# INTERNALLY RELATIVISTIC FERMION–BOSON PAIR WITH SLOWLY MOVING CENTRE OF MASS*

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The relativistic two-body wave equation proposed previously for a pair of one spin-1/2 particle and one spin-0 or spin-1 particle of equal masses is approximately specified to the case of slowly moving centre of mass of the pair (affected by external forces). A simple wave equation is found in the instance of a spin-0 constituent.

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Several years ago a new relativistic two-body wave equation was found [1] for a pair of one spin-$^1/_2$ particle and one spin-0 or spin-1 particle which, if isolated from each other, were described by the Dirac equation and the Duffin–Kemmer–Petiau equation, respectively. This can be written in the form

$$\Big\{\beta^0\Big[i\frac{\partial}{\partial t}-V(\vec{r}_{\rm D},\vec{r}_{\rm DKP},t)-\vec{\alpha}\cdot\Big(\vec{p}_{\rm D}-e_{\rm D}\vec{A}(\vec{r}_{\rm D},t)\Big)-\beta m_{\rm D}\Big] -\vec{\beta}\cdot\Big(\vec{p}_{\rm DKP}-e_{\rm DKP}\vec{A}(\vec{r}_{\rm DKP},t)\Big)-m_{\rm DKP}\Big\}\psi(\vec{r}_{\rm D},\vec{r}_{\rm DKP},t)=0\,, \tag{1}$$

where the Duffin–Kemmer–Petiau algebra for the DKP-constituent may be represented as

$$\beta^\mu=\tfrac{1}{2}(\gamma_1^\mu+\gamma_2^\mu)\,, \tag{2}$$

with $(\gamma_i^\mu)=(\beta_i,\,\beta_i\vec{\alpha}_i)$ being two commuting sets of Dirac matrices [which commute also with the set $(\gamma^\mu)=(\beta,\,\beta\vec{\alpha})$ of Dirac matrices for the D--constituent]. Here, an external Abelian gauge field $A^\mu(x)$ and an external

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and internal vector-like interaction $V(\vec{r}_{\rm D}, \vec{r}_{\rm DKP}, t)$ [including among others $e_{\rm D} A^0(\vec{r}_{\rm D}, t) + e_{\rm DKP} A^0(\vec{r}_{\rm DKP}, t)$ ] are considered.

In the case of a spin-0 DKP-constituent, the wave equation (1) can be exactly reduced [1] to the form

$$\left\{ \left[ i\frac{\partial}{\partial t} - V(\vec{r}_{\rm D}, \vec{r}_{\rm KG}, t) - \vec{\alpha} \cdot \left( \vec{p}_{\rm D} - e_{\rm D} \vec{A}(\vec{r}_{\rm D}, t) \right) - \beta m_{\rm D} \right]^2 - \left( \vec{p}_{\rm KG} - e_{\rm KG} \vec{A}(\vec{r}_{\rm KG}, t) \right)^2 - m_{\rm KG}^2 \right\} \psi(\vec{r}_{\rm D}, \vec{r}_{\rm KG}, t) = 0 \,, \tag{3}$$

where the DKP-constituent becomes a KG-constituent which, if isolated from the D-constituent, is described by the Klein–Gordon equation.

The wave equation (3) was applied to a model of up and down quarks, each composed of a flavored spin-$^1/_2$ constituent and a colored spin-0 constituent (of equal masses) bound together by a new Abelian gauge field ("ultraelectromagnetic field") [2, 3]. Such composite quarks, though expected to be neutral with regard to a new Abelian charge ("ultraelectric charge"), should display new magnetic-type moments ("ultramagnetic moments") coupled to the magnetic-type part of the Abelian field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ ("ultramagnetic field"). Because of the Abelian character of $F_{\mu\nu}$ the corresponding magnetic-type effects should also appear on the level of nucleons [2] built up of three composite quarks. However, the magnitude of new magnetic-type moments for nucleons was estimated [3] to be very tiny, on the ground of lack of its influence on hfs in $H_2$ molecules [4].

In our model of nucleon [3], three centres of mass of three composite constituent quarks $q$ move comparatively slowly within the nucleon $qqq$. In the present paper, in view of further applications, we give an improved nonrelativistic approximation for the motion of the pair centre of mass in the wave equation (3), taking this time Eq. (1) as the basic wave equation. Having in mind the composite quarks, we will consider two specific cases: *(i)* $A^\mu(x)$ is the ultraelectromagnetic field coupled to the ultraelectric charges $e_{\rm D} \equiv e$ and $e_{\rm DKP} \equiv -e$ (then $e_{\rm D} + e_{\rm DKP} = 0$), or *(ii)* $A^\mu(x)$ is the electromagnetic field coupled to the electric charges $e_{\rm D} \equiv (1,0)e$ and $e_{\rm DKP} \equiv -(^1/_3)e$ [then $e_{\rm D} + e_{\rm DKP} = (^2/_3, -^1/_3)e$].

Our starting point will be the Eq. (1) with equal masses :$m_{\rm D} = m_{\rm DKP} \equiv m$. Introducing the centre-of-mass and relative canonical variables,

$$\vec{r}_{\rm D} = \vec{R} + \tfrac{1}{2}\vec{r}, \quad \vec{r}_{\rm DKP} = \vec{R} - \tfrac{1}{2}\vec{r} \tag{4}$$

and

$$\vec{p}_{\rm D} = \tfrac{1}{2}\vec{P} + \vec{p}, \quad \vec{p}_{\rm DKP} = \tfrac{1}{2}\vec{P} - \vec{p}, \tag{5}$$

we can rewrite Eq. (1) in the case of (*i*) or (*ii*) in the form

$$\Big\{\beta^0 i\frac{\partial}{\partial t} - V(\vec{R},\vec{r},t) - \tfrac{1}{2}(\beta^0\vec{\alpha}+\vec{\beta})\cdot\left[\vec{P} - e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R},t)\right]$$
$$-(\beta^0\vec{\alpha}-\vec{\beta})\cdot\left[\vec{p}-e\vec{A}(\vec{R},t)\right] - (\beta^0\beta+1)m\Big\}\psi(\vec{R},\vec{r},t) = 0 \tag{6}$$

or

$$\Big\{\beta^0 i\frac{\partial}{\partial t} - V(\vec{R},\vec{r},t) - \tfrac{1}{2}(\beta^0\vec{\alpha}+\vec{\beta})\cdot\Big[\vec{P} - \begin{pmatrix}\frac{2}{3}\\ -\frac{1}{3}\end{pmatrix} e\vec{A}(\vec{R},t)$$
$$- \begin{pmatrix}\frac{2}{3}\\ \frac{1}{6}\end{pmatrix} e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R},t)\Big]$$
$$-(\beta^0\vec{\alpha}-\vec{\beta})\cdot\left[\vec{p} - \begin{pmatrix}\frac{2}{3}\\ \frac{1}{6}\end{pmatrix} e\vec{A}(\vec{R},t) - \begin{pmatrix}\frac{1}{6}\\ -\frac{1}{12}\end{pmatrix} e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R},t)\right]$$
$$-(\beta^0\beta+1)m\Big\}\psi(\vec{R},\vec{r},t) = 0\,, \tag{7}$$

respectively. Here, unless $\vec{A}(\vec{R}\pm{}^1\!/_2\vec{r},t)$ is already linear in $\vec{R}\pm{}^1\!/_2\vec{r}$ , we use the linear approximation in $\vec{r}$ :

$$\vec{A}(\vec{R}\pm\tfrac{1}{2}\vec{r},t) \simeq \vec{A}(\vec{R},t) \pm \tfrac{1}{2}\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R},t)\,, \tag{8}$$

justified for close internal motion corresponding to tight bound states of our pair of a D- and a DKP-constituent.

Note that Eq. (6) or (7) implies that

$$\beta^0\dot{\vec{R}} = \tfrac{1}{2}(\beta^0\vec{\alpha}+\vec{\beta})\,,\quad \beta^0\dot{\vec{r}} = \beta^0\vec{\alpha}-\vec{\beta}\,, \tag{9}$$

thus

$$\beta^0\dot{\vec{r}}_{\rm D} = \beta^0\vec{\alpha}\,,\quad \beta^0\dot{\vec{r}}_{\rm DKP} = \vec{\beta}\,, \tag{10}$$

through Eq. (4). Hence, for nonrelativistic motion of the pair centre of mass we can put approximately in Eq. (6)

$$\tfrac{1}{2}(\beta^0\vec{\alpha}+\vec{\beta})\cdot\left[\vec{P}-e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R},t)\right] + \tfrac{1}{2}(\beta^0\beta+1)M_{\rm eff}$$
$$\overset{\rm NR}{\simeq} \beta^0\frac{1}{2M_{\rm eff}}\left[\vec{P}-e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R},t)\right]^2 + \beta^0 M_{\rm eff} \tag{11}$$

or in Eq. (7)

$$
\begin{aligned}
&\tfrac{1}{2}(\beta^0\vec{\alpha}+\vec{\beta})\cdot\left[\vec{P}-\begin{pmatrix}\frac{2}{3}\\ -\frac{1}{3}\end{pmatrix}e\vec{A}(\vec{R},t)-\begin{pmatrix}\frac{2}{3}\\ \frac{1}{6}\end{pmatrix}e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R},t)\right]\\
&\quad+\tfrac{1}{2}(\beta^0\beta+1)M_{\rm eff}\\
&\quad\overset{\rm NR}{\simeq}\beta^0\frac{1}{2M_{\rm eff}}\left\{\left[\vec{P}-\begin{pmatrix}\frac{2}{3}\\ -\frac{1}{3}\end{pmatrix}e\vec{A}(\vec{R},t)-\begin{pmatrix}\frac{2}{3}\\ \frac{1}{6}\end{pmatrix}e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R},t)\right]^2\right.\\
&\quad\left.-\begin{pmatrix}\frac{2}{3}\\ -\frac{1}{3}\end{pmatrix}e\vec{\sigma}\cdot\vec{B}(\vec{R},t)\right\}+\beta^0M_{\rm eff}\,, \qquad (12)
\end{aligned}
$$

where $2m\to M_{\rm eff}$, the constant $M_{\rm eff}$ being an effective mass of our pair. Here, $\vec{B}={\rm rot}\vec{A}$ [such a spin term as in Eq. (12) and *not any other* holds in the case of a spin-0 DKP-constituent *i.e.*, a KG-constituent, provided a possible inhomogeneity of $\vec{B}$ is neglected]. The formula (11) or (12) follows from the fact that [in view of Eq. (9)] its left-hand side is the relativistic kinetic energy of the pair centre of mass (multiplied by $\beta^0$ ), so it must be approximately equal to the nonrelativistic expression for this kinetic energy (multiplied by $\beta^0$). The right-hand side of Eq. (11) or (12) gives consistently

$$
\beta^0\dot{\vec{R}}\overset{\rm NR}{\simeq}\beta^0\frac{1}{M_{\rm eff}}\left[\vec{P}-e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R},t)\right], \qquad (13)
$$

or

$$
\beta^0\dot{\vec{R}}\overset{\rm NR}{\simeq}\beta^0\frac{1}{M_{\rm eff}}\left[\vec{P}-\begin{pmatrix}\frac{2}{3}\\ -\frac{1}{3}\end{pmatrix}e\vec{A}(\vec{R},t)-\begin{pmatrix}\frac{2}{3}\\ \frac{1}{6}\end{pmatrix}e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R},t)\right], \qquad (14)
$$

respectively.

In the case of a spin-0 DKP-constituent (*i.e.* a KG-constituent), the wave equation (6) or (7) — with the nonrelativistic approximation (11) or (12) applied — can be exacly reduced [1] to the following form analogical to Eq. (3):

$$
\begin{aligned}
&\left(\left\{i\frac{\partial}{\partial t}-V(\vec{R},\vec{r},t)-M_{\rm eff}-\frac{1}{2M_{\rm eff}}\left[\vec{P}-e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R},t)\right]^2\right.\right.\\
&\qquad\left.-\vec{\alpha}\cdot\left[\vec{p}-e\vec{A}(\vec{R},t)\right]-\beta m_{\rm eff}\right\}^2\\
&\qquad\left.-\left[\vec{p}-e\vec{A}(\vec{R},t)\right]^2-m_{\rm eff}^2\right)\psi(\vec{R},\vec{r},t)=0 \qquad (15)
\end{aligned}
$$

or

$$\begin{aligned}\Bigg(\Bigg\{ & i\frac{\partial}{\partial t} - V(\vec{R},\vec{r},t) - M_{\text{eff}} \\ & - \frac{1}{2M_{\text{eff}}}\left[\vec{P} - \begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3}\end{pmatrix} e\vec{A}(\vec{R},t) - \begin{pmatrix} \frac{2}{3} \\ \frac{1}{6}\end{pmatrix} e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R},t)\right]^2 \\ & + ((\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3}\end{pmatrix} \frac{1}{2M_{\text{eff}}} e\vec{\sigma}\cdot\vec{B}(\vec{R},t) \\ & - \vec{\alpha}\cdot\left[\vec{p} - \begin{pmatrix} \frac{2}{3} \\ \frac{1}{6}\end{pmatrix} e\vec{A}(\vec{R},t) - \begin{pmatrix} \frac{1}{6} \\ -\frac{1}{12}\end{pmatrix} e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R},t)\right] - \beta m_{\text{eff}}\Bigg\}^2 \\ & - \left[\vec{p} - \begin{pmatrix} \frac{2}{3} \\ \frac{1}{6}\end{pmatrix} e\vec{A}(\vec{R},t) - \begin{pmatrix} \frac{1}{6} \\ -\frac{1}{12}\end{pmatrix} e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R},t)\right]^2 \\ & - m_{\text{eff}}^2\Bigg)\psi(\vec{R},\vec{r},t) = 0\,, \end{aligned} \tag{16}$$

respectively. Here, $m_{\text{eff}} = m - {}^1\!/_2 M_{\text{eff}}$.

Note further that we can identically write in Eq. (15)

$$\left\{\vec{\alpha}\cdot\left[\vec{p} - e\vec{A}(\vec{R},t)\right] + \beta m_{\text{eff}}\right\}^2 = \left[\vec{p} - e\vec{A}(\vec{R},t)\right]^2 + m_{\text{eff}}^2\,, \tag{17}$$

or in Eq. (16)

$$\begin{aligned} & \left\{\vec{\alpha}\cdot\left[\vec{p} - \begin{pmatrix} \frac{2}{3} \\ \frac{1}{6}\end{pmatrix} e\vec{A}(\vec{R},t) - \begin{pmatrix} \frac{1}{6} \\ -\frac{1}{12}\end{pmatrix} e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R},t)\right] + \beta m_{\text{eff}}\right\}^2 \\ & = \left[\vec{p} - \begin{pmatrix} \frac{2}{3} \\ \frac{1}{6}\end{pmatrix} e\vec{A}(\vec{R},t) - \begin{pmatrix} \frac{1}{6} \\ -\frac{1}{12}\end{pmatrix} e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R},t)\right]^2 \\ & + m_{\text{eff}}^2 - \begin{pmatrix} \frac{1}{6} \\ -\frac{1}{12}\end{pmatrix} e\vec{\sigma}\cdot\vec{B}(\vec{R},t)\,. \end{aligned} \tag{18}$$

Then, after some calculations, we can transform Eq. (15) for instance, into the following form (in the stationary case):

$$\begin{aligned}\Bigg(\Bigg\{ & E_{\text{eff}} - V - \frac{1}{2M_{\text{eff}}}\left[\vec{P} - e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}\right]^2\Bigg\}^2 \\ & - 2\left\{E_{\text{eff}} - V - \frac{1}{2M_{\text{eff}}}\left[\vec{P} - e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}\right]^2\right\}\left[\vec{\alpha}\cdot\left(\vec{p} - e\vec{A}\right) + \beta m_{\text{eff}}\right] \\ & - \left[\vec{\alpha}\cdot(\vec{p} - e\vec{A})\,,\, E_{\text{eff}} - V - \frac{1}{2M_{\text{eff}}}\left[\vec{P} - e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}\right]^2\right]\Bigg)\psi = 0\,, \end{aligned} \tag{19}$$

where $E_{\rm eff} = E - M_{\rm eff}$ and a commutator appears. Dividing Eq. (19) by the square root $W = \left\{E_{\rm eff} - V - \left[\vec{P} - e(\vec{r}\cdot\partial/\partial\vec{R})\vec{A}\right]^2/2M_{\rm eff}\right\}^{1/2}$, we obtain

$$\left\{W^2 - 2\left[\vec{\alpha}\cdot\left(\vec{p} - e\vec{A}\right) + \beta m_{\rm eff}\right] + I\right\}W\psi = 0\,. \tag{20}$$

Here, the double commutator

$$I = \left[\left[\vec{\alpha}\cdot(\vec{p} - e\vec{A})\,, W\right], W^{-1}\right], \tag{21}$$

involving $\vec{p} = -i(\partial/\partial\vec{r})$ vanishes approximately, if the consistent nonrelativistic approximation

$$W = \left\{E_{\rm eff} - V - \frac{1}{2M_{\rm eff}}\left[\vec{P} - e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}\right]^2\right\}^{1/2} \overset{\rm NR}{\simeq} \left(E_{\rm eff} - V\right)^{1/2}, \tag{22}$$

is used [ $V(\vec{R},\vec{r})$ includes a very strong pair internal interaction $V(\vec{r})$ dominating over the kinetic energy of the pair centre of mass].

In such a way, Eq. (15) or (16) transits approximately into the following wave equation (in the stationary case) with slowly moving centre of mass and relativistic internal motion:

$$\begin{aligned}&\left\{E_{\rm eff} - V(\vec{R},\vec{r}) - \frac{1}{2M_{\rm eff}}\left[\vec{P} - e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R})\right]^2\right.\\ &\left.-2\vec{\alpha}\cdot\left[\vec{p} - e\vec{A}(\vec{R})\right] - 2\beta m_{\rm eff}\right\}\sqrt{E_{\rm eff} - V(\vec{R},\vec{r})}\,\psi(\vec{R},\vec{r}) = 0\end{aligned} \tag{23}$$

or

$$\begin{aligned}&\left\{E_{\rm eff} - V(\vec{R},\vec{r}) - \frac{1}{2M_{\rm eff}}\left[\vec{P} - \begin{pmatrix}\frac{2}{3}\\ -\frac{1}{3}\end{pmatrix} e\vec{A}(\vec{R}) - \begin{pmatrix}\frac{2}{3}\\ \frac{1}{6}\end{pmatrix} e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R})\right]^2\right.\\ &+\begin{pmatrix}\frac{2}{3}\\ -\frac{1}{3}\end{pmatrix}\frac{1}{2M_{\rm eff}}e\,\vec{\sigma}\cdot\vec{B}(\vec{R}) - 2\vec{\alpha}\cdot\left[\vec{p} - \begin{pmatrix}\frac{2}{3}\\ \frac{1}{6}\end{pmatrix} e\vec{A}(\vec{R}) - \begin{pmatrix}\frac{1}{6}\\ -\frac{1}{12}\end{pmatrix} e\left(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\right)\vec{A}(\vec{R})\right]\\ &\left.-2\beta m_{\rm eff} - \begin{pmatrix}\frac{1}{6}\\ -\frac{1}{12}\end{pmatrix}\frac{1}{E_{\rm eff} - V(\vec{R},\vec{r})}e\vec{\sigma}\cdot\vec{B}(\vec{R})\right\}\sqrt{E_{\rm eff} - V(\vec{R},\vec{r})}\,\psi(\vec{R},\vec{r}) = 0\,,\end{aligned} \tag{24}$$

respectively. Here, $\vec{A}(\vec{R}) = {}^1\!/_2(\vec{B} \times \vec{R})$ and $(\vec{r} \cdot \partial\vec{A}/\partial\vec{R})(\vec{R}) = {}^1\!/_2(\vec{B} \times \vec{r})$ if $\vec{B} = \overrightarrow{\text{const}}$. For the wave function $\psi(\vec{R}, \vec{r})$, the scalar product (if existing) is defined by the weighted integral

$$\int d^3\vec{R} d^3\vec{r} \psi^+(\vec{R}, \vec{r}) \left[ E_{\text{eff}} - V(\vec{R}, \vec{r}) \right] \psi(\vec{R}, \vec{r}) , \tag{25}$$

since $\psi(\vec{R}, \vec{r})$ ought to be normalized as a Klein–Gordon wave function due to the KG-constituent.

In the case when both the ultraelectromagnetic interaction [Eq. (23)] and electromagnetic interaction [Eq. (24)] are active for our pair of a spin-${}^1\!/_2$particle and a spin-0 particle, these interactions must be taken together [then, in Eq. (23) $e \to e^{\text{ultra}}$ and $\vec{A} \to \vec{A}^{\text{ultra}}$, and also a part of $V(\vec{R}, \vec{r})$ must get the label "ultra"].

Of the equations (23) and (24), the first one (referring to the ultraelectromagnetic interaction) is particularly simple. If $\vec{B} = \overrightarrow{\text{const}}$, it assumes the form

$$\left\{ E_{\text{eff}} - V(\vec{R}, \vec{r}) - \frac{1}{2M_{\text{eff}}} \left( \vec{P} - e\frac{\vec{B} \times \vec{r}}{2} \right)^2 \right.$$
$$\left. -2\left[ \vec{\alpha} \cdot \left( \vec{p} - e\frac{\vec{B} \times \vec{R}}{2} \right) + \beta m_{\text{eff}} \right] \right\} \sqrt{E_{\text{eff}} - V(\vec{R}, \vec{r})}\psi(\vec{R}, \vec{r}) = 0 \,. \tag{26}$$

In the case of $V(\vec{R}, \vec{r}) = V^{\text{ex}}(\vec{R}) + V(\vec{r})$, the only interaction in Eq. (26) between the internal motion and the slow external motion is caused by the external $\vec{B} = \overrightarrow{\text{const}}$ coupled to $\vec{R} \times \vec{\alpha}$ and $\vec{r} \times \vec{P}$ .

The significant absence of any coupling between $\vec{\sigma}$ and $\vec{B}$ in Eq. (23) is a consequence of ultracharge neutrality of our pair $(e_{\text{D}} + e_{\text{KG}} = 0)$ and the nonrelativistic approximation used for the motion of the pair centre of mass [Eq. (11)].

In order to introduce to Eq. (23) a coupling between the spin $\vec{\sigma}$ and the external $\vec{B} = \text{rot}\vec{A}$ one must take into account the next (third) term $+{}^1\!/_8(\vec{r} \cdot \partial/\partial\vec{R})^2 \vec{A}(\vec{R}, t)$ in the expansion (8) of $\vec{A}(\vec{R} \pm {}^1\!/_2\vec{r}, t)$ in powers of $\vec{r}$. Then, Eq. (17) goes over into

$$\left\{ \vec{\alpha} \cdot \left[ \vec{p} - e\vec{A}(\vec{R}, t) - \tfrac{1}{8}e\left( \vec{r} \cdot \frac{\partial}{\partial\vec{R}} \right)^2 \vec{A}(\vec{R}, t) \right] + \beta m_{\text{eff}} \right\}^2$$
$$= \left[ \vec{p} - e\vec{A}(\vec{R}, t) - \tfrac{1}{8}e\left( \vec{r} \cdot \frac{\partial}{\partial\vec{R}} \right)^2 \vec{A}(\vec{R}, t) \right]^2$$
$$+ m_{\text{eff}}^2 - \tfrac{1}{8}e\left( \vec{r} \cdot \frac{\partial}{\partial\vec{R}} \right) \left[ \vec{\sigma} \cdot \vec{B}(\vec{R}, t) \right] , \tag{27}$$

leading approximately (after analogical calculations as before) to the modified equation (23):

$$\Big\{E_{\rm eff} - V(\vec{R},\vec{r}) - \frac{1}{2M_{\rm eff}}\Big[\vec{P} - e\Big(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\Big)\vec{A}(\vec{R})\Big]^2$$
$$- 2\vec{\alpha}\cdot\Big[\vec{p} - e\vec{A}(\vec{R}) - \frac{1}{8}e\Big(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\Big)^2\vec{A}(\vec{R})\Big] - 2\beta m_{\rm eff}$$
$$- \frac{1}{8}\frac{1}{E_{\rm eff} - V(\vec{R},\vec{r})}e\Big(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\Big)\Big[\vec{\sigma}\cdot\vec{B}(\vec{R})\Big]\Big\}\sqrt{E_{\rm eff} - V(\vec{R},\vec{r})}\psi(\vec{R},\vec{r}) = 0\,. \tag{28}$$

Note that the $\vec{\alpha}$-term here (when collaborating with the $\beta$-term) contains also a spin interaction with $\vec{B}$ as can be seen *via* its Pauli nonrelativistic approximation unveiling in Eq. (28) the term

$$+\tfrac{1}{4}\frac{1}{2m_{\rm eff}}e\Big(\vec{r}\cdot\frac{\partial}{\partial\vec{R}}\Big)\Big[\vec{\sigma}\cdot\vec{B}(\vec{R})\Big] \tag{29}$$

(this coupling gets in the hamiltonian the opposite sign). Thus, the coupling between $\vec{\sigma}$ and $\vec{B}$ may manifest itself in Eq. (28) only for an inhomogeneous field $\vec{B}$ and vanishes when the pair extension $\langle r\rangle \to 0$.

Concluding, the hypothetical ultramagnetic moments of our composite quarks and, consequently, those of nucleons (built up of the composite quarks) may be observed only in inhomogeneous ultramagnetic fields and disappear in the point-like limit of $\langle r\rangle \to 0$ [at any rate, on the ground of Eq. (28) involving our nonrelativistic approximation (11) for the motion of the quark centre of mass].

Finally, we would like to emphasize that our model of composite quarks requires necessarily relativistic internal dynamics, since in the case of a nonrelativistic one the quark magnetic coupling between $\vec{\sigma}$ and $\vec{B}$ in such a model is $-(e_{\rm D}/2m)\vec{\sigma}\cdot\vec{B}$ with $e_{\rm D} = (1,0)e$ and $e_{\rm KG} = -{}^1\!/_3\, e$.

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