# ARE THERE NEW HADRONIC STATES INVOLVING HYPOTHETICAL COLORED SCALAR CONSTITUENTS ?*

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First, we recollect the model of algebraically composite Dirac-type particles, leading to three families of leptons and quarks as well as to two hypothetical families of Yukawa scalars. Then, we briefly discuss new hadrons involving colored quark-like Yukawa scalars as their constituents. When denoting such scalars by $y$, these (colorless) hadrons are: $(y\bar{y})$, $(q\bar{y})$ (both with $B = 0$, $L = 0$) and $(yyy)$, $(qyy)$, $(qqy)$ (all three with $B = 1$, $L = 0$). The lowest-in-mass versions of them have the $J^P(I)$-signatures: $0^+(0,1)$, ${}^1\!/_2{}^+(0,1)$ and $0^-({}^3\!/_2)$, ${}^1\!/_2{}^+({}^1\!/_2)$, $0^+({}^1\!/_2)$ [or $1^+({}^1\!/_2, {}^3\!/_2)$], respectively. Among them, $(q\bar{y})$ may be the only states stable in strong interactions (for adequate masses).

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## 1. Introduction

Recently, we developed a model of three families of leptons and quarks [1] and two families of Yukawa bosons [2], all being solutions to Dirac-type equations based on new composite representations of Dirac algebra (subject to a generalized exclusion principle). This gave an explanation of the puzzling phenomenon of three fundamental-fermion families and, at the same time, a prediction of two Yukawa-boson families. Our argument went as follows.

First of all, we can observe that the sequence $N = 1, 2, 3, \ldots$ of Clifford algebras

$$\left\{\gamma_i^\mu, \gamma_j^\nu\right\} = 2\delta_{ij} g^{\mu\nu} \quad (i, j = 1, 2, \ldots, N), \tag{1}$$

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defines the sequence $N = 1, 2, 3, \ldots$ of composite representations

$$\Gamma^\mu = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \gamma_i^\mu \,, \tag{2}$$

for the Dirac algebra

$$\{\Gamma^\mu, \Gamma^\nu\} = 2g^{\mu\nu} \tag{3}$$

and, consequently, the sequence $N = 1, 2, 3, \ldots$ of Dirac-type equations

$$[\Gamma \cdot (p - gA) - M]\,\psi = 0 \,. \tag{4}$$

Here, $A_\mu(x)$ symbolize the standard-model gauge fields including SU(3) $\otimes$ SU$_L$(2) $\otimes$ U(1) coupling matrices, $\lambda$'s, $\tau$'s, $Y$ and $\Gamma^5 = i\Gamma^0\Gamma^1\Gamma^2\Gamma^3$, and so implying the familiar 16 standard–model states for any $\psi(x)$ in the sequence $N = 1, 2, 3, \ldots$ of solutions to Eqs (4).

The matrix $\Gamma_1^\mu \equiv \Gamma^\mu$ together with $N-1$ other Jacobi-type independent combinations of $\gamma_i^\mu$, *viz.*

$$\Gamma_2^\mu = \frac{1}{\sqrt{2}}\left(\gamma_1^\mu - \gamma_2^\mu\right) \,, \; \Gamma_3^\mu = \frac{1}{\sqrt{6}}\left(\gamma_1^\mu + \gamma_2^\mu - 2\gamma_3^\mu\right) \,,$$
$$\ldots \,, \; \Gamma_N^\mu = \frac{1}{\sqrt{N(N-1)}}\left[\gamma_1^\mu + \ldots + \gamma_{N-1}^\mu - (N-1)\gamma_N^\mu\right] \,, \tag{5}$$

defines the Clifford algebras

$$\left\{\Gamma_i^\mu, \Gamma_j^\nu\right\} = 2\delta_{ij}g^{\mu\nu} \qquad (i, j = 1, 2, \ldots, N) \tag{6}$$

isomorphic to those in Eq. (1) for any $N = 1, 2, 3, \ldots$. Thus, the composite representations (2) may be realized in the convenient reduced forms

$$\Gamma^\mu \equiv \Gamma_1^\mu = \gamma^\mu \otimes \underbrace{\mathbf{1} \otimes \cdots \otimes \mathbf{1}}_{(N-1)\text{ times}} \,, \tag{7}$$

with $\gamma^\mu$ and $\mathbf{1}$ being the usual $4 \times 4$ Dirac matrices (in the case of $N = 2$, for instance, one can write $\Gamma_1^\mu = \gamma^\mu \otimes \mathbf{1}$, $\Gamma_2^\mu = \gamma^5 \otimes i\gamma^5\gamma^\mu$ with $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$; when multiplied $\otimes\mathbf{1}$ they give $\Gamma_1^\mu$ and $\Gamma_2^\mu$ for the case $N = 3$ and then $\Gamma_3^\mu = \gamma^5 \otimes \gamma^5 \otimes \gamma^\mu$). In the representation (7), the Dirac-type equation (4) for any $N$ reads

$$[\gamma \cdot (p - g\,A) - M]_{\alpha_1\beta_1}\, \psi_{\beta_1\alpha_2\ldots\alpha_N} = 0 \,, \tag{8}$$

where $M_{\alpha_1\beta_1} = M\delta_{\alpha_1\beta_1}$. For $N = 1$ it is, of course, the usual Dirac equation, while for $N = 2$ it is known as the Dirac form [3] of the Kähler equation [4]. For $N = 3, 4, 5, \ldots$ we get new Dirac-type equations. Here, the wave function or the field $\psi(x) = \left(\psi_{\alpha_1\alpha_2\ldots\alpha_N}(x)\right)$ carries $N$ Dirac bispinor indices $\alpha_i = 1, 2, 3, 4$ $(i = 1, 2, \ldots, N)$ of which only $\alpha_1$ is affected by the standard-model gauge fields (in particular, the electromagnetic field), whereas the rest of them, $\alpha_2, \ldots, \alpha_N$, are free. So, only $\alpha_1$ may be "visible" in these fields, while $\alpha_2, \ldots, \alpha_N$ are "hidden".

The Dirac-type equation (4) lead to the conserved, fully-relativistic Dirac-type currents

$$j_D^\mu = \eta_N \psi^+ \Gamma_1^0 \Gamma_2^0 \ldots \Gamma_N^0 \Gamma_1^\mu \psi\,, \tag{9}$$

but *only for N odd* (here, the phase factors $\eta_N$ make the products $\eta_N \Gamma_2^0 \ldots \Gamma_N^0$ Hermitian, in particular, $\eta_1 = 1$, $\eta_3 = i$, $\eta_5 = i^2$). If $A_\mu$ do not include $\Gamma_1^5 \equiv \Gamma^5$, what may be the case *only for N even* (corresponding to integer total spins), there certainly exist the conserved, fully relativistic Klein–Gordon-type currents

$$j_{KG}^\mu = \eta_N \psi^+ \Gamma_1^0 \Gamma_2^0 \ldots \Gamma_N^0 \left(i \overleftrightarrow{\partial^\mu} - g A^\mu\right) \psi \tag{10}$$

with $\overleftrightarrow{\partial^\mu} = 1/2 \left(\partial^\mu - \overleftarrow{\partial^\mu}\right)$ (here, $\eta_N$ make $\eta_N \Gamma_1^0 \Gamma_2^0 \ldots \Gamma_N^0$ Hermitian, in particular, $\eta_2 = i$, $\eta_4 = i^2$). It is so, as then Eqs (4) imply the second-order equations of the form

$$\left\{(p - gA)^2 - M^2 - i\, g \tfrac{1}{4} \left[\Gamma_1^\mu, \Gamma_1^\nu\right] F_{\mu\nu}\right\} \psi = 0\,, \tag{11}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i\, g\, [A_\mu, A_\nu]$.

In the case of $N$ odd, the Dirac-type density $j_D^0 = \eta_N \psi^+ \Gamma_2^0 \ldots \Gamma_N^0 \psi$ should be always positive-definite. This imposes on $\psi(x)$ the constraint

$$\eta_N \Gamma_2^0 \ldots \Gamma_N^0 \psi = \psi\,, \tag{12}$$

which is consistent with the Dirac-type equation (4) since for $N$ odd both $j_D^\mu$ and $\psi^+ \Gamma_1^0 \Gamma_1^\mu \psi$ are conserved currents [the latter is a fully relativistic vector solely under the constraint (12)]. The constraint (12) in turn, not being in general fully relativistic, becomes such (effectively) if $\psi$ is a superposition of a *scalar and pseudoscalar only*, with respect to the *hidden* bispinor indices $\alpha_2, \ldots, \alpha_N$ (as carrying also the visible bispinor index $\alpha_1$, they are bispinors).

In the case of $N$ even, if the conserved current $j_{KG}^\mu$ exists [in the absence of fermion sources on the right-hand side of Eq. (11)], the Klein–Gordon-

-type density $j^0_{KG} = \eta_N \psi^+ \Gamma^0_1 \Gamma^0_2 \ldots \Gamma^0_N \left( i \overleftrightarrow{\partial}^0 - gA^0 \right) \psi$ should be positive-definite for positive-energy modes. This imposes on $\psi(x)$ the constraint

$$\eta_N \Gamma^0_1 \Gamma^0_2 \ldots \Gamma^0_N \psi = \psi \,, \tag{13}$$

what is consistent with the second-order equation (11) if for $N$ even both $j^\mu_{KG}$ and $\psi^+ \left( i \overleftrightarrow{\partial}^\mu - gA^\mu \right) \psi$ are conserved currents [the latter being a fully relativistic vector solely under the constraint (13)]. The constraint (13), on the other hand, becomes fully relativistic (effectively) if $\psi$ is a superposition of *a scalar and a pseudoscalar only*, with respect to the *visible and hidden* bispinor indices $\alpha_1, \alpha_2, \ldots, \alpha_N$. Then, $\psi^+ \left[\Gamma^0_1, \Gamma^\nu_1\right] \psi = 0$ in Eq. (11), implying indeed the conservation of $\psi^+ \left( i \overleftrightarrow{\partial}^\mu - gA^\mu \right) \psi$.

Note that in the case of $N = 3$ and $N = 2$, for instance, one can write in Eqs (12) and (13) $\eta_3 \Gamma_2 \Gamma_3 = 1 \otimes \gamma^0 \otimes \gamma^0$ and $\eta_2 \Gamma_1 \Gamma_2 = \gamma^5 \gamma^0 \otimes \gamma^5 \gamma^0$, respectively [*cf.* the comment made just after Eq. (7)], where in the chiral representation $\gamma^5 = \mathrm{diag}(\mathbf{1}^P, -\mathbf{1}^P)$ and $\gamma^0 = \mathrm{antidiag}(\mathbf{1}^P, \mathbf{1}^P)$ with Pauli unit matrix $\mathbf{1}^P$.

Treating the particle's hidden bispinor indices $\alpha_2, \ldots, \alpha_N$ as undistinguishable degrees of freedom obeying the Fermi statistics along with the Pauli exclusion principle (then $\psi_{\alpha_1, \alpha_2, \ldots, \alpha_N}$ are fully antisymmetric with respect to $\alpha_2, \ldots, \alpha_N$), we can conclude that the sequence $N = 1, 2, 3, \ldots$ of Dirac-type equations (4) must *terminate at* $N = 5$.

## 2. Three fermion families versus two hypothetical boson families

Thus, in the case of $N$ odd, there are only three Dirac-type equations (4) corresponding to $N = 1, 3, 5$. As was shown in Ref. [1], the constraint (12), when required to be fully relativistic (effectively), implies that for each of these three $N = 1, 3, 5$ there exists one and only one Dirac-type particle which in this case carries spin $1/2$ [for any of 16 standard-model signatures: $(1+3\mathrm{colors}) \times 2(\mathrm{up/down}) \times 2(L/R) = 16\mathrm{states}$]. These particles are described by three visible bispinors :

$$\psi^{(1)}_{\alpha_1} \equiv \psi_{\alpha_1} \,, \tag{14}$$

$$\psi^{(3)}_{\alpha_1} \equiv \tfrac{1}{4} \left( C^{-1} \gamma^5 \right)_{\alpha_2 \alpha_3} \psi_{\alpha_1 \alpha_2 \alpha_3} = \psi_{\alpha_1 12} = \psi_{\alpha_1 34} \,, \tag{15}$$

$$\psi^{(5)}_{\alpha_1} \equiv \tfrac{1}{24} \varepsilon_{\alpha_2 \alpha_3 \alpha_4 \alpha_5} \psi_{\alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5} = \psi_{\alpha_1 1234} \,, \tag{16}$$

where $C$ is the usual charge-conjugation matrix, while the bispinor indices are defined in the chiral representation with $\gamma^5 = \mathrm{diag}(\mathbf{1}^P, -\mathbf{1}^P) =$

diag(1, 1, −1, −1). Then, one may write $C^{-1} = \mathrm{diag}(i\sigma_2^P, -i\sigma_2^P)$ in terms of Pauli matrix $\sigma_2^P$. Here, in the case of $N = 3$ the hidden pseudoscalar $(C^{-1})_{\alpha_2\alpha_3}\psi_{\alpha_1\alpha_2\alpha_3} \equiv 0$ due to the constraint (12) with $\eta_3 = i$ (the roles of $C^{-1}\gamma^5$ and $C^{-1}$ are interchanged if $\eta_3 = -i$), while in the case of $N = 5$ it is obvious that only the hidden scalar (16) exists: it fixes the phase factor $\eta_5 = i^2$ in the constraint (12) which then is satisfied identically.

These three states were interpreted in Ref. [1] as three replicas of a fundamental fermion (for any of 16 standard-model signatures), responsible for the phenomenon of three families of leptons and quarks. Note that the states (14), (15), (16) all correspond to the positive eigenvalue of total hidden chirality $\Gamma_2^5 \ldots \Gamma_N^5$.

The Reader may compare the last Ref. [1] for a semiempirical, numerical discussion of lepton and quark masses. There, the value $m_\tau = 1776.80$ MeV based on the input of experimental $m_e$ and $m_\mu$ is predicted.

In the case of $N$ even, there is a room only for two Dirac-type equations (4) corresponding to $N = 2, 4$. As was discussed in Ref. [2], they give two visible-hidden double bispinors :

$$\psi^{(2)}_{\alpha_1\alpha_2} \equiv \psi_{\alpha_1\alpha_2}\,, \tag{17}$$

$$\psi^{(4)}_{\alpha_1\alpha_2} \equiv \tfrac{1}{6}\varepsilon_{\beta_2\beta_3\beta_4\alpha_2}\psi_{\alpha_1\beta_2\beta_3\beta_4}\,, \tag{18}$$

where our Pauli exclusion principle does not apply to the pairs of indices $\alpha_1, \alpha_2$ as they are of different sorts: one visible and one hidden. Now, if the conserved current $j^\mu_{KG}$ exists, the constraint (13), when required to be fully relativistic (effectively), causes that for each of these two $N = 2, 4$ there is one and only one Dirac-type particle which in this case has spin 0 [for any of 8 standard-model signatures : (1 + 3colors) × 2(up/down) = 8states]. These particles are described by two visible-hidden scalars:

$$S^{(2)} \equiv \tfrac{1}{4}(C^{-1}\gamma^5)_{\alpha_1\alpha_2}\psi^{(2)}_{\alpha_1\alpha_2} = \tfrac{1}{2}\left(\psi_{12} - \psi_{21}\right) = \tfrac{1}{2}\left(\psi_{34} - \psi_{43}\right)\,, \tag{19}$$

$$S^{(4)} \equiv \tfrac{1}{4}(C^{-1}\gamma^5)_{\alpha_1\alpha_2}\psi^{(4)}_{\alpha_1\alpha_2} = \tfrac{1}{2}\left(\psi_{1341} - \psi_{2432}\right) = \tfrac{1}{2}\left(\psi_{3123} - \psi_{4214}\right)\,. \tag{20}$$

Here, the visible-hidden pseudoscalars $(C^{-1})_{\alpha_1\alpha_2}\psi^{(2,4)}_{\alpha_1\alpha_2} \equiv 0$ due to the constraint (13) with $\eta_2 = i$ and $\eta_4 = i^2$ (in the case of $N = 2$ and/or $N = 4$ the roles of $C^{-1}\gamma^5$ and $C^{-1}$ are interchanged if $\eta_2 = -i$ and/or $\eta_2 = -i^2$, respectively).

These two states may be interpreted as two replicas of a fundamental Yukawa scalar (for any of 8 standard-model signatures), leading to a new phenomenon of two families of Yukawa scalars (including possibly a weak-isospin doublet of Higgs scalars). Note that the states (19) and (20) both correspond to the positive eigenvalue of total visible-hidden chirality $\Gamma_1^5\Gamma_2^5 \ldots \Gamma_N^5$ .

## 3. Hadrons involving colored scalars as their constituents

Each of two hypothetical families $N = 2,4$ of Yukawa scalar bosons consists of two weak-isospin doublets: one colorless lepton-like (with weak hypercharge $Y = -1$) and one colored quark-like (with weak hypercharge $Y = {}^1/_3$). Let us denote these doublets by $x$ and $y$, respectively [or, more precisely, by $x^{(2,4)}$ and $y^{(2,4)}$, where $x^{(2)}$ might be the weak-$G$-conjugate Higgs doublet $h^G = (h^0, -h^-)$]. Their electric charge $Q$ is given by the formula $2Q = 2I_3^{(W)} + Y$, where $\vec{I}^{(W)}$ is the weak isospin. The conjecture that masses of both $x$ and $y$ are high enough seems natural. Evidently, beside colored quarks $q$, the colored quark-like scalars $y$ are coupled to the usual gluons and obey the extended QCD based on the standard color SU(3) symmetry (including now new scalar sector of $y$'s).

In contrast to quarks $q$ appearing in 6 flavors: $2I_3 = -1, 1, St = -1, Ch = 1$, $Bo = -1$ and $To = 1$ (?) in obvious notation, the quark-like scalars $y$, as belonging to two families, can develop *only 4 flavors.* It will be very attractive for us to assume here that these 4 flavors *are identical* with those observed in the case of quarks $q$ belonging to *two first families* $N = 1,3$ *i.e.*, with $2I_3 = -1, 1, St = -1$ and $Ch = 1$. (Then, the third family $N = 5$ of leptons and quarks, as the only one having no counterpart among Yukawa scalars, is responsible for the lack of formal symmetry between fundamental fermions and hypothetical Yukawa scalar bosons.) Thus, under our assumption, the strong isospin $\vec{I}$, the strangeness $St$ and the charm $Ch$ are well-defined and (hopefully) conserved in strong interactions, both for quarks $q$ and quark-like scalars $y$. [The bottomness $Bo$ and the topness $To$ are taken equal to zero for $y$, and so they are also well-defined and (hopefully) conserved in strong interactions for $q$ and $y$.] Therefore, the familiar formula

$$2Q = 2I_3 + St + Ch + Bo + To + B\,, \tag{21}$$

holds in general for the electric charge $Q$ of strongly interacting particles. This defines the baryon number $B$ . Thus, the quark-like scalars $y$ carry baryon number $B = {}^1/_3$. Note that they have lepton number $L = 0$. Also in the case of lepton-like scalars $x$ there is no reason for ascribing them nonzero value of the lepton number $L$ conserved in electroweak interactions.

In analogy to quarks $q$ forming colorless bound states $(q\bar{q})$ as well as $(qqq)$, the colored quark-like scalars $y$ would imply the following new colorless bound states: $(y\bar{y})$ and $(q\bar{y})$ with baryon numbers $B = 0$, as well as $(yyy)$, $(qyy)$ and $(qqy)$ with baryon number $B = 1$. All these states would have lepton number $L = 0$. Because of the Bose statistics of $y$'s and Fermi statistics of $q$'s the lowest-in-mass versions of the above hadrons (when built of $y$ and $q$ with no higher flavors: $St = Ch = Bo = To = 0$) should get the following $J^P(I)$-signatures : $0^+(0,1)$ and ${}^1\!/_2{}^+(0,1)$ (with $B=0, L=0$), as well as $0^-({}^3\!/_2)$, ${}^1\!/_2{}^+({}^1\!/_2)$ and $0^+({}^1\!/_2)$ or $1^+({}^1\!/_2, {}^3\!/_2)$ (with $B=1$, $L=0$), respectively. We can see that here integer and half-integer strong isospin $I$ (not ordinary spin $J$!) would be related to baryon number $B$ equal to 0 and 1, respectively. Note that the realistic, lowest-in-mass versions of two hadrons $(y\bar{y})$ and $(q\bar{y})$ should contain, in general, also some contributions from $y\bar{y}$ and $q\bar{y}$ pairs with neutralized higher flavors, first of all the strangeness (in fact, in a very similar way the familiar $\eta$, $\eta'$ or $\omega$, $\phi$ [5] contain contributions from $s\bar{s}$ due to the approximate flavor SU(3) symmetry). In the case of the so called "ideal mixing" [5] such contributions would be negligible (as for $\omega$, when $\phi = s\bar{s}$).

Among the five new hadrons, only the $0^+(0,1)$-hadron $(y\bar{y})$ (with $B=0, L=0$) and ${}^1\!/_2{}^+({}^1\!/_2)$-hadron $(qyy)$ (with $B=1$, $L=0$) have familiar signatures, identical to those for $f_0$ or $a_0$ mesons [5] and nucleons, respectively. The signatures of three others are unfamiliar as they cannot be realized for purely quark states. Especially interesting are here the ${}^1\!/_2{}^+(0,1)$-hadron $(q\bar{y})$ (with $B=0, L=0$) and $0^+({}^1\!/_2)$-hadron $(qqy)$ (with $B=1$, $L=0$). The $0^-({}^3\!/_2)$-hadron $(yyy)$ (with $B=1$, $L=0$), because of the Bose statistics of $y$'s, requires an antisymmetric spatial part of wave function constructed from three orbital angular momenta 1 (of course, the color part of its wave function is antisymmetric). So, it is orbital-excited *per se*.

The $0^+(0,1)$–hadron $(y\bar{y})$ (with $B=0, L=0$) can decay strongly through the two-gluon annihilation process into $\pi\pi$ or $\pi\eta$ (for example), like $f_0$ or $a_0$ mesons [5].

For convenience, let us denote by $\xi = (\xi^+, \xi^0, \xi^-)$ or $\zeta$ the ${}^1\!/_2{}^+(1)$- or ${}^1\!/_2{}^+(0)$-hadron $(q\bar{y})$ (with $B=0$, $L=0$), respectively. For adequate masses, $\xi$ and $\zeta$ hadrons may be stable in strong interactions, while the hadrons $(qqy)$, $(qyy)$ and $(yyy)$ may decay strongly into $N$ plus $\bar{\xi}$ or $\bar{\zeta}$, $N$ plus a pair of $\bar{\xi}$ or $\bar{\zeta}$ and $N$ plus a triple of $\bar{\xi}$ or $\bar{\zeta}$, respectively (it may happen, however, that for other masses one or more of these three hadrons is also stable in strong interactions). If there were "non-gluon-mediated" strong virtual transitions $qq' \leftrightarrow yy'$ and $q\bar{y}' \leftrightarrow \bar{q}'y$ (allowed by all known selection rules including the baryon-number conservation), the dominating strong decays for these hadrons would be

$$(qqy) \to N\bar{\xi} \text{ or } N\bar{\zeta}\,,\ (qyy) \to N\pi \text{ or } N\eta\,,\ (yyy) \to N\bar{\xi}\,, \tag{22}$$

for adequate masses. Such additional strong virtual transitions, if appearing, might be referred to as "gluino-mediated".

Evidently, when $m_\zeta > m_\xi + m_\pi$ or $m_\xi > m_\zeta + m_\pi$ the strong decays $\zeta \to \xi^\pm\pi^\mp$, $\zeta \to \xi^0\pi^0$ or $\xi^\pm \to \zeta\pi^\pm$, $\xi^0 \to \zeta\pi^0$ should always appear. When, however,

$m_\xi + m_\pi > m_\zeta > m_\xi + m_e + m_{\nu_e}$ or $m_\zeta + m_\pi > m_\xi > m_\zeta + m_e + m_{\nu_e}$, the weak and electromagnetic processes $\zeta \to \xi^+ e^- \bar{\nu}_e$, $\zeta \to \xi^- e^+ \nu_e$, $\zeta \to \xi^0 \gamma$ or $\xi^+ \to \zeta e^+ \nu_e$, $\xi^- \to \zeta e^- \bar{\nu}_e$, $\xi^0 \to \zeta\gamma$ should be dominating.

Thus, in the "minimal case", only the spin-$^1/_2$hadron $\zeta$ (with $B = 0$, $L = 0$, $Q = 0$, $\vec{I} = 0$ and higher flavors $= 0$) may be stable (both in strong and electroweak interactions) among all new hadrons predicted in this paper. Then, a number of fascinating questions arises, first of all, whether $\zeta$ can form bound states with nucleons (through, say, two-pion-exchange forces and one-$\eta$-exchange forces caused by the effective coupling $g_{\zeta\xi\pi}\bar{\zeta}\gamma^5\vec{\xi}\cdot\vec{\pi}$+h.c. and $g_{\zeta\zeta\eta}\bar{\zeta}\gamma^5\zeta\eta$, respectively) and further, whether during the evolution of the universe there were good chances for forming such or similar bound systems involving $\zeta$. One may also ask the related question, whether $\zeta$, if abundant enough in the universe as a free phase (in spite of its annihilation in pairs), can be a candidate for the dark matter. We hope to return to these questions elsewhere. It is worth while to emphasize here that, in contrast to observed stable fermions, $\nu_e, e^-, \nu_\mu, \nu_\tau$ and $p$, our $\zeta$ is an unusual one in carrying $B = 0, L = 0$. So, if there were gluino-mediated virtual transitions, it might pass into a Majorana fermion that could strongly annihilate in particle–particle pairs and also be so produced. (Beside $\zeta$, also $\xi$ fermion has $B = 0, L = 0$, and the same is true for hypothetical supersymmetric partners of observed gauge bosons.) Evidently, $\zeta$ and $\xi$ may be always produced in particle–antiparticle pairs or jointly with the hadrons $(qqy)$ in strong interactions of the usual hadrons. The corresponding strong annihilation processes are allowed at low relative kinetic energies if masses are adequate. Note that, in another option of the "minimal case", only the spin-$^1/_2$ hadron $\xi^0$ may be stable among all new hadrons.

There should also exist higher-in-mass versions of our five types of hadrons with active or neutralized higher flavors $St, Ch, Bo$ and $To$ . Radial and orbital excited states in all flavor versions should appear as well. The Reader may easily realize a full classification of states for five new types of hadrons involving (beside quarks from three families) color quark-like scalars from two hypothetical families of Yukawa scalar bosons.

Finally, a remark concerning notation is due. In this paper which is the first presentation of the new hypothetical hadrons we avoided to give them assignments, except for $\xi = (q\bar{y})$ with $J^P(I) = {^1/_2}^+(1)$ and $\zeta = (q\bar{y})$ with ${^1/_2}^+(0)$ (where $q$ and $y$ carried no higher flavors). However, the notation $\xi_y = (y\bar{y})$ with $J^P(I) = 0^+(1)$ and $\zeta_y = (y\bar{y})$ with $0^+(0)$ for the $B = 0$ states, as well as ${}^1N_y = (qqy)$ with $J^P(I) = 0^+(^1/_2)$, ${}^3N_y = (qqy)$ with $1^+(^1/_2)$, $\Delta_y = (qqy)$ with $1^+(^3/_2)$, $N_{yy} = (qyy)$ with ${^1/_2}^+(^1/_2)$ and $\Delta_{yyy} = (yyy)$ with $0^-(^3/_2)$ for the $B = 1$ states might be practical (here also, $q$ and $y$ carry no higher flavors). The colored quark-like Yukawa scalars $y$ could be called "yukawions". Then, in general, one could speak of yukawions in 4 flavors: down, up, strange and charmed yukawions which, if specified, might be denoted by $y_d, y_u, y_s$ and $y_c$, respectively. In this way, the lowest-in-mass versions of new strange hadrons (for instance) might be denoted as follows:

$$\begin{aligned} K_y &= (y\bar{s}) \quad \text{with} \quad J^P(I) = \tfrac{1}{2}^+ \left(\tfrac{1}{2}\right) , \\ K_{\bar{y}_s} &= (q\bar{y}_s) \quad \text{with} \quad \tfrac{1}{2}^+ \left(\tfrac{1}{2}\right) , \\ K_{y\bar{y}_s} &= (y\bar{y}_s) \quad \text{with} \quad 0^+ \left(\tfrac{1}{2}\right) , \end{aligned}$$

all three being $B = 0$ (colorless) states, and

$$\begin{aligned}
\left.\begin{matrix} {}^1\Lambda_y \\ {}^3\Lambda_y \end{matrix}\right\} &= (qys) \quad \text{with} \quad J^P(I) = \left\{\begin{matrix} 0^+(0) \\ 1^+(0) \end{matrix}\right. , \\
\left.\begin{matrix} {}^1\Sigma_y \\ {}^3\Sigma_y \end{matrix}\right\} &= (qys) \quad \text{with} \quad \left\{\begin{matrix} 0^+(1) \\ 1^+(1) \end{matrix}\right. , \\
\Lambda_{yy} &= (yys) \quad \text{with} \quad \tfrac{1}{2}^+(0) , \\
\Lambda_{y_s} &= (qqy_s) \quad \text{with} \quad 0^+(0) , \\
\Sigma_{y_s} &= (qqy_s) \quad \text{with} \quad 1^+(1) , \\
\Lambda_{yy_s} &= (qyy_s) \quad \text{with} \quad \tfrac{1}{2}^+(0) , \\
\Sigma_{yy_s} &= (qyy_s) \quad \text{with} \quad \tfrac{1}{2}^+(1) , \\
\Lambda_{yyy_s} &= (yyy_s) \quad \text{with} \quad 0^+(0) , \\
\Xi_y &= (yss) \quad \text{with} \quad 1^+ \left(\tfrac{1}{2}\right) , \\
\left.\begin{matrix} {}^1\Xi_{y_s} \\ {}^3\Xi_{y_s} \end{matrix}\right\} &= (qy_s s) \quad \text{with} \quad \left\{\begin{matrix} 0^+ \left(\tfrac{1}{2}\right) \\ 1^+ \left(\tfrac{1}{2}\right) \end{matrix}\right. , \\
\Xi_{yy_s} &= (yy_s s) \quad \text{with} \quad \tfrac{1}{2}^+ \left(\tfrac{1}{2}\right) , \\
\left.\begin{matrix} {}^2\Xi_{y_s y_s} \\ {}^4\Xi_{y_s y_s} \end{matrix}\right\} &= (qy_s y_s) \quad \text{with} \quad \left\{\begin{matrix} \tfrac{1}{2}^+ \left(\tfrac{1}{2}\right) \\ \tfrac{3}{2}^+ \left(\tfrac{1}{2}\right) \end{matrix}\right. , \\
\Omega_{y_s} &= (y_s ss) \quad \text{with} \quad 1^+(0) , \\
\left.\begin{matrix} {}^2\Omega_{y_s y_s} \\ {}^4\Omega_{y_s y_s} \end{matrix}\right\} &= (y_s y_s s) \quad \text{with} \quad \left\{\begin{matrix} \tfrac{1}{2}^+(0) \\ \tfrac{3}{2}^+(0) \end{matrix}\right. , \\
\Omega_{y_s y_s y_s} &= (y_s y_s y_s) \quad \text{with} \quad 0^-(0) ,
\end{aligned}$$

all describing $B = 1$ (colorless) states. Here, ${}^{2,4}\Xi_{y_s y_s}, {}^{2,4}\Omega_{y_s y_s}$ and $\Omega_{y_s y_s y_s}$ require two, two and three orbital angular momenta 1, respectively, due to the Bose statistics of $y_s$'s [the last of these *per se* orbital-excited states is somewhat analogous to $\Delta_{yyy} = (yyy)$ with $J^P(I) = 0^-\left({}^3\!/_2\right)$ ]. Similarly, $\phi_{y_s} = (y_s\bar{s})$ with $J^P(I) = {}^1\!/_2{}^+(0)$ is a $B = 0$ (colorless) state with neutralized strangeness. In the above states $q$ and $y$ carry no higher flavors, so they stand for $u$ or $d$ and $y_u$ or $y_d$, respectively. Note that $\xi$ might be symbolized also by $\pi_{\bar{y}} = (q\bar{y})$ with $J^P(I) = 0^-(1)$, and then $\xi_y$ — by $\pi_{y\bar{y}} = (y\bar{y})$ with $0^+(1)$.

Eventually, we should remark that, while the existence of three fundamental-fermion families is an unavoidable consequence of the Dirac-type equations (4) (subject to the extended exclusion principle), the appearance of two Yukawa-boson families is conditioned by there being the conserved Klein–Gordon-type current (10) [in the absence of fermion sources on the right-hand side of the second-order equation (11)]. Certainly, this is the case for $N$ even, if then $\Gamma_1^5 \equiv \Gamma^5$ is absent from the standard-model coupling in the Dirac-type equations (4) (another case, where such a current exists, is considered in the second Ref. [2]).

## REFERENCES

[1] W. Królikowski, *Acta Phys. Pol.* **B21**, 871 (1990); *Phys. Rev.* **D45**, 3222 (1992); *Acta Phys. Pol.* **B23**, 933 (1992); *Acta Phys. Pol.* **B23**, 1245 (1992).
[2] W. Królikowski, *Acta Phys. Pol.* **B23**, 785 (1992); *Acta Phys. Pol.* **B23**, 861 (1992); *Phys. Rev.* **D46**, 5188 (1992).
[3] T. Banks, Y. Dothan, D. Horn, *Phys. Lett.* **B117**, 413 (1982).
[4] E. Kähler, *Rendiconti di Matematica* **21**, 425 (1962); *cf.* also D. Ivanenko, L. Landau, *Z. Phys.* **48**, 341 (1928).
[5] Particle Data Group, Review of Particle Properties, *Phys. Rev.* **D45**, Part 2 (June 1992).