mass. This is a very useful property of the systems with one heavy and one or two light quarks. In this picture a heavy quark is treated as a point source of the colour Coulomb-like field. The colour magnetic field is created by the magnetic moment of the heavy quark, which has the value $1/2m_Q$. Gluon self-coupling is neglected and replaced by boundary conditions on the quark fields at the surface of the bag:

$$\sum_i \hat{r} \cdot \vec{E}_i^a|_s = 0\,, \tag{2}$$

$$\sum_i \hat{r} \times \vec{B}_i^a|_s = 0\,, \tag{3}$$

where $\vec{E}_i^a$, $\vec{B}_i^a$ are the colour electric and magnetic fields of the i-th quark, and a is a colour index. In this approximation the colour fields become eight independent static electric and magnetic fields. In lowest order in α_s, the colour electric energy E_E and the colour magnetic energy E_M are given by [7]:

$$E_E = -\frac{2\alpha_s\lambda}{3R}\sum_{i>j} f(m_iR, m_jR)\,, \tag{4}$$

$$E_M = 2\lambda\alpha_s\sum_{i>j}\frac{\mu(m_i,R)\mu(m_j,R)}{3R^3}I(m_iR,m_jR)\vec{\sigma}_i\vec{\sigma}_j\,, \tag{5}$$

where $\lambda = 2$ for mesons, $\lambda = 1$ for baryons and

$$\alpha_s = \frac{2\pi}{9}\frac{b}{\ln(1+1/\Lambda R)}\,,$$

b is an additional phenomenological constant that defines the magnitude of α_s, $\Lambda = 0.42$ GeV [7]. The R dependence of α_s is taken from paper [8]. $\vec{\sigma}$ is the vector of Pauli matrixes. Sums in the above equations run over quarks contained in the bag. i, j = Q, q, s; where Q stands for heavy quark c or b, q for quark u or d and s is the strange quark.

$$I(m_iR,m_jR) = 1 + \frac{2R^3}{\mu(m_i,R)\mu(m_j,R)}\int_0^R\frac{dr}{r^4}\mu(m_i,r)\mu(m_j,r)\,, \tag{6}$$

where for the light quarks:

$$\mu(m_i,r)\vec{\sigma} = \tfrac{1}{2}\int_{K(0,r)} d^3r'\vec{r'}\times\vec{j_i} \tag{7}$$

and for heavy quark ([7]): $\mu(m_Q,r) = 1/2m_Q$. $\vec{j_i}$ is the vector current of a free Dirac particle contained in the bag. Function f is defined as:

$$f(m_iR,m_jR) = R\int_0^R\frac{dr}{r^2}\rho_i(r)\rho_j(r)\,, \tag{8}$$

where for the light quarks:

$$\rho_i(r) = \int_{K(0,r)} d^3r' j_i^0 \tag{9}$$

and for heavy quark ([7]): $\rho(r) = 1$. j_i^0 is the zero component of the current of a free Dirac particle contained in the bag. We neglect the self energies of the quarks.

3. The model

The colour electric and the colour magnetic energy are included in the renormalizations of the other constants. We express these energies in the form:

$$E_E = -\frac{2\lambda}{3} b \sum_{i>j} \left[A_{ij}(m) + \frac{\tilde{A}_{ij}(m)}{r} + \frac{4\pi}{3}\tilde{\tilde{A}}_{ij}(m) r^3 \right] , \tag{10}$$

$$E_M = 2\lambda b \sum_{i>j} 2\mu_{ij} [D_{ij}(m) + \frac{\tilde{D}_{ij}(m)}{r} + \frac{4\pi}{3}\tilde{\tilde{D}}_{ij}(m) r^3] \vec{\sigma}_i \vec{\sigma}_j , \tag{11}$$

where m is the mass of light quarks q or s, $\mu_{ij} = 1/2m_Q$ if i or $j = Q$ and $1/2$ — otherwise. This means that in our effective model the mass formula changes into the form:

$$M = m_Q^{\text{eff}} + E_k + \tfrac{4}{3}\pi B^{\text{eff}} R^3 + \frac{Z^{\text{eff}}}{R} , \tag{12}$$

where:

$$m_Q^{\text{eff}} = m_Q - \frac{2\lambda}{3} b \sum_{i>j} A_{ij}(m) + 2\lambda b \sum_{i>j} 2\mu_{ij} D_{ij}(m) \vec{\sigma}_i \vec{\sigma}_j , \tag{13}$$

$$Z^{\text{eff}} = Z - \frac{2\lambda}{3} b \sum_{i>j} \tilde{A}_{ij}(m) + 2\lambda b \sum_{i>j} 2\mu_{ij} \tilde{D}_{ij}(m) \vec{\sigma}_i \vec{\sigma}_j , \tag{14}$$

$$B^{\text{eff}} = B - \frac{2\lambda}{3} b \sum_{i>j} \tilde{\tilde{A}}_{ij}(m) + 2\lambda b \sum_{i>j} 2\mu_{ij} \tilde{\tilde{D}}_{ij}(m) \vec{\sigma}_i \vec{\sigma}_j , \tag{15}$$

where m is the mass of a light quark. The coefficients $A_{ij}(m)$ and $D_{ij}(m)$ can be fitted from equations (4), (10) and (5), (11). This was done for three independent interactions: heavy–light quark (Qs or Qq), light quark–light quark for same type of quarks (qq or ss) and light quark–light quark for different types of quarks (qs). The formulas for the parameters $A_{ij}(m)$ and $D_{ij}(m)$ can be found in the Appendix. In the figures the exact expressions for the energy is compared with our fit. As a typical example we choose for comparison the energy of interaction between the heavy and the light quark. One can see that fit is very good for the magnetic energy but less good for the electric energy. The difference between our fit and the exact expression is less then 0.3 MeV. Within the accuracy of the Model there is no difference whether we use the exact expression, or our approximation.

One can compute the "exact" bag radius from equation (12). We prefer another treatment, first suggested in paper [7]. The radius of the bag is

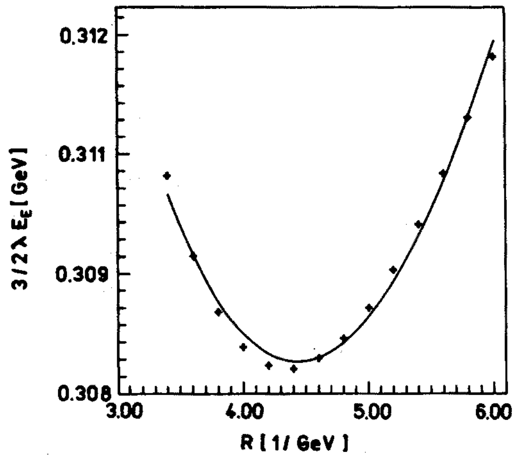

Fig. 1. Dependence of the electric energy on the bag radius for heavy quark-light quark interactions at light quark mass equal 250 MeV. The points denoted by crosses are calculated from formula (4). The curve is the fit given in the Appendix.

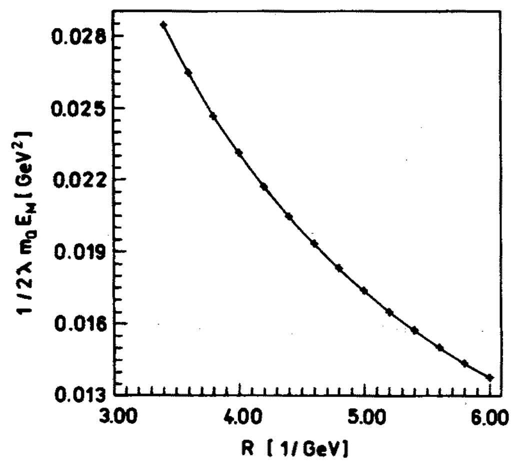

Fig. 2. Dependence of the magnetic energy on the bag radius for heavy quark-light quark interactions at light quark mass equal 250 MeV. The points denoted by crosses are calculated from formula (5). The curve is the fit given in the Appendix.

computed by minimizing the expression:

$$M = m_Q + E_k + \tfrac{4}{3}\pi B R^3 + \frac{Z}{R}. \tag{16}$$

This is the zero order approximation for bag radius R. We checked that this simplification has little effect on the results.

4. The model parameters and numerical results

In our effective model there are five free parameters: Z, b, m_c, m_b, m_s. We fit them using as input the measured masses of D, D_s, D^*, B, and Λ_c. Our results for the masses of the mesons and baryons with no orbital excitations are given in Table I and Table II. The input values are underlined. The results are compared with experimental data, where available, and with the predictions of the two other versions of the bag model described in Refs [5], [7]. The best values of the parameters are: $Z = -0.73$, $b = 0.679$, $m_c = 1.7683$ GeV, $m_b = 5.1126$ GeV and $m_s = 0.291$ GeV. We used $B^{(1/4)} = 0.1383$ GeV ([7]).

TABLE I

Masses of mesons containing quark c or b with the light quark in 1S-state.

J^P			our model	bag [5] GeV	bag [7] GeV	mass exp [9] MeV
0^-	D	$c\bar{q}$	1.8646	1.800	1.830	1864.5 ± 0.5
0^-	D_s	$c\bar{s}$	1.9688	1.957	1.92	1968.8 ± 0.7
1^-	D^*	$c\bar{q}$	2.0070	2.009	2.01	2007.1 ± 1.4
1^-	D_s^*	$c\bar{s}$	2.1052	2.141	2.09	2110.3 ± 2.0
0^-	B	$b\bar{q}$	5.2787	5.232	5.27	5278.7 ± 2.1
0^-	B_s	$b\bar{s}$	5.3800	5.372	5.36	–
1^-	B^*	$b\bar{q}$	5.3280	5.299	5.34	5324.6 ± 2.1
1^-	B_s^*	$b\bar{s}$	5.4272	5.431	5.42	–

TABLE II

Masses of baryons containing quark c or b with the diquark in 1S-state.

J^P			our model	bag [5] GeV	bag [7] GeV	mass exp [9] MeV
1	2	3	4	5	6	7
$\frac{1}{2}^+$	Λ_c	cqq	2.284	2.243	2.28	2284.9 ± 0.6
$\frac{1}{2}^+$	Σ_c	cqq	2.347	2.38	2.380	2452.9 ± 0.8
$\frac{3}{2}^+$	Σ_c^*	cqq	2.425	2.481	2.49	–
$\frac{1}{2}^+$	Ω_c	css	2.586	2.60	2.61	–
$\frac{3}{2}^+$	Ω_c^*	css	2.659	2.764	2.71	–
$\frac{1}{2}^+$	Ξ_c^A	csq	2.423	2.425	2.43	2466.8 ± 2.4
$\frac{1}{2}^+$	Ξ_c^S	csq	2.467	2.530	2.50	–
$\frac{3}{2}^+$	Ξ_c^*	csq	2.543	2.624	2.60	–

TABLE II continued

1	2	3	4	5	6	7
$\frac{1}{2}^+$	Λ_b	bqq	5.629	5.555	5.64	5641 ± 50
$\frac{1}{2}^+$	Σ_b	bqq	5.725	5.735	5.78	-
$\frac{3}{2}^+$	Σ_b^*	bqq	5.752	5.912	5.82	-
$\frac{1}{2}^+$	Ω_b	bss	5.962	6.022	6.01	-
$\frac{3}{2}^+$	Ω_b^*	bss	5.987	6.051	6.05	-
$\frac{1}{2}^+$	Ξ_b^A	bsq	5.767	5.736	5.79	-
$\frac{1}{2}^+$	Ξ_b^S	bsq	5.844	5.88	5.90	-
$\frac{3}{2}^+$	Ξ_b^*	bsq	5.870	5.912	5.94	-

Our effective model predicts the masses of the heavy mesons within a few MeV. For baryons the agreement is less good, but this seems to be a common feature of all bag models.

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Appendix

In this appendix we present our fits to the functions A, $\tilde{A}, \tilde{\tilde{A}}$ from formula (10) and for the function D, $\tilde{D}$ from formula (11). For all the cases considered we find $\tilde{\tilde{D}} \simeq 0$.

For interactions of a heavy quark Q with a light quark q of mass $0 \leq m \leq 0.35$ GeV:

$$A_{Qq}(m) = 0.19843 + 0.36997m - 0.0081159m^2 - 0.25011m^3 + 0.075525m^4$$

$$\tilde{A}_{Qq}(m) = 0.19564 - 0.53335m - 0.0016301m^2 + 0.68698m^3 - 0.31299m^4$$

$$\tilde{\tilde{A}}_{Qq}(m) = 0.0000942172m - 0.0000596538m^2 - 0.000419322m^3 + 0.000627383m^4$$

$$D_{Qq}(m) = -0.0064657 - 0.0019086m - 0.014457m^2 - 0.01691m^3 + 0.048572m^4$$

$$\tilde{D}_{Qq}(m) = 0.12184 - 0.0017827m + 0.020143m^2 + 0.038468m^3 - 0.12118m^4$$

For the interaction of two light quarks of equal mass $0 \leq m \leq 0.35$ GeV:

$$A_{qq}(m) = 0.083535 + 0.18985m + 0.044029m^2 - 0.090646m^3 - 0.045491m^4$$

$$\tilde{A}_{qq}(m) = 0.082362 - 0.27378m - 0.10119m^2 + 0.25305m^3 + 0.069553m^4$$

$$\tilde{\tilde{A}}_{qq}(m) = 0.0000480589m + 0.000028494m^2 - 0.000332181m^3 + 0.000370684m^4$$

$$D_{qq}(m) = 0.017705 - 0.033594m + 0.0039342m^2 - 0.012688m^3 + 0.051793m^4$$

$$\tilde{D}_{qq}(m) = 0.017457 + 0.048573m - 0.0086966m^2 + 0.033452m^3 - 0.13921m^4$$

For the interaction of a massles quark with a light quark with mass $0 \leq m \leq 0.35$ GeV:

$$A_{sq}(m) = 0.083534 + 0.094899m - 0.020313m^2 - 0.065949m^3 + 0.041707m^4$$

$$\tilde{A}_{sq}(m) = 0.082361 - 0.13684m + 0.03841m^2 + 0.17974m^3 - 0.137m^4$$

$$\tilde{\tilde{A}}_{sq}(m) = 0.0000242047m - 0.0000347732m^2 - 0.0000576727m^3 + 0.00012225m^4$$

$$D_{sq}(m) = 0.017705 - 0.016812m - 0.0032713m^2 - 0.0097438m^3 + 0.028741m^4$$

$$\tilde{D}_{sq}(m) = 0.017456 + 0.024333m + 0.0063569m^2 + 0.026645m^3 - 0.0793m^4$$

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